ELEMENTARY ENGINEERING DRAWING

[PLANE AND SOLID GEOMETRY]

By the same author

MACHINE DRAWING

GEOMETRICAL AND MACHINE DRAWING

[FOR DIPLOMA STUDENTS]

ELEMENTARY ENGINEERING DRAWING

[PLANE AND SOLID GEOMETRY]

by

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SEVENTH REVISED AND ENLARGED EDITION

[WITH MORE THAN 580 DIAGRAMS AND NUMEROUS EXERCISES]

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Dedicated

to

My students,

past and present

FOREWORD

It gives me pleasure to introduce this text-book on Engineering Drawing by Prof. N. D. Bhatt of the Birla Vishvakarma Mahavidyalaya to students of engineering. Prof. Bhatt has been teaching this subject for over twenty years and has deservedly earned the reputation of being one of the best teachers in the subject.

This book covers the syllabus usually prescribed for the Pre-Engineering and First Year of the Degree and Diploma courses in Engineering and deals with the fundamental principles of this basic subject which have been treated by Prof. Bhatt with his characteristic lucidity. I am sure the book will prove a boon to students and help them to acquire a sound knowledge of the subject without which a really satisfactory progress cannot be achieved in any branch of Engineering.

POONA
June 6, 1958

S. B. Junnarkar,
M.B.E., B.A., B.Sc. Hons. (Eng.), London

PREFACE

From the very early days, man realized that if he had to construct any structure or machine correctly and methodically, he must first record his ideas before starting construction work. These recorded ideas become more vivid and forceful if they are shown on paper in form of a drawing of the structure or machine. Such a drawing will be of very great help to the man who looks after the construction of this structure or machine.

Indeed, '*technical drawing is the language of engineering*'. Without the good knowledge of drawing, an engineer is nowhere and he could not have constructed the various magnificent structures or intricate machines. Evidently, any one, connected in any way, with engineering construction must understand this language of engineers. Technical drawing is, therefore, indispensable to-day and shall continue to be so as long as engineering and technology continue to be of use in the activities of man.

By means of drawing, the shape, size, finish, colour and construction of any object (no matter how complex) can be described accurately and clearly. The engineer should develop his skill, in two phases of technical drawing; first, he must be able to draw clearly and rapidly, the freehand technical sketches; secondly, he must be proficient in drawing to scale the instrumental drawing. The purpose of the present volume is to give the basic principles of the instrumental drawing only.

The book covers the syllabii in drawing of many University Colleges and Polytechnics in India and has been written keeping in view the difficulties of a beginner in the subject of Engineering Drawing. I am quite hopeful that this book will serve its purpose very well for young and budding engineers.

I am highly indebted to Principal S. B. Junnarkar for his valuable guidance and for his kindness to write a suitable foreword for the book. I am also thankful to Prof. V. B. Priyani of Birla Vishvakarma Mahavidyalaya for going through the initial manuscript and for offering constructive suggestions. Finally, I feel grateful to the following:

(i) The authorities of the Universities of Bombay, Poona and Gujarat, and the Department of Technical Education, Bombay, for their kind permission to include a few questions set at their examinations. (ii) Mr. N. M. Panchal and Mr. M. D. Bhatt for their help in preparing pencil sketches. (iii) Mr.

L. D. Bhatt for preparing the excellent typed manuscript. (iv) Mr. Ramanbhai C. Patel of Charotar Book Stall for careful proof-reading and for his efforts to see the book out in proper time. (v) The Anand Press authorities for the care and interest shown in the printing and get-up of this book. (vi) The Prabhat Process Studio for the promptness and good work of block-making.

Any suggestion to improve the value of this book will be gratefully received and will be incorporated in subsequent editions of this book after due scrutiny.

June 30, 1958

N. D. Bhatt

PREFACE TO THE SIXTH EDITION

Suggestions for inclusion of chapters on Perspective Projection and Conversion of Pictorial Views into Orthographic Views were received from many quarters. These two chapters have been incorporated in this edition. Construction of Comparative scales, Vernier scales and Scale of chords has also been added in the chapter on Scales.

The author is indebted to Shri P. A. Soni, B. Arch., Lecturer in Civil Engineering, B. V. Mahavidyalaya, for his constructive and valuable suggestions on the chapter on Perspective Projection.

The book was completely converted from British system to Metric system of length measurement in its fifth edition. The same has been continued. Approximate values of centimetres in inches have been given side by side, but within brackets. 'Inches to millimetres' conversion table is given at the end of the book for exact conversion.

March 3, 1965

N. D. B.

PREFACE TO THE SEVENTH EDITION

In this edition, third-angle projection has been introduced along with first-angle projection.

Construction of two more curves, viz. Evolute and Logarithmic spiral has been added in the chapter on 'Curves used in engineering practice.'

"Ālekhan"
Vallabh Vidyanagar
August 7, 1966

N. D. B.

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ELEMENTARY ENGINEERING DRAWING

CHAPTER 1

DRAWING INSTRUMENTS AND THEIR USES

Drawing instruments are used to prepare drawings easily and accurately. The accuracy of the drawings depends largely on the quality of instruments. With instruments of good quality, desirable accuracy can be attained with ease. It is, therefore, essential to procure instruments of as superior quality as possible.

Below is the minimum list of drawing instruments and other drawing materials which every student must possess:

1. Drawing board
2. T-square
3. Set-squares — 45° and 30°-60°
4. Instrument box, containing:
   - (i) Large-size compass with inter-changeable pencil and pen legs
   - (ii) Large-size divider
   - (iii) Small bow pencil
   - (iv) Small bow pen
   - (v) Small bow divider
   - (vi) Lengthening bar
   - (vii) Inking pen
5. Scale
6. Protractor
7. Drawing paper
8. Pencils
9. Rubber eraser
10. Drawing pins
11. Sand-paper block
12. Duster.

Drawing board (fig. 1-1): Drawing board is rectangular in shape and is made of strips of well-seasoned soft wood about 2·5 cm (1″) thick. It is cleated at the back by two battens to prevent warping. One of the edges of the board is used as the *working edge*, on which the T-square is made to slide. It should, therefore, be perfectly straight. In some boards, this edge is grooved throughout its length and a perfectly straight ebony edge is fitted inside this groove. This provides a true and more durable guide for the T-square to slide on.

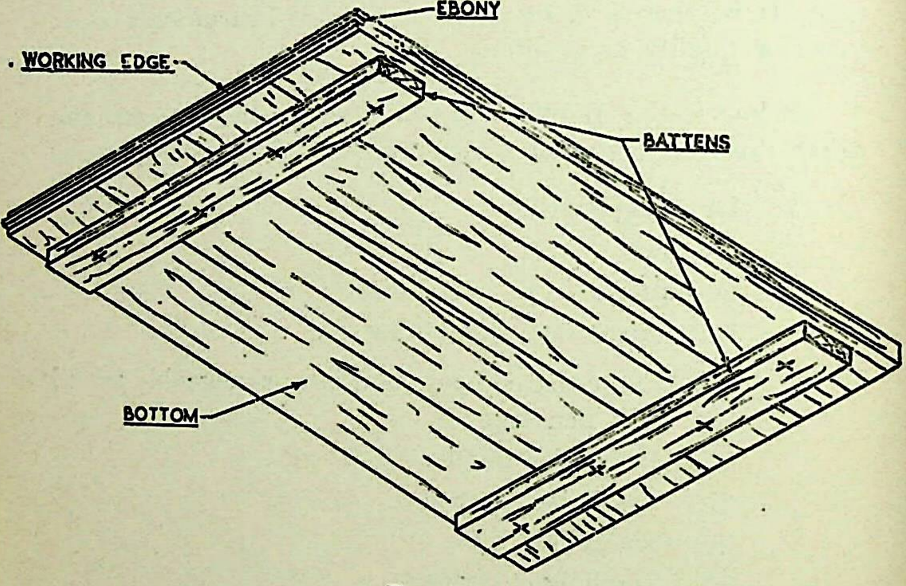


FIG. 1-1

Drawing board is made in various sizes. Its selection depends upon the size of the drawing paper to be used.

The kinds and measurements of the drawing boards generally used are given below:

Kind	Measurement
1. Half-imperial size	60 cm × 40 cm (23″ × 16″)
2. Imperial size	80 cm × 60 cm (31″ × 23″)
3. Double-elephant size	105 cm × 75 cm (42″ × 29″)
4. Antiquarian size	140 cm × 85 cm (54″ × 33″)

For use in schools and colleges, the first two sizes are more convenient. Large-size boards are used in drawing offices of engineers and engineering firms.

The drawing board is placed on the table in front of the student, with its working edge on his left side. It is more convenient if the table-top is sloping downwards towards the student. If such a table is not available, the necessary slope can be obtained by placing a suitable block of wood under the distant longer edge of the board.

T-square (fig. 1-2): The T-square should be of hard-quality wood. It consists of two parts — the stock and the blade — joined together at right angles to each other by means of screws and pins. The stock is placed adjoining

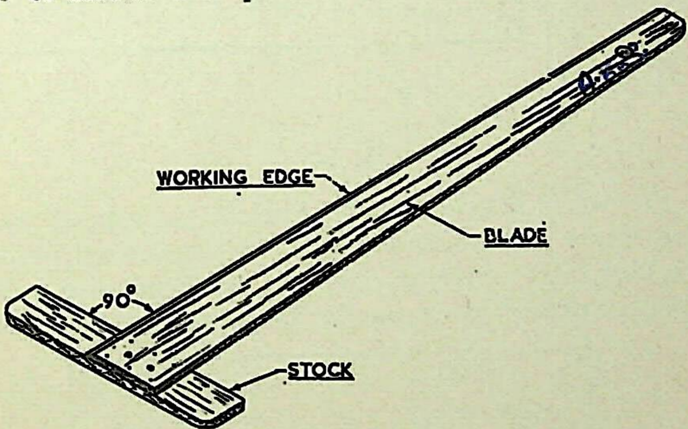


FIG. 1-2

the working edge of the board and is made to slide on it as and when required. The blade lies on the surface of the board. Its distant edge which is generally bevelled, is used as the working edge and hence, it should be perfectly straight. The nearer edge of the blade is never used. The length of the blade is selected so as to suit the size of the drawing board.

The T-square is used for drawing horizontal lines. The stock of the T-square is held firmly with the left hand against the working edge of the board, and the line is drawn from left to right as shown in fig. 1-3. The pencil should

be held slightly inclined in the direction of the line (i.e., to the right) while the pencil point should be as close as possi-

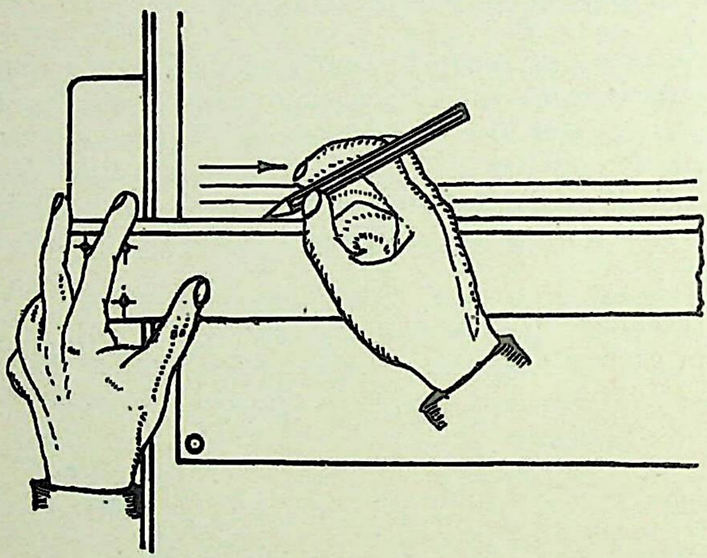


FIG. 1-3

ble to the working edge of the blade. Horizontal parallel lines are drawn by sliding the stock to the desired positions. The working edge of the T-square is also used as a base for set-squares to draw vertical, inclined or mutually parallel lines.

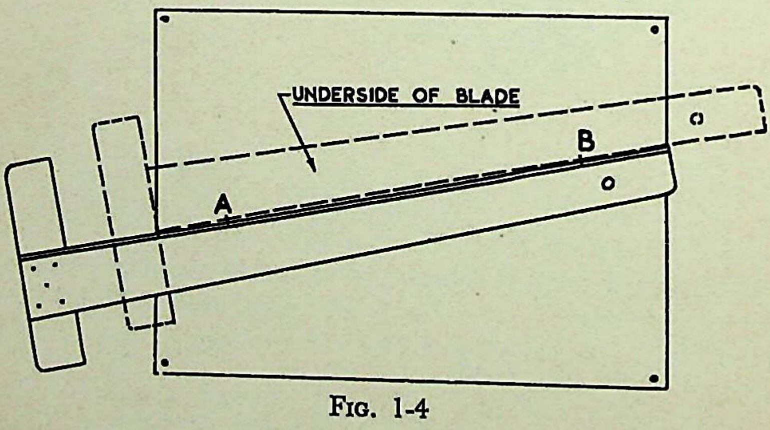


FIG. 1-4

The T-square should never be used on edge other than the working edge of the board. It should always be kept on the board even when not in use.

Testing the straightness of the working edge of the T-square:

Mark any two points *A* and *B* (fig. 1-4) spaced wide apart and through them, carefully draw a line with the working edge. Turn the T-square upside down as shown by dashed lines and with the same edge, draw another line passing through the same two points. If the edge is defective the lines will not coincide. The error should be rectified by planing or sand-papering the defective edge.

Set-squares: The set-squares are made of wood, tin, celluloid or plastic. Those made of transparent celluloid or plastic are commonly used as they retain their shape and accuracy for longer time. Two forms of set-squares are in general use. They are triangular in shape with one corner in each, a right angle. The 30°-60° set-square of 25 cm (10″) length and 45° set-square of 20 cm (8″) length are convenient sizes for use in schools and colleges (fig. 1-5).

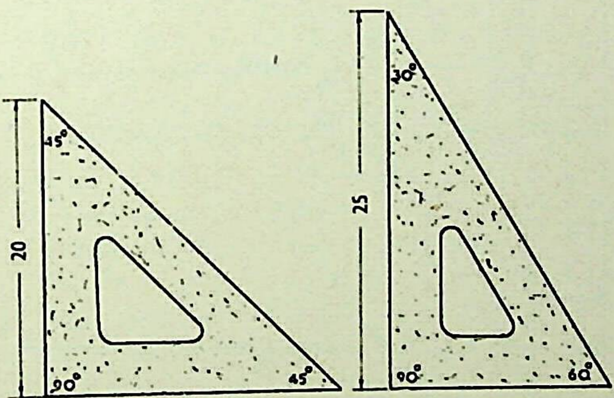


FIG. 1-5

Set-squares are used for drawing all straight lines except the horizontal lines which are usually drawn with the T-square. Vertical lines can be drawn with the T-square and the set-square.

In combination with the T-square, lines at 30° or 60° angle with vertical or horizontal lines can be drawn with 30°-60° set-square and at 45° angle with 45° set-square. The two set-squares used simultaneously along with the T-square will produce lines making angles of 15°, 75°, 105° etc.

Parallel straight lines in any position, not very far apart, as well as lines perpendicular to any line from any given point within or outside it, can also be drawn with the two set-squares.

Problem 1:

To draw a line perpendicular to a given horizontal line from a given point within it.

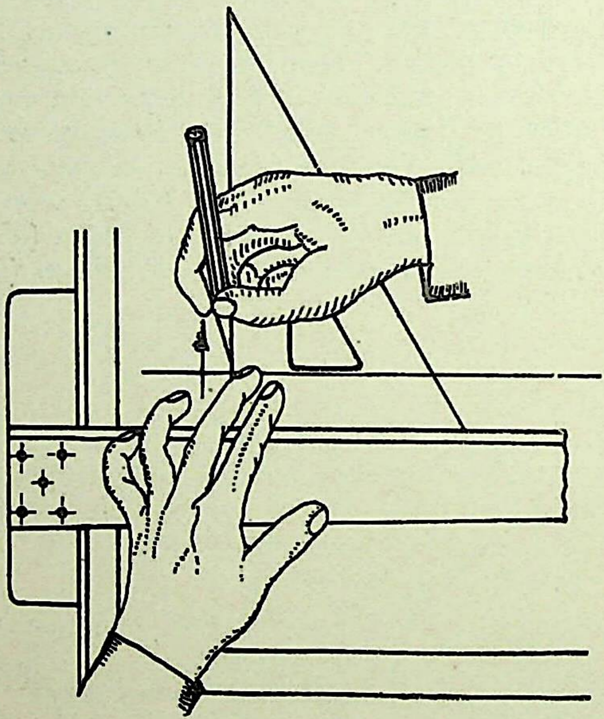


Fig. 1-6

Place the T-square a little below the given line (fig. 1-6). Arrange any one set-square with one of the edges containing the right angle touching the working edge of the T-square, and the other edge passing through the given point. Hold the T-square and the set-square in this position firmly with the left hand. With the right hand, draw the required line through the given point in the upward direction as shown by the arrow. The pencil point should always be in contact with the edge of the set-square. A

perpendicular from any given point outside the line can also be drawn in the same manner. Vertical parallel lines may be drawn by sliding the set-square along the edge of the T-square to the required positions.

Problem 2:

To draw a line inclined at 45°, 30° or 60° to a given horizontal line from a given point.

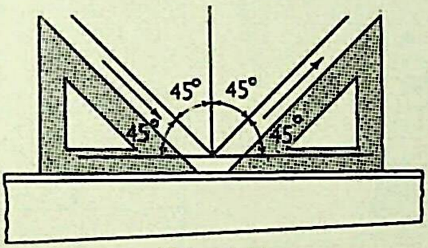


FIG. 1-7

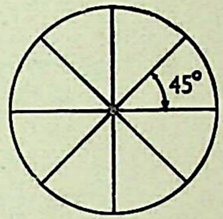


FIG. 1-8

Place the edge containing the right angle of the 45° set-square on the edge of the T-square (fig. 1-7). Slide it so that its longest edge (hypotenuse) passes through the given point and then draw the required line. The same line will make 45° angle with the vertical line passing through that point. By turning the set-square upside down, the line making 45° angle in the other direction will be drawn. The lines can also be drawn by placing the set-square so that its longest edge coincides with the edge of the T-square and the other edge passes through the given point. A circle can similarly be divided into eight equal parts by lines passing through its centre (fig. 1-8).

Lines inclined at 30° or 60° to a given horizontal line

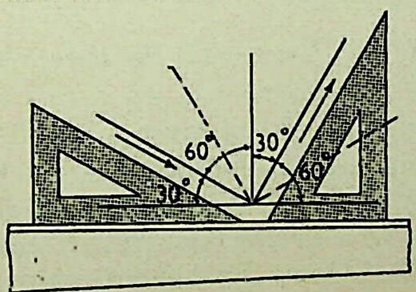


FIG. 1-9

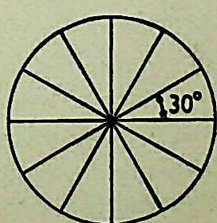


FIG. 1-10

can similarly be drawn with the aid of a 30°-60° set-square (fig. 1-9). A circle may be divided into twelve equal divisions in the same manner (fig. 1-10).

Problem 3:

To draw a line inclined at 15° to a given horizontal line from a given point.

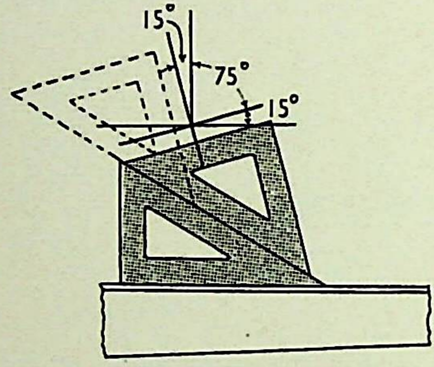


FIG. 1-11

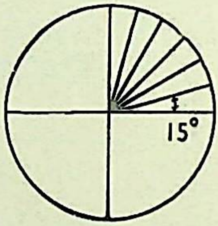


FIG. 1-12

Place the 30°-60° set-square with its longer edge containing the right angle, coinciding with the edge of the T-square (fig. 1-11). Arrange the 45° set-square with its longest edge on the longest edge of the 30°-60° set-square. Slide the 45° set-square so that one of its edges containing the right angle passes through the given point, and draw the required line. The line drawn with the other edge will make 15° angle with the vertical line and 105° or 75° angles with the horizontal line. A circle may thus be divided into 24 equal parts with the aid of the two set-squares (fig. 1-12).

Problem 4:

To draw a line parallel to a given straight line through a given point.

The line *AB* and the point *P* are given (fig. 1-13).

Arrange an edge of a set-square coinciding with *AB*. Place the other set-square as a base for the first. Hold the second set-square firmly and slide the first, till its arranged edge is along the point *P*. Draw the line *CD* through *P*. *CD* is the required parallel line.

By keeping the edge of the T-square as base for the set-square, parallel lines, long distances apart, can be drawn.

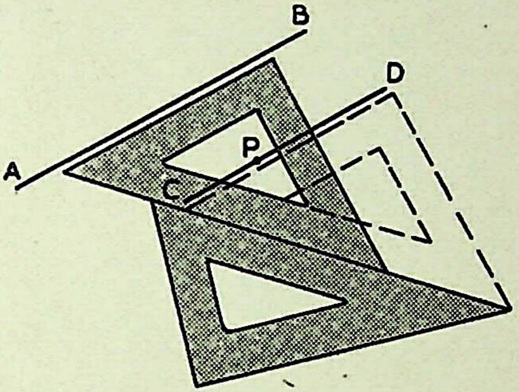


FIG. 1-13

Problem 5:

To draw a line perpendicular to a given line through a point within or outside it.

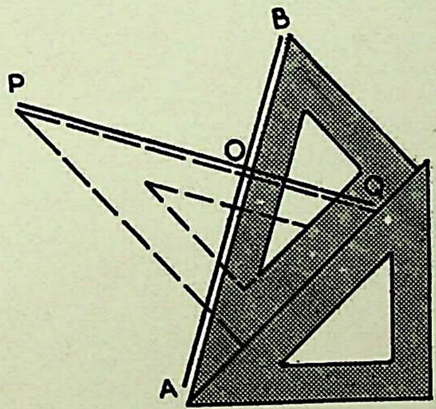


FIG. 1-14

The line *PQ* and the point *O* are given (fig. 1-14).

Method I: Arrange the longest edge of one set-square along *PQ*. Place the second set-square or T-square as base along one of the edges containing the right angle. Holding the base set-square firmly, turn over the first set-square so that its other edge containing the right angle coincides with the edge of the base set-square. Slide the first set-square

till its longest edge is on the point *O* and draw the required line *AB*.

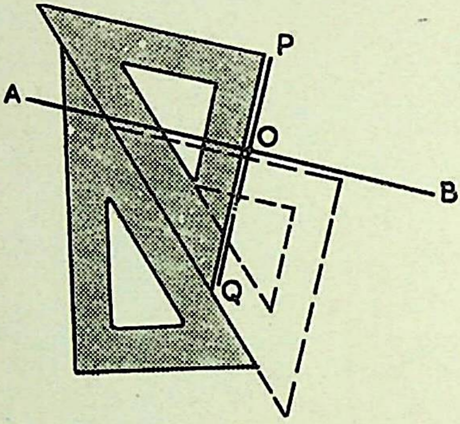


FIG. 1-15

Method II: (fig. 1-15): Arrange one set-square with an edge containing the right angle along the line *PQ*. Place the second set-square or T-square as a base under the longest edge. Slide the first set-square on the second till the other edge containing the right angle is on the point *O* and draw the required line *AB*.

Problem 6:

To draw a line parallel to a given straight line at a given distance, say 1 cm from it (fig. 1-16).

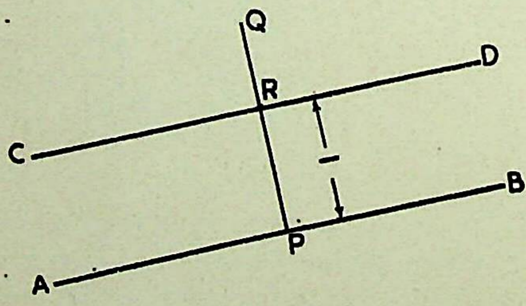


FIG. 1-16

The line *AB* is given. From any point *P* in *AB*, draw a line *PQ* perpendicular to *AB* (Prob. 5). Mark a point *R*

so that $PR = 1$ cm. Through R, draw the required line CD parallel to AB (Prob. 4).

Compass (fig. 1-17): The compass is used for drawing circles and arcs of circles. It consists of two legs hinged together at its upper end. A pointed needle is fitted at the lower end of one leg, while a pencil lead is inserted at the end of the other leg. The lower part of the pencil leg is detachable and it can be interchanged with a similar piece containing an inking pen. Both the legs are provided with knee joints. Circles upto about 12 cm (5″) diameter can be drawn with the legs of the compass kept straight. For drawing larger circles, both the legs should be bent at the knee joints so that they are perpendicular to the surface of the paper (fig. 1-18).

As the needle is required to be inserted slightly inside the paper, it is kept longer than the lead point. The setting of the pencil-lead relative to the needle, and the shape to which the lead should be ground are shown in fig. 1-19.

Fig. 1-17

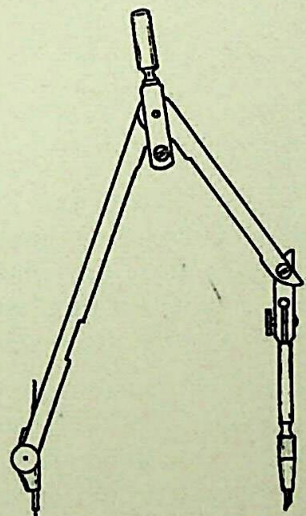


Fig. 1-18

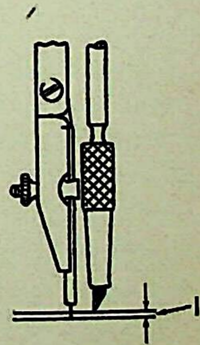


Fig. 1-19

To draw a circle, adjust the opening of the legs of the compass to the required radius. Hold the compass with the thumb and the first two fingers of the right hand and

place the needle point lightly on the centre, with the help of the left hand. Bring the pencil point down on the paper and swing the compass about the needle-leg with a twist of the thumb and the two fingers, in clockwise direction, until the circle is completed. The compass should be kept slightly inclined in the direction of its rotation. While drawing concentric circles, beginning should be made with the smallest circle.

Circles of more than 15 cm (6″) radius are drawn with the aid of the *lengthening bar*. The lower part of the pencil leg is detached and the lengthening bar is inserted in its place. The detached part is then fitted at the end of the lengthening bar, thus increasing the length of the pencil leg (fig. 1-20). For drawing large circles, it is often necessary

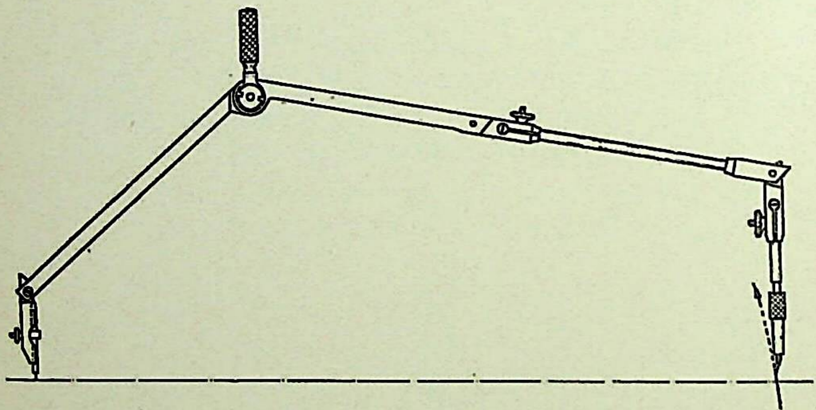


FIG. 1-20

to guide the pencil leg with the other hand. For drawing small circles and arcs of less than 2·5 cm (1″) radius and particularly, when a large number of small circles of the same diameter are to be drawn, small bow compass is used (fig. 1-21).

Curves drawn with the compass should be of the same darkness as that of the straight lines. It is difficult to exert the same amount of pressure on the lead in the compass as on a pencil. It is, therefore, desirable to use slightly softer variety of lead (about one grade lower) in the compass than the pencil used for drawing straight lines, to maintain uniform darkness in all the lines.

Divider: The divider has two legs hinged at the upper end and is provided with steel points at both the lower

ends, but it does not have the knee joints (fig. 1-22). In most of the instrument boxes, a needle attachment is also provided which can be interchanged with the pencil part of the compass, thus converting it into a divider.

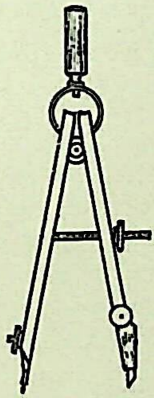


FIG. 1-21

FIG. 1-22

The dividers are used (i) to divide curved or straight lines into desired number of equal parts, (ii) to transfer dimensions from one part of the drawing to another part and (iii) to set-off given distances from the scale to the drawing. They are very convenient for setting off points at equal distances around a given point or along a given line.

Small bow divider is adjusted by a nut and is very convenient for marking minute divisions and large number of short equal distances.

Problem 7:

To divide a straight line into a number of equal parts — say 3.

The straight line *AB* is given (fig. 1-23).

Set the legs of the divider so that the steel points are approximately $\frac{1}{3}$ of the length of the line apart. Step this

distance lightly from one end of the line, say *B*, turning the divider first in one direction and then in the other. If the last division falls short, increase the set distance by approximately $\frac{1}{3}$ of the difference by means of the nut *N*, keeping

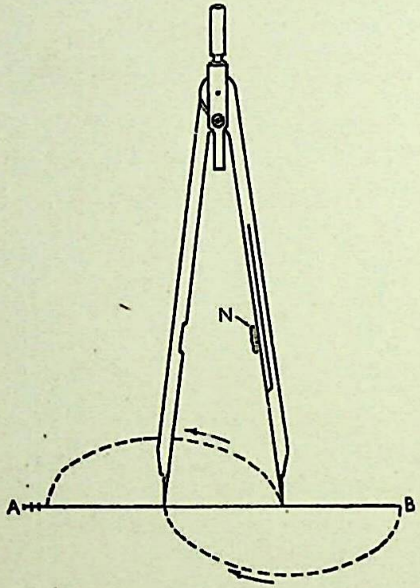


FIG. 1-23

the other point of the divider on the paper. If the last division goes beyond the end of the line, decrease the set distance by $\frac{1}{3}$ of the difference. Re-space the line, beginning from the starting point and, adjusting the divider until the required setting is obtained.

With some practice, it will be possible to obtain the desired result with less trials and in short time. The trial divisions should be set off as lightly as possible so that the paper is not pricked with large and unnecessary holes.

Any arc or a circle can similarly be divided into any number of equal divisions.

Scales: The scales are made of wood, steel, celluloid or plastic. They are usually 30 cm (12″) in length and are either flat or triangular. Flat scales if more than 1·5 mm ($\frac{1}{16}$″) thick are generally bevelled at both the edges which help in marking measurements from the scale to the drawing paper

accurately. Full-size flat scale with $\frac{1}{16}''$ divisions on one edge and $\frac{1}{20}''$ divisions on the other edge (fig. 1-24) or with millimetre divisions on both edges is quite suitable for beginners. Various other types of scales are described in Chapter IV.

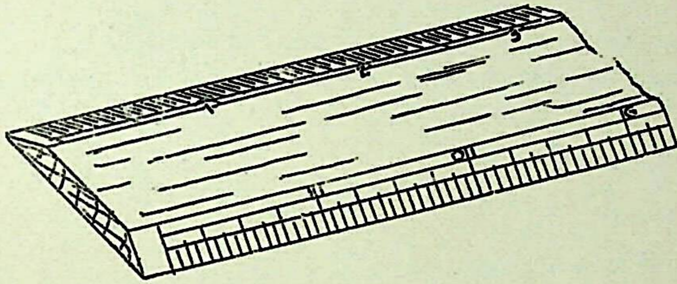


FIG. 1-24

The scale is used to transfer the true or relative dimensions of an object to the drawing. It is placed with its edge on the line on which measurements are to be marked and, looking from exactly above the required division, the marking is done with a fine pencil point. *The scale should never be used as a straight-edge for drawing lines.*

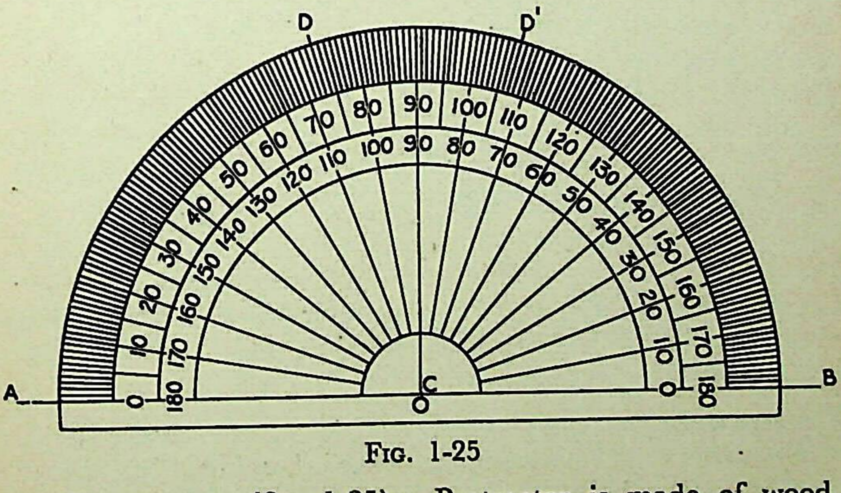


FIG. 1-25

Protractor (fig. 1-25): Protractor is made of wood, tin or celluloid. Protractors of transparent celluloid are in common use. They are flat and circular or semi-circular in shape. The commonest type of protractor is semi-circular,

and of about 10 cm (4″) diameter; its circumferential edge is graduated to 1° divisions, is numbered at every 10° interval and is readable from both the ends. The diameter of the semi-circle (viz. straight line 0-180) is called the base of the protractor and its centre O is marked by a line perpendicular to it.

The protractor is used to draw or measure such angles as cannot be drawn with the set-squares. A circle can be divided into any number of equal parts by means of the protractor.

Problem 8:

To draw a line making an angle of 73° with a given line through a given point in it.

Let AB be the line and C the point in it. Set the protractor with its base coinciding with AB (fig. 1-25) and its centre exactly on the point C. Mark a point D opposite the 73° division and join CD. Then $\angle ACD = 73°$ (fig. 1-26).

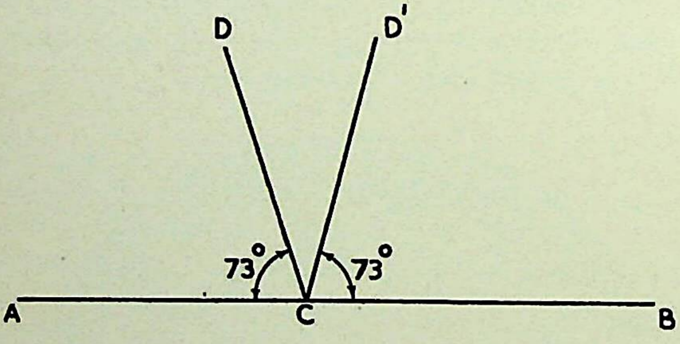


FIG. 1-26

Another point D' can also be marked against the 73° division reading from the other side. In this case $\angle BCD' = 73°$ while $\angle ACD' = 107°$.

Drawing papers: Drawing papers are available in many varieties. For ordinary pencil-drawings, the paper selected should be tough and strong. It should be uniform in thickness and as white as possible. When the rubber eraser is used on it, its fibres should not disintegrate. Good quality of paper with smooth surface should be selected for

drawings which are to be inked and preserved for a long time. It should be such that the ink does not spread. Thin and cheap-quality paper may be used for drawings from which tracings are to be prepared.

Drawing papers are available in bundles (sheets) and in rolls. The rolls are 75 cm (30″) to 180 cm (72″) in width and generally 20 metres (22 yards) in length. The standard sizes of drawing papers and their names are given below:

Kind	Measurement
Half-imperial size	56 cm × 38 cm (22″ × 15″)
Imperial size	76 cm × 56 cm (30″ × 22″)
Double-elephant size	102 cm × 69 cm (40″ × 27″)
Antiquarian size	135 cm × 79 cm (53″ × 31″)

Imperial size and half-imperial size papers are very commonly used in schools and colleges.

Pencils: The accuracy and appearance of a drawing depend very largely on the quality of the pencils used. With cheap and low-quality pencils, it is very difficult to draw lines of uniform shade and thickness. The grade of a pencil lead is usually shown by figures and letters marked at one of its ends. Letters HB denote the medium grade. The increase in hardness is shown by the value of the figure put in front of the letter H, viz. 2H, 3H, 4H etc. Similarly, the grade becomes softer according to the figure placed in front of the letter B, viz. 2B, 3B, 4B etc.

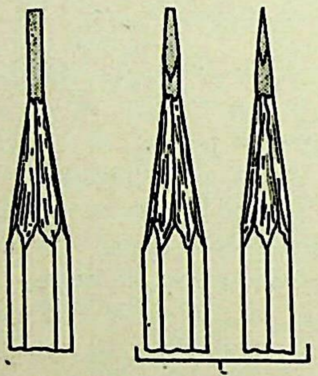
Beginning of a drawing should be made with H or 2H pencil using it very lightly, so that the lines are faint, and unnecessary or extra lines can be easily erased. The final fair work may be done with harder pencils, e.g. 3H and upwards. Lines of uniform thickness and darkness can be more easily drawn with hard-grade pencils.

H and HB pencils are more suitable for lettering and dimensioning. For freehand sketching, where considerable erasing is required to be done, soft-grade pencils such as HB should be used.

Great care should be taken in mending the pencil and sharpening the lead, as the uniformity in thickness of lines

depends largely on this. The lead may be sharpened to two different forms: (i) conical point and (ii) chisel edge. The conical point is used in sketch work and for lettering, etc. With the chisel edge, long thin lines of uniform thickness can be easily drawn and hence, it is suitable for drawing work.

To prepare the pencil lead for drawing work, the wood around the lead from the end, other than that on which the grade is marked, is removed with a pen-knife, leaving about 1 cm ($\frac{3}{8}''$) of lead projecting out, as shown in fig. 1-27(*a*). The chisel edge [fig. 1-27(*b*)] is prepared by rubbing the lead on a sand-paper block, making it flat, first on one side and then on the other side by turning the pencil through half circle. For making the conical end [fig. 1-27(*c*)], the pencil should be rotated between the thumb and fingers, while rubbing the lead.



(*a*) (*b*) (*c*)

FIG. 1-27

The pencil lead should occasionally be rubbed on the sand-paper block (while doing the drawing work) to maintain the same thickness of the chisel edge or the pointed end.

Sand-paper block: This consists of a wooden block about 15 cm × 5 cm × 1·2 cm thick (6″ × 2″ × $\frac{1}{2}''$ thick) with a piece of sand-paper pasted or nailed on about half of its length, as shown in fig. 1-28. The sand-paper, when it becomes dirty or worn out, should be replaced by another. This block should always be kept within easy reach for sharpening the lead every few minutes.

Eraser: Soft India-rubber is the most suitable kind of eraser for pencil drawings. It should be such as not to spoil the surface of the paper. The frequent use of rubber should be avoided by careful planning.

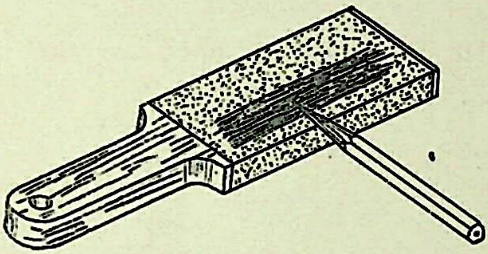


FIG. 1-28

Duster: Duster should preferably be of towel cloth larger in size than the drawing board. Before starting work, all instruments and materials should be thoroughly cleaned with the duster. The rubber crumbs formed after the use of the rubber should be swept away by the duster and not by hand. The underside of the T-square and the set-squares which continuously rub against the paper should be frequently cleaned.

Inking pen (fig. 1-29): This is used for drawing straight lines and non-circular arcs in ink. It consists of a pair of steel nibs fitted to a holder made of metal or ivory. Ink is filled between the two nibs to about 6 mm ($\frac{1}{4}''$) length by means of a quill which is usually fitted to the cork of the ink bottle. The gap between the nibs through which the ink flows and upon which the thickness of the line depends is adjusted by means of the screw *S*.

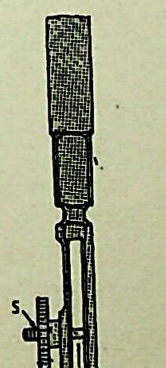


FIG. 1-29

The pen should be kept sloping at about 60° with the paper in the direction of drawing the line and the ends of the nibs should be slightly

away from the edge of the T-square or set-square. The screw should be on the side, farther from the T-square.

As the ink dries rapidly, the pen should be used immediately after it is filled. The inside faces of the nibs should be frequently cleaned for the ink to flow freely and to maintain uniformity in thickness of lines. Ink should never be allowed to dry within the pen. There should be no ink on the outside of the nibs and hence, the pen should never be dipped in ink.

For drawing large circles and circular arcs, inking attachment should be fitted in place of the pencil leg in the compass. Ink bow pens are used for drawing small circles and arcs.

HOW TO BEGIN DRAWING WORK:

General: Clean the drawing board and the T-square and place them on the table, with the working edge of the board on your left-hand side and the stock of the T-square attached to that working edge. Clean all other instruments and materials and place them on a neat piece of paper by the side of the board.

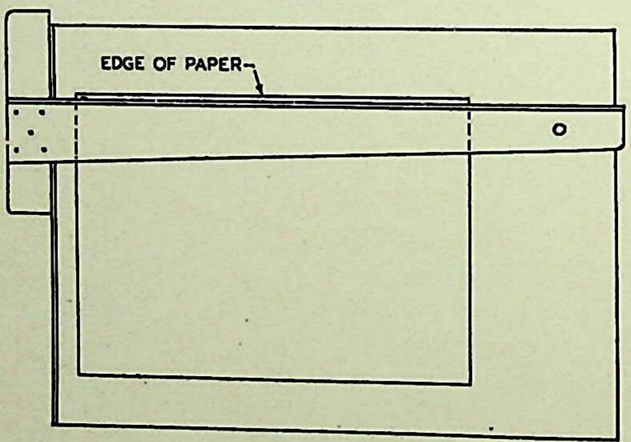


Fig. 1-30

Pinning the paper to the drawing board: Place the paper at about equal distances from the top and bottom edges of the board and one of its shorter edges at about 2·5 cm (1″) from the working edge of the board. When

the paper is of a much smaller size than that of the board, it may be placed with its lower edge at about 5 cm (2″) from the bottom edge of the board. Insert a pin on the left-hand top corner of the paper and at about 1 cm (½″) from its edges. Adjust the paper with the right hand, bringing its upper edge in line with the working edge of the T-square (fig. 1-30). Stretch the paper gently to make it perfectly flat and insert the second pin at the right-hand bottom corner. In the same manner, fix two more pins at the remaining corners. Push the pins down firmly till their heads touch the surface of the paper.

Border lines: Perfectly rectangular working space is determined by drawing the border lines. These may be drawn at equal distances of about 2 cm (¾″) to 2·5 cm (1″) from the top, bottom and right-hand edges of the paper and at about 2·5 cm (1″) to 4 cm (1½″) from the left-hand edge. More space on the left-hand side is provided to facilitate binding of the drawing sheets in a book-form, if so desired.

To draw the border lines (fig. 1-31): Mark points along the left-hand edge of the paper at required distances

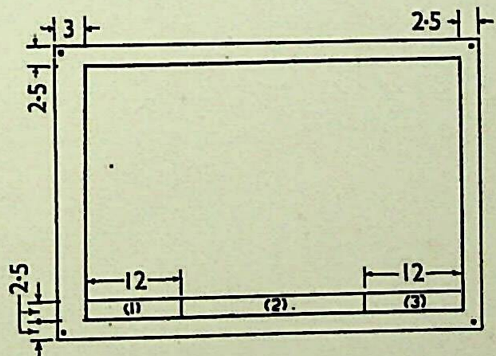


Fig. 1-31

from the top and bottom edges and through them, draw horizontal lines with the T-square. Along the upper horizontal line, mark two points at required distances from the left-hand and right-hand edges, and draw vertical lines through them with the aid of T-square and a set-square. Erase the extra lengths of lines beyond the points of intersection.

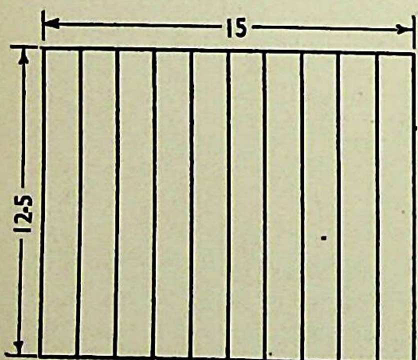
One more horizontal line at about 2·5 cm (1″) from the bottom border line may also be drawn and the space divided into three blocks in which (i) name of the institution, (ii) title of the drawing and (iii) name, class etc. of the student may be written.

The border lines are generally drawn thicker and more black than all other lines.

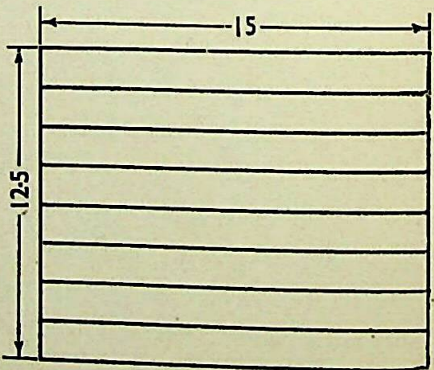
Spacing of drawings: When only one drawing or figure is to be drawn on a sheet, it should be drawn in the centre of the working space. For more than one figure, the space should be divided into suitable blocks and each figure should be drawn in the centre of its respective block.

EXERCISES I

(1) In a half-imperial size sheet, copy fig. 1-32(*a*) to (*g*) as per layout shown in fig. 1-33.



(*a*)



(*b*)

Fig. 1-32(*a*) and (*b*)

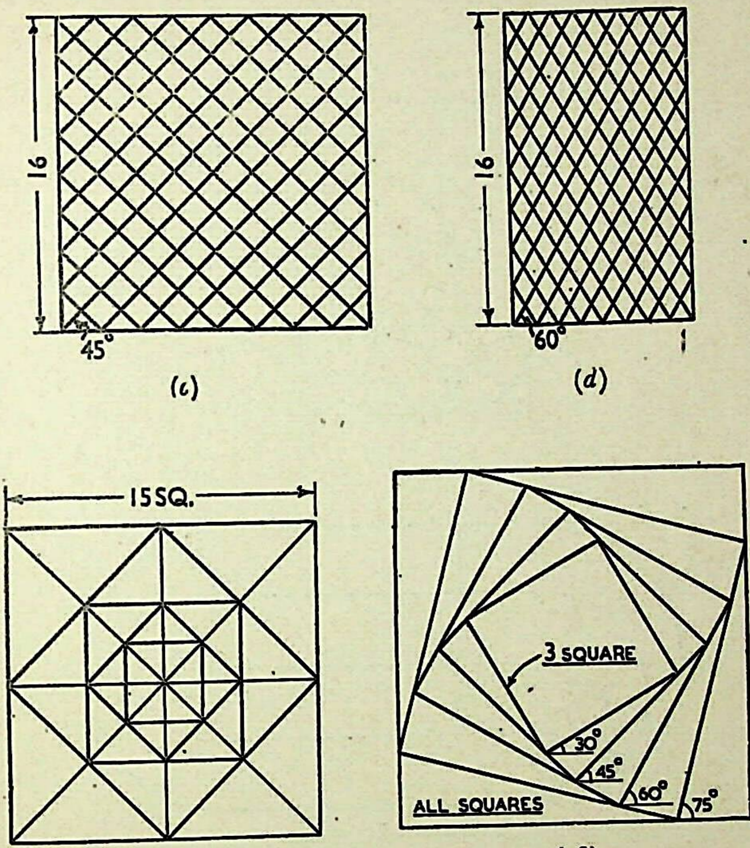


FIG. 1-32(c) to (f)

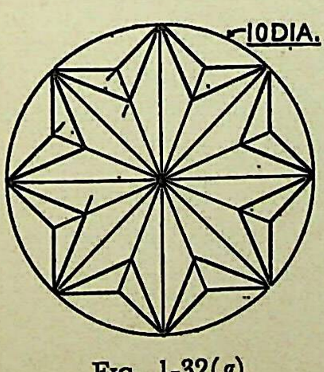


FIG. 1-32(g)

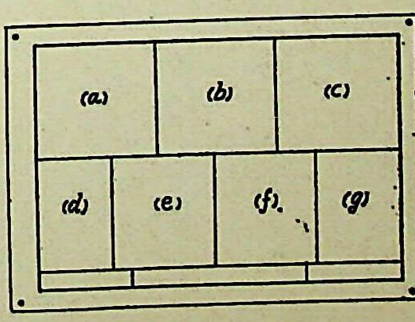
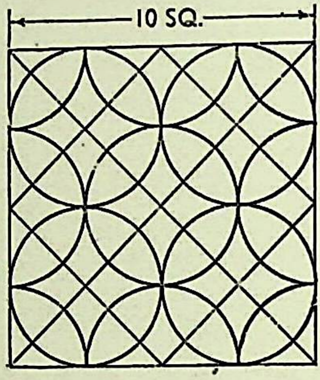
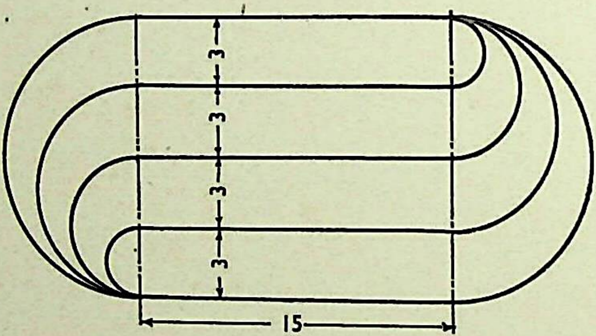


FIG. 1-33

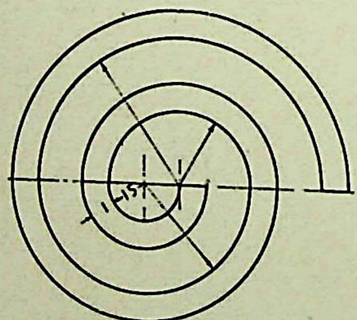
(2) Copy fig. 1-34 (*a*) to (*d*) as per layout shown in fig. 1-35.



(*a*)



(*b*)



(*c*)

FIG. 1-34(*a*), (*b*) and (*c*)

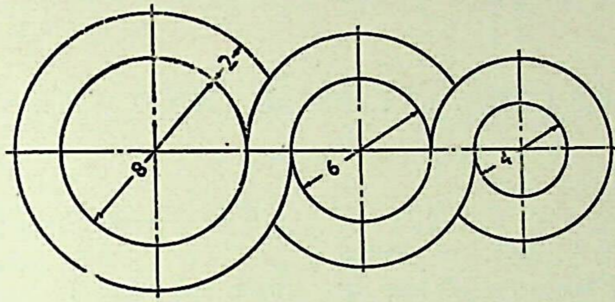


FIG. 1-34(*d*)

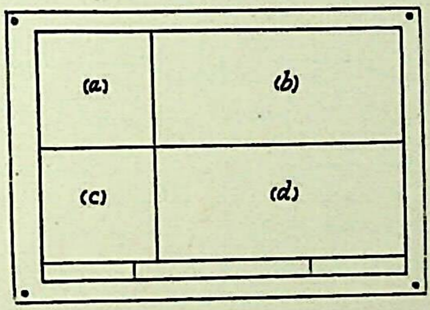


FIG. 1-35

(3) Draw a circle of 7·5 cm (3″) diameter. With the aid of T-square and set-squares only, draw lines passing through its centre, dividing it into (i) eight, (ii) twelve and (iii) twenty four equal parts.

(4) Without using a protractor, draw triangles having following base angles on a 7·5 cm (3″) long line as base: (i) 75° and 15°, (ii) 60° and 75°, (iii) 135° and 15°, (iv) 105° and 45°.

(5) Draw a line 12·5 cm (5″) long and divide it into seven equal parts by means of a divider.

(6) Draw lines (using the protractor) meeting at end *A* of a line *AB* and making with it the following angles:

27°, 49°, 115°, 151°.

CHAPTER II

TYPES OF LINES, LETTERING AND DIMENSIONING

The various types of lines used in engineering drawing are shown in figs. 2-1 and 2-7.

1. Principal lines or outlines: Lines drawn to represent visible edges and surface boundaries of objects are called outlines. In geometrical drawing, these lines show the final shape of the required figures. They are continuous lines.

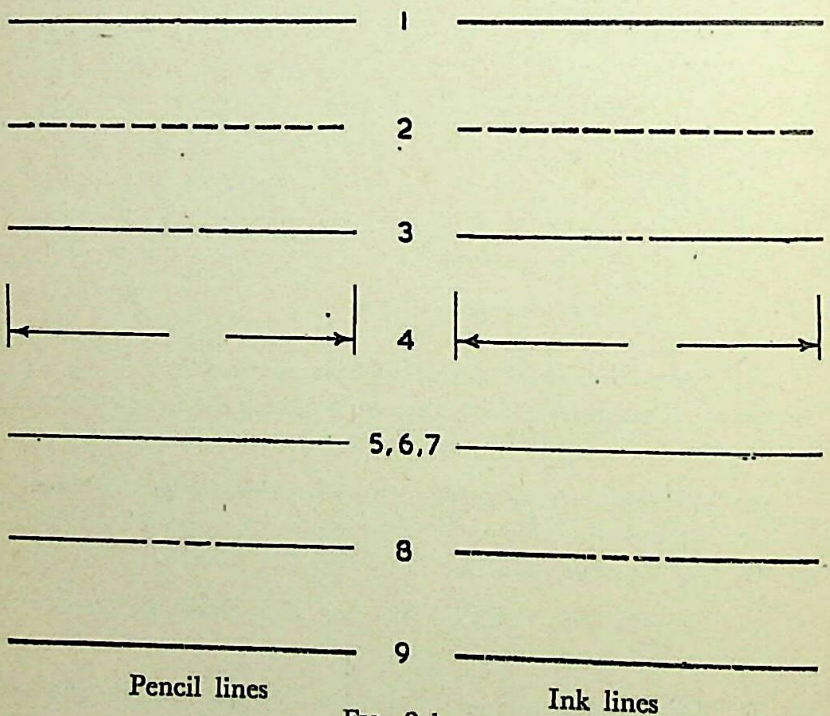


FIG. 2-1

2. Dotted or dashed lines: Hidden edges are shown in a drawing by dotted lines. These lines are made up of short dashes of approximately equal lengths of about 3 mm ($\frac{1}{8}''$) and spaced at equal distances of about 1 mm ($\frac{1}{32}''$). When a dotted line meets or intersects another dotted line

or the outlines, their points of meeting or intersection should be clearly shown.

3. **Centre lines:** Centre lines are drawn in the centres of figures which are symmetrical on two or all the four sides of their centres. They are usually extended about 1 cm ($\frac{1}{2}''$) beyond the boundary of the figure. They are composed of alternately long and short dashes, spaced approximately 1·5 mm ($\frac{1}{16}''$) apart. The short dashes are about 3 mm ($\frac{1}{8}''$) in length and the longer dashes about 6 to 8 times the short dashes. The point of intersection between two centre lines must be clearly indicated.

4. **Dimension lines:** These lines are continuous but broken at a suitable place with sufficient gap for inserting dimensions. They are terminated at the outer ends by pointed arrow-heads, partly filled-in and touching the outlines or extension lines.

5. **Extension lines:** These are continuous lines. A gap of about 1·5 mm ($\frac{1}{16}''$) is kept between these lines and the outlines of the drawing. They extend by about 3 mm ($\frac{1}{8}''$) beyond the dimension lines.

6. **Construction lines:** These lines are drawn for constructing drawings and they are shown in geometrical drawings only. They are continuous lines.

7. **Section lines:** These are continuous lines inclined at 45° and spaced uniformly about 1·5 mm ($\frac{1}{16}''$) apart.

8. **Cutting-plane lines:** The location of a cutting plane is shown by a line made up of alternately a long dash and two short dashes in the ratio of about 8 : 1, the short dashes being approximately 3 mm ($\frac{1}{8}''$) long, with uniform gap of about 1·5 mm ($\frac{1}{16}''$) between them.

9. **Border lines:** These are continuous lines thicker than the outlines of a drawing.

THICKNESS AND SHADE OF LINES:

In *pencil drawings* all lines are drawn thin. The outlines should be *intensely black* and they should form an outstanding feature on any drawing. The dotted lines should be black and all the other lines grey.

In ink-drawings, the outlines are drawn thick, dotted lines of medium thickness and all the other lines thin, the ratio of their thicknesses being 3 : 1·5 : 1.

In this book, in addition to the above lines, other lines made up of alternately a dash and a dot, or a dash and two dots etc. have been drawn in some figures, merely to distinguish them from other lines.

Lettering: Writing of titles, dimensions, notes and other important particulars on a drawing is called *lettering*. It is an important part of a drawing. However accurate and neat a drawing may be drawn, its appearance is spoiled and sometimes, its usefulness is impaired by poor lettering. Lettering should, therefore, be done properly in clear, legible and uniform style. It should be in plain and simple style so that it could be done *freehand* and *speedily*. Use of drawing instruments in lettering takes considerable time and hence, it should be avoided. Efficiency in the art of lettering can be achieved by careful and continuous practice.

Single-stroke letters: These are the simplest forms of letters and are usually employed in most of the engineering drawings. The word single-stroke should not be taken to mean that the letter should be made in one stroke without lifting the pencil. It actually means that the thickness of the line of the letter should be such as is obtained in one stroke of the pencil. The horizontal lines of letters should be drawn from left to right and vertical or inclined lines, from top to bottom.

Single-stroke letters are of two types: (i) vertical and (ii) inclined. Inclined letters lean to the right, the slope being $67\frac{1}{2}°$ with the horizontal. The size of a letter is described by its height. The ratio of height to width varies but in case of most of the letters it is 6 : 5.

Lettering is generally done in capital letters. Different sizes of letters are used for different purposes. The main titles are generally written in 1 cm ($\frac{3}{8}''$) to 1·2 cm ($\frac{1}{2}''$) size, sub-titles in 3 mm ($\frac{1}{8}''$) to 6 mm ($\frac{1}{4}''$) size, while notes, dimensions figures etc. in 3 mm ($\frac{1}{8}''$) to 4 mm ($\frac{5}{32}''$) size. In dimension figures, the overall height of the fraction is kept twice that of the integer.

Fig. 2-2 shows single-stroke vertical capital letters and figures with approximate proportions.

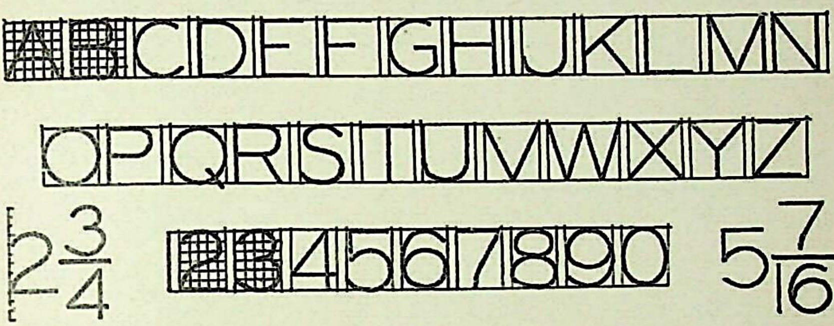


Fig. 2-2

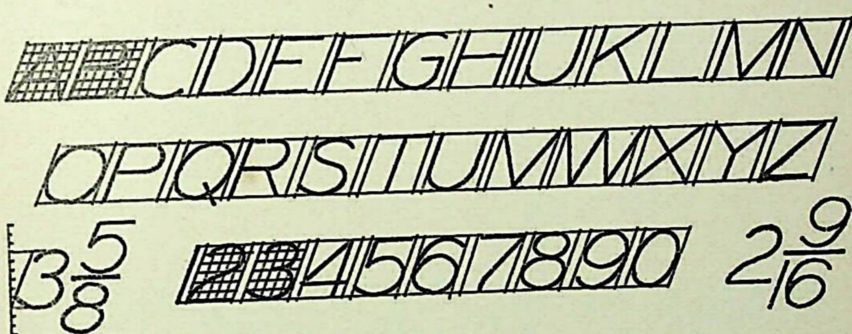


Fig. 2-3

Single-stroke inclined capital letters and figures are shown in fig. 2-3. The lower-case letters are usually used in architectural drawings. Vertical and inclined lower-case alphabets are shown in fig. 2-4 and fig. 2-5 respectively. The width of the majority of letters is equal to the height.

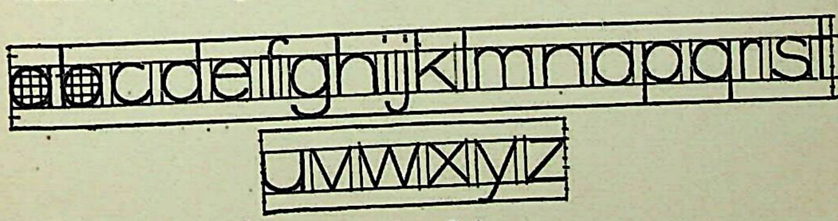


Fig. 2-4

All letters should be uniform in shape, slope, size, shade and spacing. The shape and slope of every letter should

be uniform throughout a drawing. For maintaining uniformity in size, thin and light guide-lines may first be drawn, and lettering may then be done between them. The shade of every letter must be the same as that of the outlines of drawings, i.e., intensely black. The spacing between two

FIG. 2-5

letters should not necessarily be equal. The letters should be so spaced that they do not appear too close together or too much apart. Judging by the eye, the back-ground areas between the letters should be kept approximately equal. The distance between the words must be uniform and at least equal to the height of the letters.

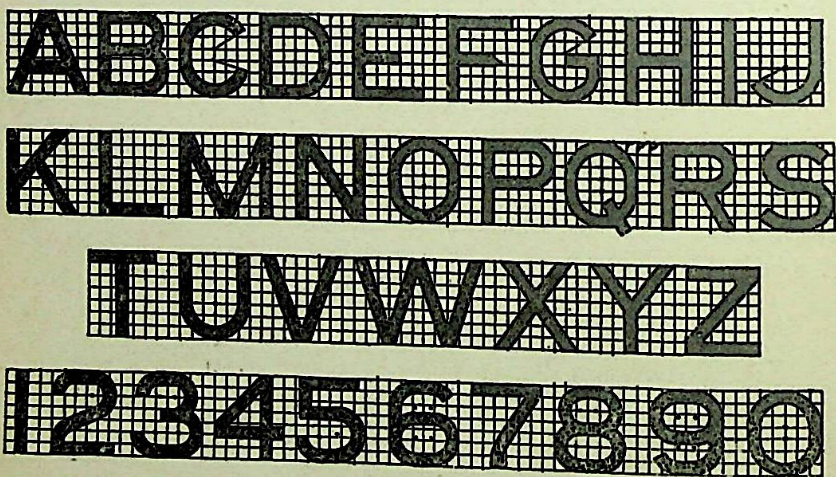


FIG. 2-6

Lettering should be so done as can be read from the front with the main title horizontal. All sub-titles should be placed below but not too close to the respective views.

Lettering, except the dimension figures, should be underlined to make them more prominent.

Gothic letters: Stems of single-stroke letters, if given more thickness, form what are known as *gothic* letters. These are mostly used for main titles of ink-drawings. The outlines of the letters are first drawn with the aid of instruments and then filled in with ink. The thickness of the stem may vary from $\frac{1}{5}$ to $\frac{1}{10}$ of the height of the letters. Fig. 2-6 shows the alphabet and figures in gothic with thickness equal to $\frac{1}{7}$ of the height.

Dimensioning: The technique of dimensioning is described in detail in the book "Machine Drawing" by this author. A few important points, useful in dimensioning the geometrical figures, are given below (fig. 2-7):

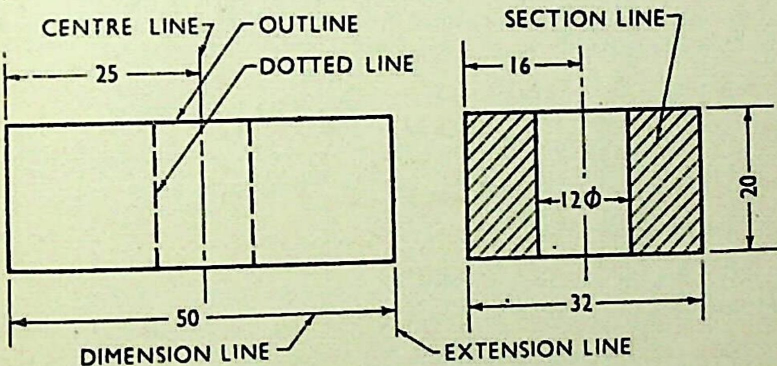


Fig. 2-7

(i) Dimensions should be placed outside the views except when they are clearer and more easily readable inside.

(ii) Dimension lines should not cross each other.

(iii) Dimensions should not be shown between dotted lines as far as possible.

(iv) Dimension lines should be placed at least 6 mm ($\frac{1}{4}''$) from the outlines and from one another.

(v) Arrow-head should be pointed and filled in. It should be about 3 mm ($\frac{1}{8}''$) long and its maximum width should be about $\frac{1}{3}$ of its length. The arrow-head is drawn

freehand with two strokes made in the direction of the point and the space between them neatly filled up.

(vi) Dimension figures are inserted in the break provided in the dimension lines. They are usually placed perpendicular to the dimension lines, and in such a manner that they can be read from the bottom or right-hand side. The dividing line of fraction is drawn in line with the dimension line.

EXERCISES II

(1) Write freehand, in single-stroke (i) vertical capital letters and (ii) inclined capital letters of 3 mm ($\frac{1}{8}''$) height, the following paragraph from page 29:

"All letters should............height of the letters."

(2) Write freehand, in single-stroke vertical lower-case letters of 3 mm ($\frac{1}{8}''$) height, the following paragraph from page 28:

"Lettering: Writing of ....practice."

(3) Write free-hand, the paragraph stated in Ex. (2) in single-stroke inclined lower-case letters of 3 mm ($\frac{1}{8}''$) height.

(4) Print in gothic letters of 1·3 cm ($\frac{1}{2}''$) height, the titles of Chapters I, II and III.

CHAPTER III

GEOMETRICAL CONSTRUCTION

In this chapter, we shall deal with problems on geometrical construction which are mostly based on plane geometry and which are very essential in the preparation of engineering drawings.

BISECTING A LINE:

Problem 1:

To bisect a given straight line (fig. 3-1).

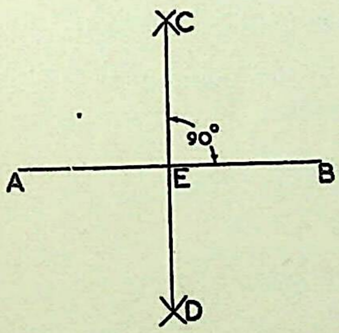


FIG. 3-1

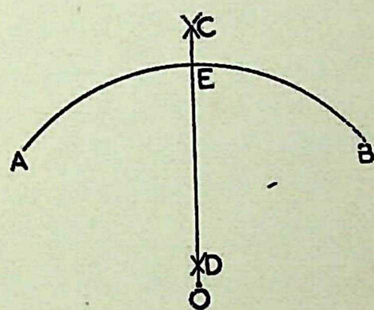


FIG. 3-2

Let AB be the given line. With centre A and radius greater than half AB, draw arcs on both sides of AB.

With centre B and the same radius, draw arcs intersecting the previous arcs at C and D. Draw a line joining C and D and cutting AB at E.

Then, $AE = EB = \frac{1}{2}AB$. Further, CD bisects AB at right angles.

Problem 2:

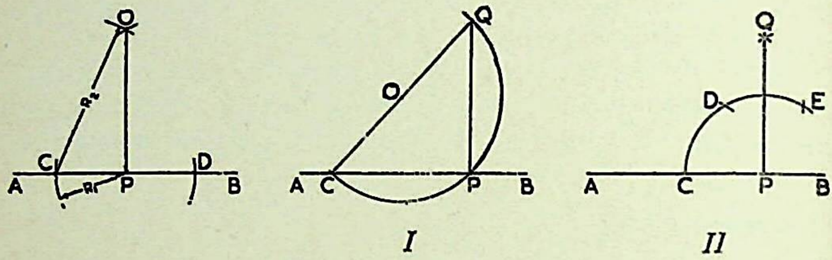
To bisect a given arc (fig. 3-2).

Let AB be the arc drawn with centre O. Adopt the same method as shown in Problem 1. The bisector CD, if produced, will pass through the centre O.

PERPENDICULARS:

Problem 3:

To draw a perpendicular to a given line from a point within it (fig. 3-3).



(*a*) (*b*)

Fig. 3-3

(a) When the point is near the middle of the line.

Let *AB* be the given line and *P* the point in it. With *P* as centre and any convenient radius R_1, draw an arc cutting *AB* at *C* and *D*.

With any radius R_2 greater than R_1 and centres *C* and *D*, draw arcs intersecting each other at *O*.

Join *P* and *O*. Then *PO* is the required perpendicular.

(b) When the point is near an end of the line.

Let *AB* be the given line and *P* the point in it.

Method I: With any point *O* as centre, and radius equal to *OP*, draw an arc greater than the semi-circle, cutting *AB* at *C*. Draw a line joining *C* and *O*, and produce it to cut the arc at *Q*.

Join *P* and *Q*. Then *PQ* is the required perpendicular.

Method II: With *P* as centre and any convenient radius, draw an arc cutting *AB* at *C*.

With the same radius, cut (from the arc) two equal divisions *CD* and *DE*. Again with the same radius and centres *D* and *E*, draw arcs intersecting each other at *Q*.

Join *P* and *Q*. Then *PQ* is the required perpendicular.

Problem 4:

To draw a perpendicular to a given line from a point outside it (fig. 3-4).

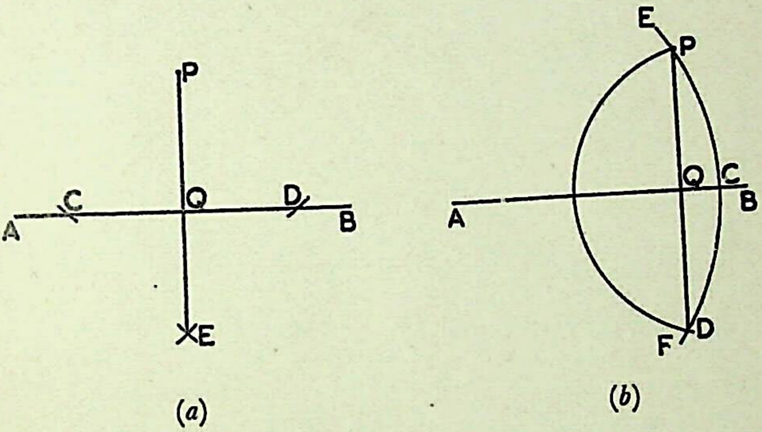


FIG. 3-4

(a) When the point is nearer the centre than the end of the line.

Let *AB* be the given line and *P* the point. With centre *P* and any convenient radius, draw an arc cutting *AB* at *C* and *D*.

With any radius greater than half *CD* and centres *C* and *D*, draw arcs intersecting each other at *E*. Draw a line joining *P* and *E* and cutting *AB* at *Q*.

Then *PQ* is the required perpendicular.

(b) When the point is nearer the end than the centre of the line.

Let *AB* be the given line and *P* the point.

With centre *A* and radius equal to *AP*, draw an arc *EF* cutting *AB* or *AB*-produced, at *C*.

With centre *C* and radius equal to *CP*, draw an arc cutting *EF* at *D*. Draw a line joining *P* and *D* and intersecting *AB* at *Q*. Then *PQ* is the required perpendicular.

PARALLEL LINES:

Problem 5:

To draw a line through a given point, parallel to a given straight line (fig. 3-5).

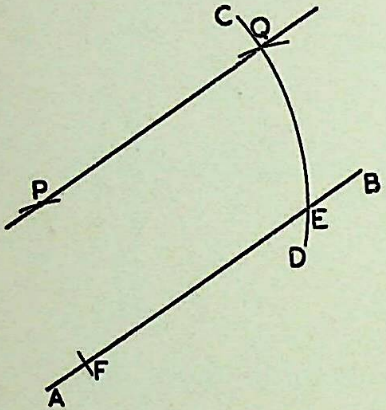


FIG. 3-5

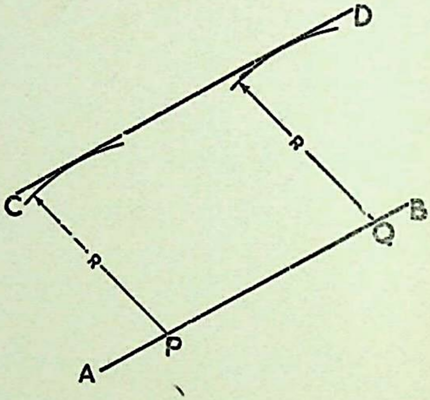


FIG. 3-6

Let *AB* be the given line and *P* the point.

With centre *P* and any convenient radius, draw an arc *CD* cutting *AB* at *E*.

With centre *E* and the same radius, draw an arc cutting *AB* at *F*.

With centre *E* and radius equal to *FP*, draw an arc to cut *CD* at *Q*.

Draw a straight line through *P* and *Q*. Then this is the required line.

Problem 6:

To draw a line parallel to and at a given distance from a given straight line (fig. 3-6).

Let *AB* be the given line and *R* the given distance.

Mark points *P* and *Q* on *AB*, as far apart as convenient. With *P* and *Q* as centres and radius equal to *R*, draw arcs on the same side of *AB*. Draw the line *CD*, just touching the two arcs.

CD is the required line.

DIVISION OF LINE:

Problem 7:

To divide a given straight line into any number of equal parts (fig. 3-7).

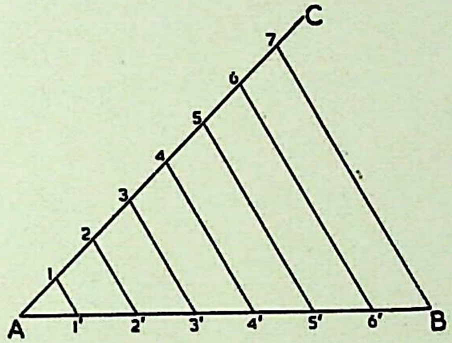


FIG. 3-7

Let *AB* be the given line to be divided into say, seven equal parts.

Draw a line *AC* of any length inclined at some convenient angle to *AB* (preferably an acute angle).

From *A* and along *AC*, cut off with a divider seven equal divisions of any convenient length. Join *B* and 7.

With the aid of two set-squares (Prob. 4, Ch. I) draw lines through 1, 2, 3 etc. parallel to *B*7, intersecting *AB* at points 1′, 2′, 3′ etc., thus dividing it into seven equal parts.

ANGLES:

Problem 8:

To bisect a given angle (fig. 3-8).

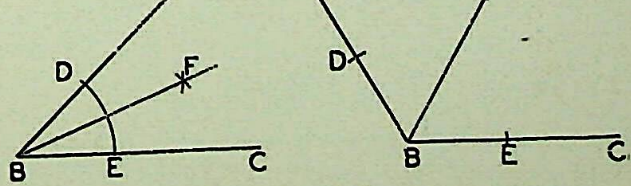


FIG. 3-8

Let *ABC* be the given angle.

With B as centre and any radius, draw an arc cutting AB at D and BC at E.

With centres D and E and the same or any convenient radius, draw arcs intersecting at F. Draw a line joining B and F; BF bisects the angle ABC, i.e., $\angle ABF = \angle FBC$.

Problem 9:

To draw a line inclined to a given line at an angle equal to a given angle (fig. 3-9).

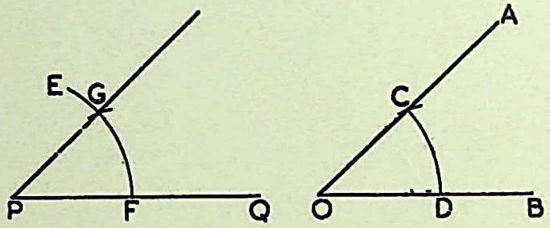


Fig. 3-9

Let PQ be the given line and AOB the given angle.

With O as centre and any radius, draw an arc cutting OA at C and OB at D. With the same radius and centre P, draw an arc EF cutting PQ at F.

With F as centre and radius equal to CD, draw an arc cutting the arc EF at G.

Draw a line through P and G; PG is the required line.

Problem 10:

To trisect a given right angle (fig. 3-10).

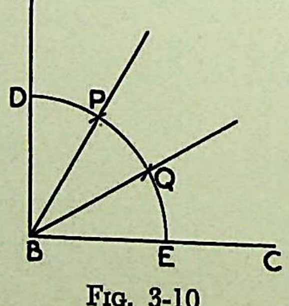


Fig. 3-10

Let ABC be the given right angle.

With centre B and any radius, draw an arc cutting AB at D and BC at E.

With the same radius and centres D and E, draw arcs cutting the arc DE at points P and Q. Draw lines joining B with P and Q; BP and BQ trisect the right angle ABC.

Thus, $\angle ABP = \angle PBQ = \angle QBC = \frac{1}{3} \angle ABC.$

ARCS OF CIRCLES:

Problem 11:

To find the centre of a given arc (fig. 3-11).

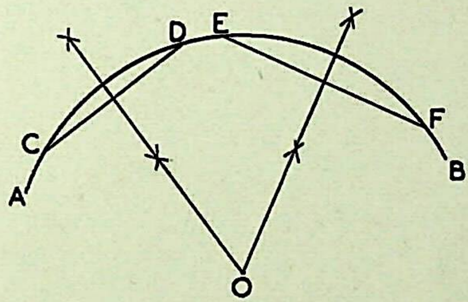


FIG. 3-11

Let AB be the given arc.

In AB, draw two chords CD and EF of any lengths.

Draw perpendicular bisectors of CD and EF intersecting at O. Then O is the required centre.

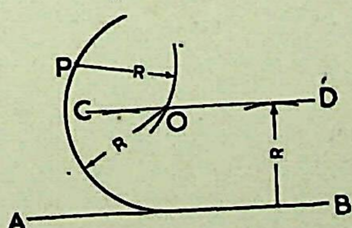


FIG. 3-12

Problem 12:

To draw an arc of a given radius, touching a given straight line and passing through a given point (fig. 3-12).

Let AB be the given line, P the point and R the radius. Draw a line CD parallel to and at a distance equal to R from AB (Prob. 6).

With P as centre and radius equal to R, draw an arc cutting CD at O.

With O as centre, draw the required arc.

Problem 13:

To draw an arc of a given radius touching two given straight lines at right angles to each other (fig. 3-13).

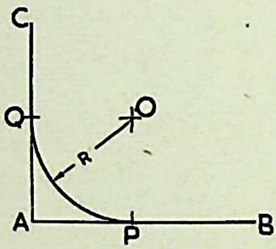


FIG. 3-13

Let AB and AC be the given lines and R the given radius.

With centre A and radius equal to R, draw arcs cutting AB at P and AC at Q.

With P and Q as centres and the same radius, draw arcs intersecting each other at O.

With O as centre and radius equal to R, draw the required arc.

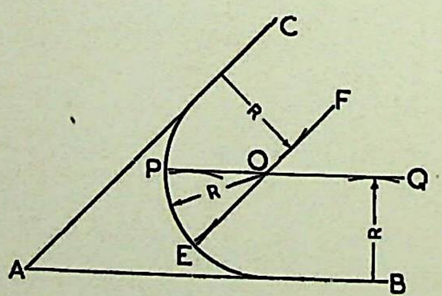


FIG. 3-14(a)

Problem 14:

To draw an arc of a given radius touching two given straight lines which make any angle between them [figs. 3-14(a) and 3-14(b)].

Let AB and AC be the lines and R, the radius. Draw a line PQ parallel to and at a distance equal to R from AB. Similarly, draw a line EF parallel to and at a distance equal to R from AC, intersecting PQ at O. With O as centre and radius equal to R, draw the required arc.

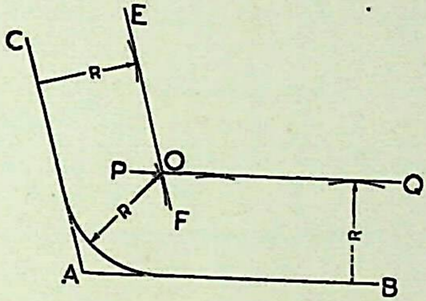


FIG. 3-14(b)

Problem 15:

To draw an arc of a given radius touching a given arc and a given straight line.

Case I [fig. 3-15(a)]: Let AB be the given line, CD the given arc drawn with centre O and radius equal to R_1, and R_2 the given radius.

With O as centre and raidus equal to $(R_1 - R_2)$, draw an arc EF.

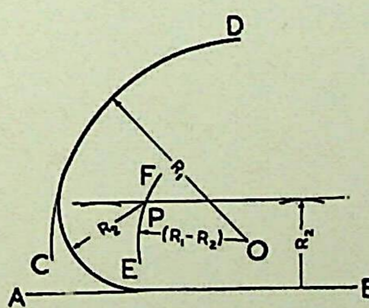


FIG. 3-15(a)

Draw a line parallel to and at a distance equal to R_2 from AB and intersecting EF at a point P.

With P as centre and radius equal to R_2, draw the required arc.

Case II [fig. 3-15(*b*)]: Let *AB* be the given line, *CD* the given arc drawn with centre *O* and radius equal to R_1, and R_2 the given radius.

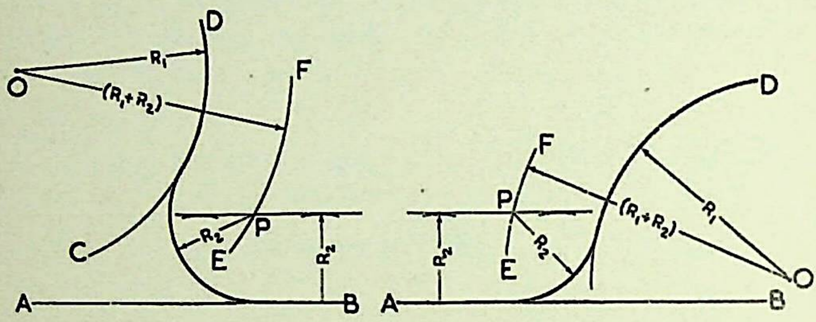


FIG. 3-15(*b*)

With *O* as centre and radius equal to $(R_1 + R_2)$, draw an arc *EF*. Draw a line parallel to and at a distance equal to R_2 from *AB* and intersecting *EF* at a point *P*.

With *P* as centre and radius equal to R_2, draw the required arc.

Problem 16:

To draw an arc of a given radius touching two given arcs (fig. 3-16).

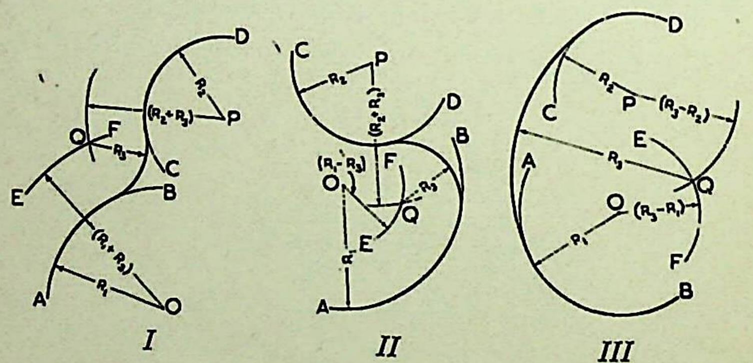


FIG. 3-16

Let *AB* be the given arc drawn with centre *O* and radius equal to R_1; *CD* the arc drawn with centre *P* and radius equal to R_2, and R_3 the given radius.

Case I: With O as centre and radius equal to $(R_1 + R_3)$, draw an arc EF. With P as centre and radius equal to $(R_2 + R_3)$, draw an arc intersecting EF at a point Q.

With Q as centre and radius equal to R_3, draw the required arc.

Case II: With O as centre and radius equal to $(R_1 - R_3)$, draw an arc EF.

With P as centre and radius equal to $(R_2 + R_3)$, draw an arc intersecting EF at a point Q.

With Q as centre and radius equal to R_3, draw the required arc.

Case III: With O as centre and radius equal to $(R_3 - R_1)$, draw an arc EF.

With P as centre and radius equal to $(R_3 - R_2)$, draw an arc intersecting EF at a point Q.

With Q as centre and radius equal to R_3, draw the required arc.

Problem 17:

To draw an arc passing through three given points not in a straight line (fig. 3-17).

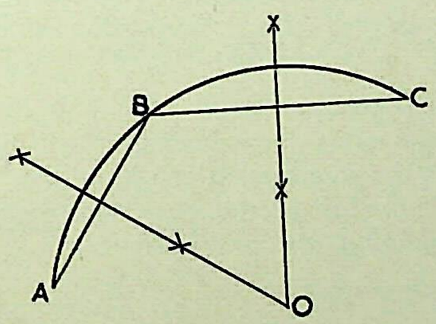


FIG. 3-17

Let A, B and C be the given points.

Draw lines joining B with A and C.

Draw perpendicular bisectors of AB and BC intersecting each other at a point O.

With O as centre and radius equal to OA or OB or OC, draw the required arc.

Problem 18:

To draw a continuous curve of circular arcs passing through any number of given points not in a straight line (fig. 3-18).

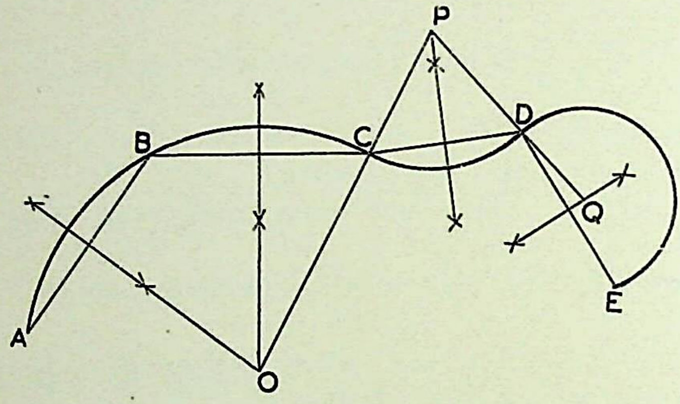


Fig. 3-18

Let A, B, C, D and E be the given points.

Draw lines joining A with B, B with C, C with D etc.

Draw perpendicular bisectors of AB and BC intersecting at O. With O as centre and radius equal to OA, draw an arc ABC. Join O and C. Draw perpendicular bisector of CD intersecting OC or OC-produced, at P.

With P as centre and radius equal to PC, draw an arc CD. Repeat the same construction. Note that the centre of the arc is at the intersection of the perpendicular bisector and the line, or the line-produced, joining the previous centre with the last point of the previous arc.

EQUILATERAL TRIANGLES:

Problem 19:

To construct an equilateral triangle, given the length of the side (fig. 3-19).

(a) With T-square and set-square only.

With T-square, draw a line AB of given length.

With 30°-60° set-square and T-square, draw a line through A making 60° angle with AB.

Similarly, through B, draw a line making the same angle with AB and intersecting the first line at C.

Then ABC is the required triangle.

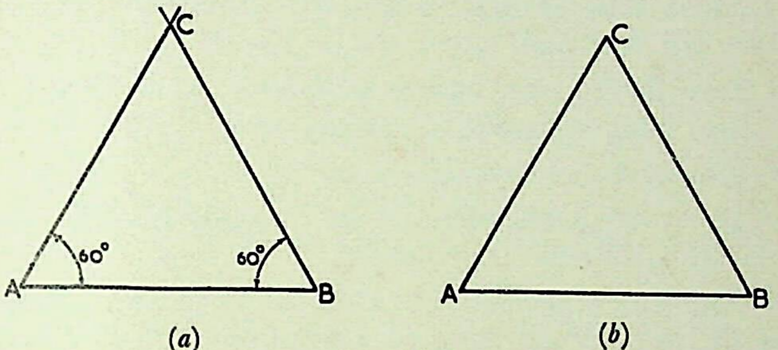


FIG. 3-19

(b) With the aid of a compass.

With centres A and B and radius equal to AB, draw arcs intersecting each other at C.

Join C with A and B.

Then ABC is the required triangle.

Problem 20:

To construct an equilateral triangle of a given altitude (fig. 3-20).

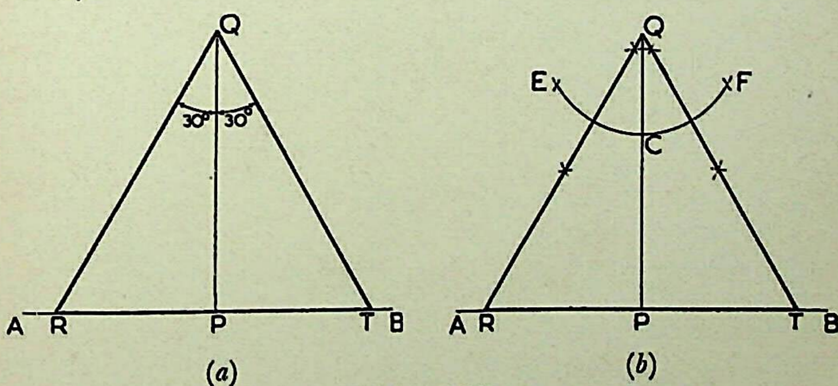


FIG. 3-20

(a) With T-square and set-square only.

With the T-square, draw a line AB of any length.

From a point P in AB, draw with a set-square, the vertical PQ equal to the given altitude.

With T-square and 30°-60° set-square, draw lines through Q on both sides of and making 30° angles with PQ and cutting AB in R and T.

Then QRT is the required triangle.

(b) With the aid of a compass.

Draw a line AB of any length.

At any point P in AB, draw the perpendicular PQ equal to the given altitude (Prob. 3).

With centre Q and any radius, draw an arc intersecting PQ at C. With centre C and the same radius, draw arcs cutting the first arc at E and F.

Draw bisectors of CE and CF to intersect AB at R and T respectively. Then QRT is the required triangle.

SQUARES:

Problem 21:

To construct a square, length of a side given (fig. 3-21).

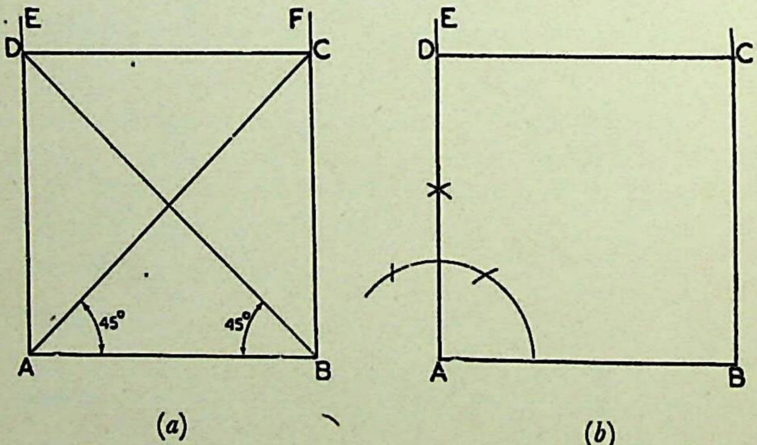


FIG. 3-21

(a) With T-square and set-square only.

With the T-square, draw a line AB equal to the given length.

At A and B, draw verticals AE and BF. Draw a line AC inclined at 45° to AB, cutting BF at C. Draw a line BD inclined at 45° to AB, cutting AE at D. Join C with D.

Then $ABCD$ is the required square.

(b) With the aid of a compass.

Draw a line AB equal to the given length. At A, draw a line AE perpendicular to AB.

With centre A and radius AB, draw an arc cutting AE at D. With centres B and D and the same radius, draw arcs intersecting at C. Draw lines joining C with B and D.

Then $ABCD$ is the required square.

REGULAR POLYGONS:

Problem 22:

To construct a regular polygon, given the length of its side.

Let the number of sides of polygon be seven.

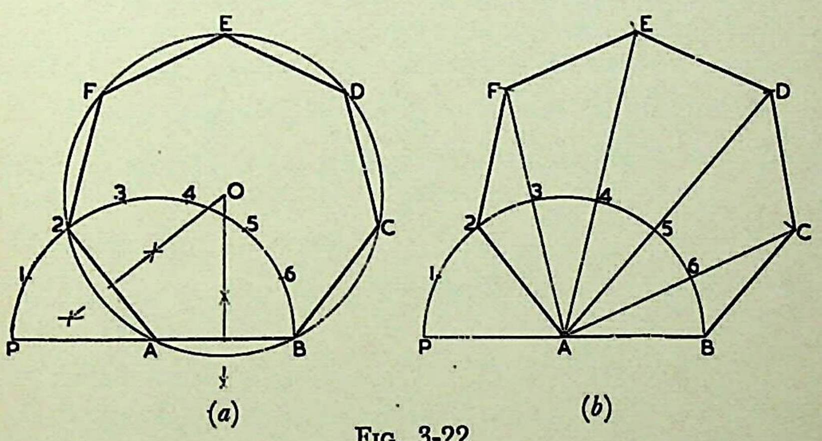


FIG. 3-22

Method I (fig. 3-22):

Draw a line AB equal to the given length.

With centre A and radius AB, draw a semi-circle BP.

With a divider, divide the semi-circle into seven equal parts (same as the number of sides). Number the division-points as 1, 2 etc. starting from P.

Join A with the second division-point 2.

(a) Draw perpendicular bisectors of *A*2 and *AB* intersecting each other at *O*.

With centre *O* and radius *OA*, describe a circle.

With radius *AB* and starting from *B*, cut the circle at points *C*, *D*......2.

Draw lines *BC*, *CD* etc. thus completing the required heptagon.

(b) With centre *B* and radius *AB*, draw an arc cutting *A*6-produced at *C*. With centre *C* and the same radius, draw an arc cutting *A*5-produced at *D*.

Find points *E* and *F* in a similar manner. Draw lines *BC*, *CD* etc. and complete the heptagon.

Method II (fig. 3-23):

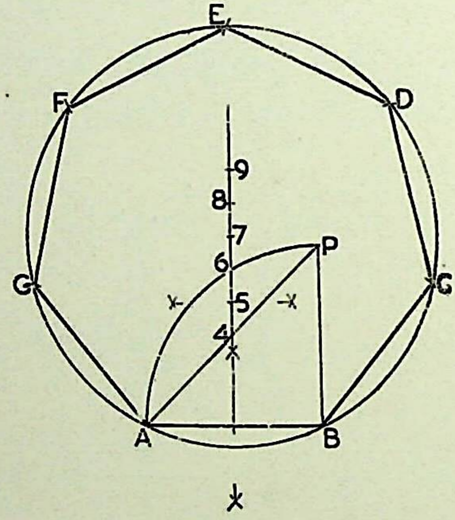


FIG. 3-23

Draw a line *AB* equal to the given length.

At *B*, draw a line *BP* perpendicular and equal to *AB*.

Draw a line joining *A* with *P*.

With centre *B* and radius *AB*, draw the quadrant *AP*.

Draw the perpendicular bisector of *AB* to intersect the line *AP* in 4 and the arc *AP* in 6.

A square of a side equal to *AB* can be inscribed in the circle drawn with centre 4 and radius *A*4.

A regular hexagon of a side equal to *AB* can be inscribed in the circle drawn with centre 6 and radius *A*6. The mid-point 5 of the line 4-6 is the centre of the circle of radius *A*5 in which a regular pentagon of a side equal to *AB* can be inscribed.

To locate centre 7 for the regular heptagon of side *AB*, step-off a line 6-7 equal to the line 5-6.

With centre 7 and radius equal to *A*7, draw a circle. Starting from *B*, cut it in seven equal divisions with radius equal to *AB*.

Draw lines *BC*, *CD* etc. and complete the heptagon.

Regular polygons of any number of sides can be drawn by this method.

Alternative method:

On *AB* as diameter, describe a semi-circle. With either *A* or *B* as centre and *AB* as radius, describe an arc on the same side as the semi-circle. Draw a perpendicular bisector of *AB* cutting the semi-circle at point 4 and the arc at point 6.

Obtain points 5, 7, 8 etc. as shown in Method II.

SPECIAL METHODS OF DRAWING SOME REGULAR POLYGONS, GIVEN THE LENGTH OF A SIDE:

Problem 23: *Pentagon.*

Method I (fig. 3-24): Draw a line *AB* equal to the given length.

With centre *A* and radius *AB*, describe a circle-1.

With centre *B* and the same radius, describe a circle-2 cutting circle-1 at *C* and *D*.

With centre *C* and the same radius, draw an arc to cut circle-1 and circle-2 at *E* and *F* respectively. Draw a perpendicular bisector of the line *AB* to cut the arc *EF* at *G*.

Draw a line *EG* and produce it to cut circle-2 at *P*.

Draw a line *FG* and produce it to cut circle-1 at *R*.

With *P* and *R* as centres and *AB* as radius, draw arcs intersecting each other at *Q*.

Draw lines *BP*, *PQ*, *QR* and *RA*, thus completing the pentagon.

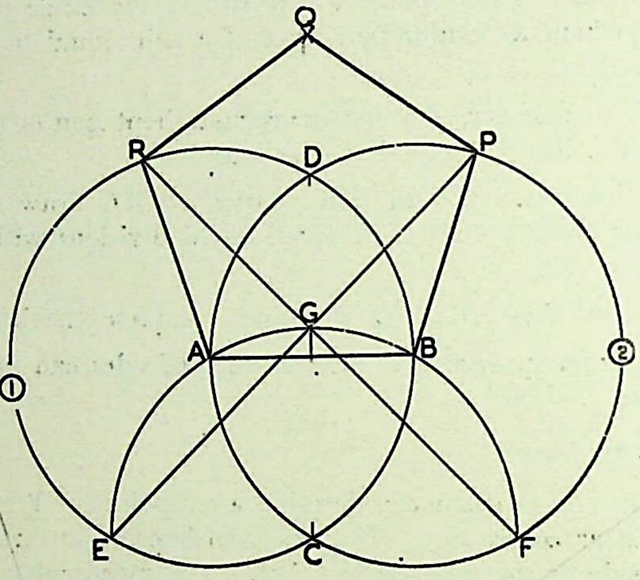


Fig. 3-24

Method II (fig. 3-25): Draw a line *AB* equal to the given length.

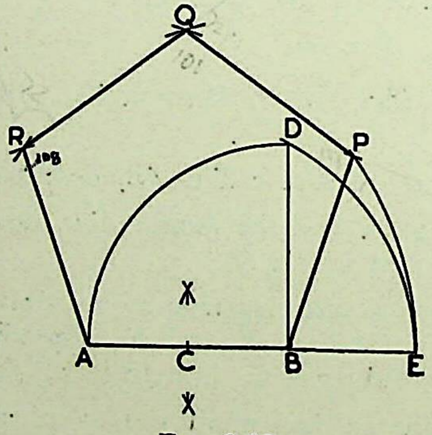


Fig. 3-25

Bisect *AB* in a point *C*.

Draw a line *BD* perpendicular and equal to *AB*.

With centre C and radius CD, draw an arc to intersect AB-produced at E. Then AE is the length of the diagonal of the pentagon.

Therefore, with centre A and radius AB, draw an arc intersecting the arc drawn with centre B and radius AE at R.

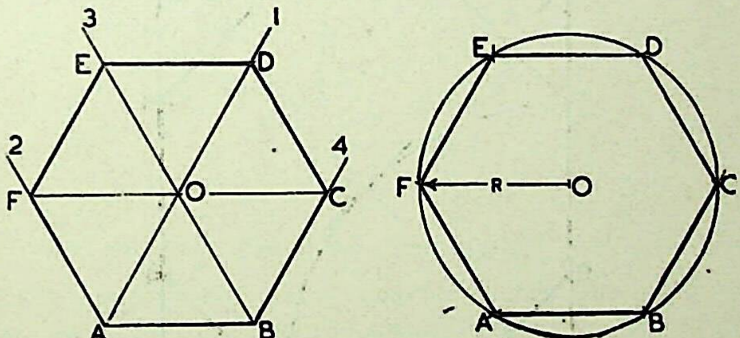
Again with centre A and radius AE, draw an arc intersecting the arc drawn with centre B and radius AB at P.

With centres A and B and radius AE, draw arcs intersecting each other at Q.

Draw lines BP, PQ, QR and RA, thus completing the pentagon.

Problem 24:

Hexagon (fig. 3-26).



(*a*) (*b*)

FIG. 3-26

(a) With T-square and 30°-60° set-square only.

Draw a line AB equal to the given length.

From A, draw lines $A1$ and $A2$ making 60° and 120° angles respectively with AB.

From B, draw lines $B3$ and $B4$ making 60° and 120° angles respectively with AB.

From O the point of intersection of $A1$ and $B3$, draw a line parallel to AB and intersecting $A2$ at F and $B4$ at C.

From F, draw a line parallel to BC and intersecting $B3$ at E.

From C, draw a line parallel to AF and intersecting $A1$ at D. Draw a line joining E and D.

Then $AB....F$ is the required hexagon.

(b) With any point O as centre and radius equal to the given length, draw a circle.

With the same radius and starting from any point on the circle, set off six divisions on the circle.

Join the division-points in proper sequence and complete the hexagon.

Problem 25:

To inscribe a regular octagon in a given square (fig. 3-27).

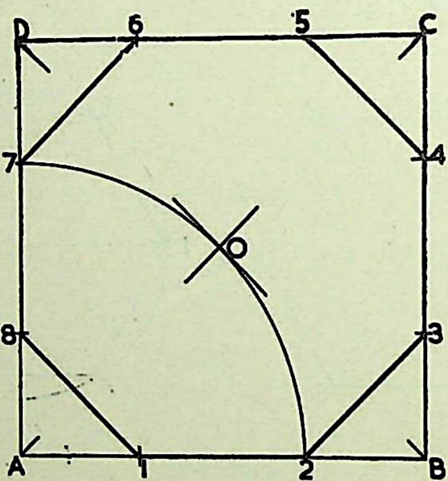


FIG. 3-27

Draw the given square $ABCD$.

Draw diagonals AC and BD intersecting each other at O.

With centre A and radius AO, draw an arc cutting AB at 2 and AD at 7.

Similarly, with centres B, C and D and the same radius, draw arcs and obtain points 1, 3, 4 etc. as shown.

Draw lines 2-3, 4-5, 6-7 and 8-1, thus completing the octagon.

REGULAR POLYGONS INSCRIBED IN CIRCLES:

Problem 26:

To inscribe a regular polygon of any number of sides, say 5, in a given circle (fig. 3-28).

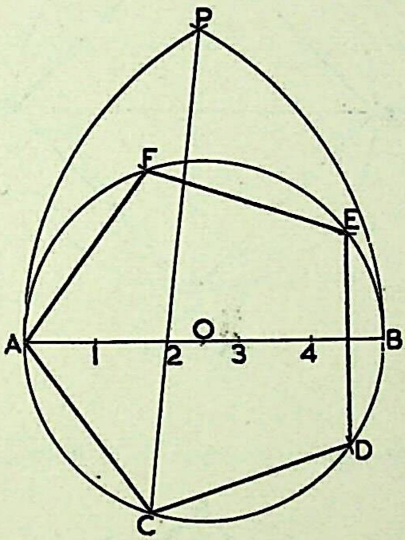


FIG. 3-28

With centre O, draw the given circle.

Draw a diameter AB and divide it into five equal parts (same number of parts as the number of sides), and number them as shown.

With centres A and B and radius AB, draw arcs intersecting each other at P.

Draw a line $P2$ and produce it to meet the circle at C.

Then AC is the length of the side of the pentagon. Starting from C, step-off on the circle, divisions CD etc. equal to AC. Draw lines CD, DE etc., thus completing the pentagon.

Problem 27:

To inscribe a square in a given circle (fig. 3-29).

With centre O, draw the given circle.

Draw diameters AB and CD perpendicular to each other.

Draw lines *AC*, *CB*, *BD* and *DA*, thus completing the square.

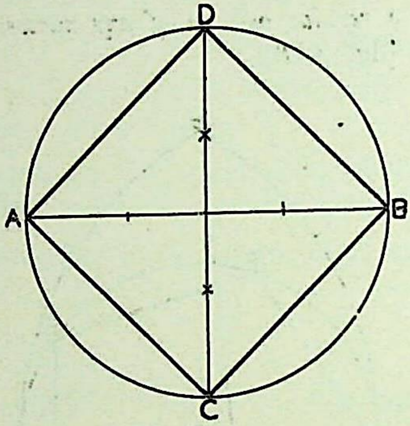


FIG. 3-29

Problem 28:

To inscribe a regular pentagon in a given circle (fig. 3-30).

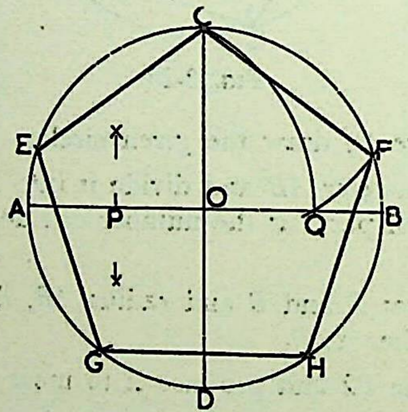


FIG. 3-30

With centre *O*, draw the given circle.

Draw diameters *AB* and *CD* perpendicular to each other. Bisect *AO* in a point *P*. With centre *P* and radius *PC*, draw an arc cutting *OB* in *Q*.

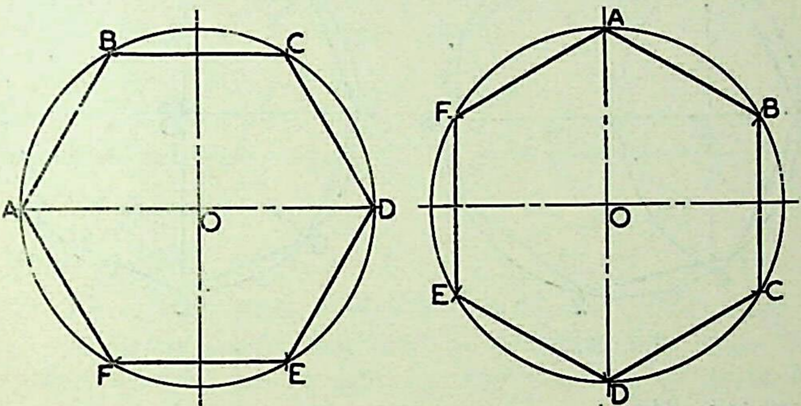
With centre *C* and radius *CQ*, draw an arc cutting the circle in *E* and *F*.

With centres E and F and the same radius, draw arcs cutting the circle in G and H respectively.

Draw lines CE, EG, GH, HF and FC, thus completing the required pentagon.

Problem 29:

To inscribe a regular hexagon in a given circle (fig. 3-31).



(*a*) (*b*)

FIG. 3-31

Apply the same method as shown in Prob. 24(*b*).

Note:

(a) When two sides of the hexagon are required to be horizontal, the starting point for stepping off equal divisions should be on an end of the horizontal diameter.

(b) If they are to be vertical, the starting point should be on an end of the vertical diameter.

In either case, the points should be joined with the aid of a T-square and 30°-60° set-square, to avoid inaccuracy.

Problem 30:

To inscribe a regular heptagon in a given circle (fig. 3-32).

With centre O, draw the given circle.

Draw a diameter AB. With centre A and radius AO, draw an arc cutting the circle at E and F.

Draw a line EF, cutting AO in G.

Then EG or FG is the length of the side of the heptagon.

Therefore, from any point on the circle, say A, step-off divisions equal to EG, around the circle.

Join the division-points and obtain the heptagon.

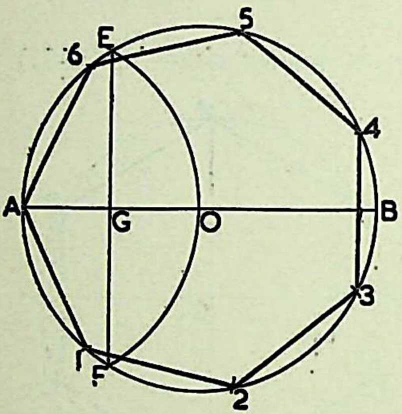


FIG. 3-32

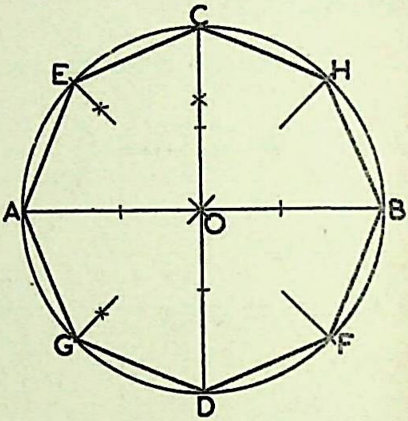


FIG. 3-33

Problem 31:

To inscribe a regular octagon in a given circle (fig. 3-33).

With centre O, draw the given circle. Draw diameters AB and CD at right angles to each other.

Draw diameters EF and GH bisecting angles AOC and COB. Draw lines AE, EC etc. and complete the octagon.

REGULAR FIGURES ABOUT A GIVEN CIRCLE WITH THE AID OF T-SQUARE AND SET-SQUARES ONLY:

Problem 32:

To describe an equilateral triangle about a given circle (fig. 3-34).

With centre O, draw the given circle. Draw a vertical radius OA.

Draw radii OB and OC with a 30°-60° set-square, such that $\angle AOB = \angle AOC = 120°$.

At A, B and C, draw tangents to the circle, i.e., a horizontal line EF through A, and lines FG and GE through B and C respectively with a 30°-60° set-square.

Then *EFG* is the required triangle.

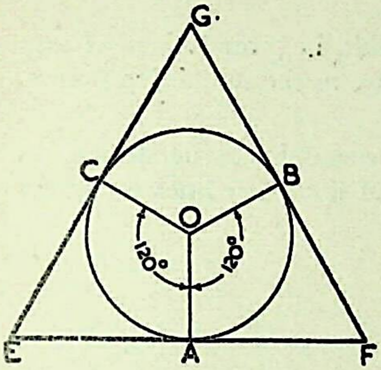


FIG. 3-34

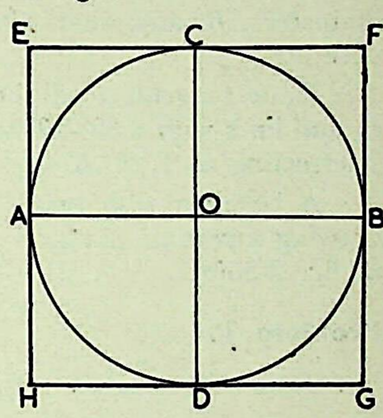


FIG. 3-35

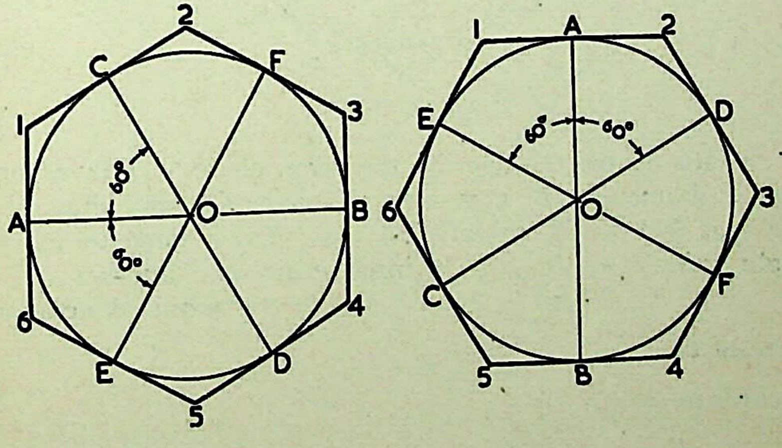
Problem 33:

To draw a square about a given circle (fig. 3-35).

With centre *O*, describe the given circle. Draw diameters *AB* and *CD* at right angles to each other as shown.

At *A* and *B*, draw vertical lines, and at *C* and *D*, draw horizontal lines intersecting at *E*, *F*, *G* and *H*.

EFGH is the required square.



(*a*) FIG. 3-36 (*b*)

Problem 34:

To describe a regular hexagon about a given circle [fig. 3-36(*a*)].

With centre O, draw the given circle. Draw horizontal diameter AB, and diameters CD and EF making 60° angle with AB.

Draw tangents at all the six ends, i.e., verticals at A and B, and lines with a 30°-60° set-square at the remaining points, intersecting at 1, 2..6.

A hexagon with two sides horizontal can be drawn by drawing a vertical diameter AB and the other lines as shown in fig. 3-36(*b*).

Problem 35:

To describe a regular octagon about a given circle (fig. 3-37).

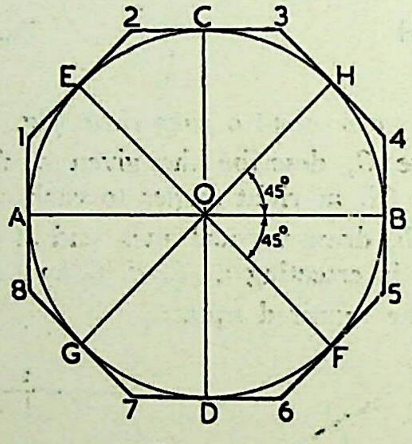


FIG. 3-37

With centre O, describe the given circle. Draw a horizontal diameter AB, a vertical diameter CD and diameters EF and GH at 45° to the first two. Draw tangents at the eight points A, B......H intersecting one another at 1, 2........8. Then 1, 2........8 is the required octagon.

TANGENTS:

Problem 36:

To draw a tangent to a given circle at any point on it (fig. 3-38).

With centre O, draw the given circle and mark a point P on it.

Draw a line joining O and P. Produce OP to Q so that $PQ = OP$.

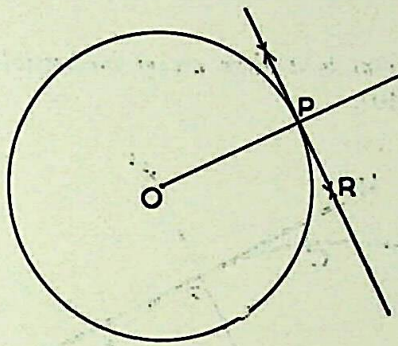


FIG. 3-38

With centres O and Q and with any convenient radius, draw arcs intersecting each other at R. Draw a line through P and R. Then this line is the required tangent.

Problem 37:

To draw a tangent to a given circle from any point outside it (fig. 3-39).

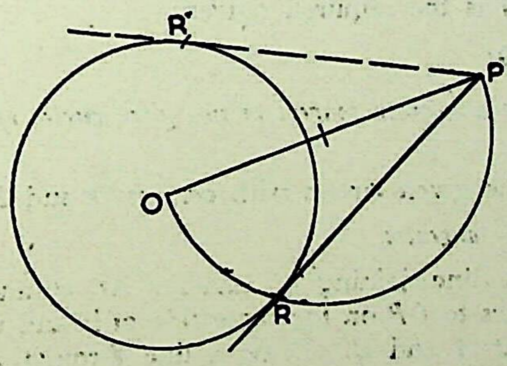


FIG. 3-39

With centre O, draw the given circle. Mark a point P outside it.

Draw a line joining O and P. With OP as diameter, draw a semi-circle cutting the given circle at R. Draw a line through P and R. Then this line is the required

tangent. The line through *P* and *R'* is the other tangent which can be drawn from the same point.

Problem 38:

To draw a tangent to a given arc of inaccessible centre at any point on it (fig. 3-40).

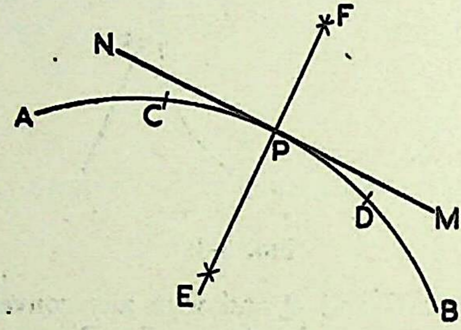


FIG. 3-40

Let *AB* be the given arc and *P* the point on it.

With centre *P* and any radius, draw arcs cutting the arc *AB* at *C* and *D*. Draw *EF*, the bisector of the arc *CD*. It will pass through *P*. Through *P*, draw *NM* perpendicular to *EF*. *NM* is the required tangent.

Problem 39:

To draw a common tangent to two given circles of equal radii (fig. 3-41).

Draw the given circles with centres *O* and *P*.

(a) *External tangents:*

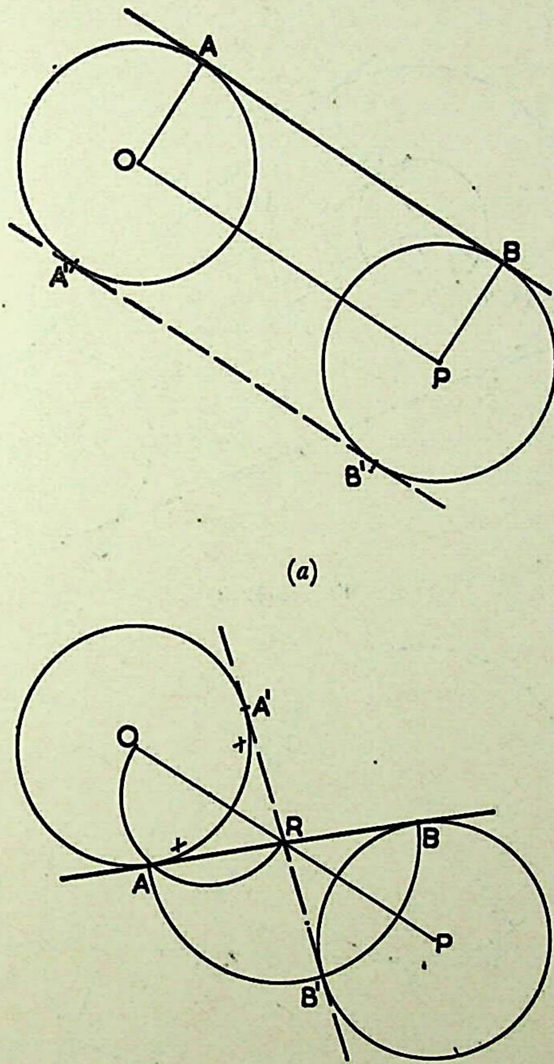
Draw a line joining *O* and *P*. At *O* and *P*, erect perpendiculars to *OP* on the same side of it and intersecting the circles at *A* and *B*. Draw a line through *A* and *B*. This line is the required tangent.

A'B' is the other tangent.

(b) *Internal tangents:*

Draw a line joining *O* and *P*. Bisect *OP* in *R*. Draw a semi-circle with *OR* as diameter to cut the circle at *A*. With centre *R* and radius *RA*, draw an arc to intersect

the other circle on the other side of OP at B. Draw a line through A and B. This line is the required tangent.



(*b*)

FIG. 3-41

The other tangent through A' and B' can also be similarly drawn.

Problem 40:

To draw a common tangent to two given circles of unequal radii (fig. 3-42).

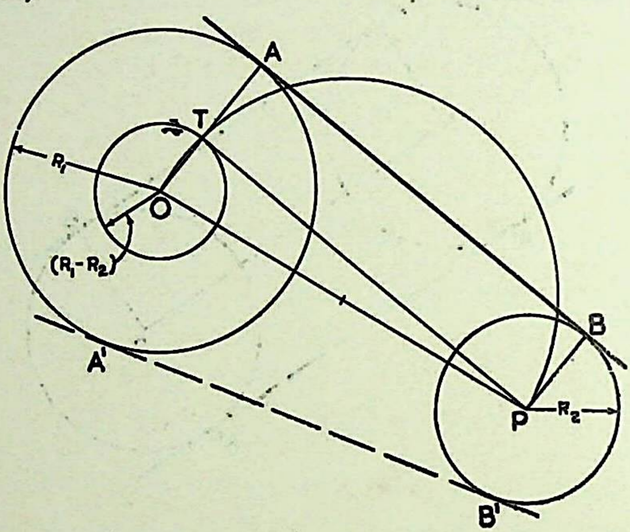


FIG. 3-42(*a*)

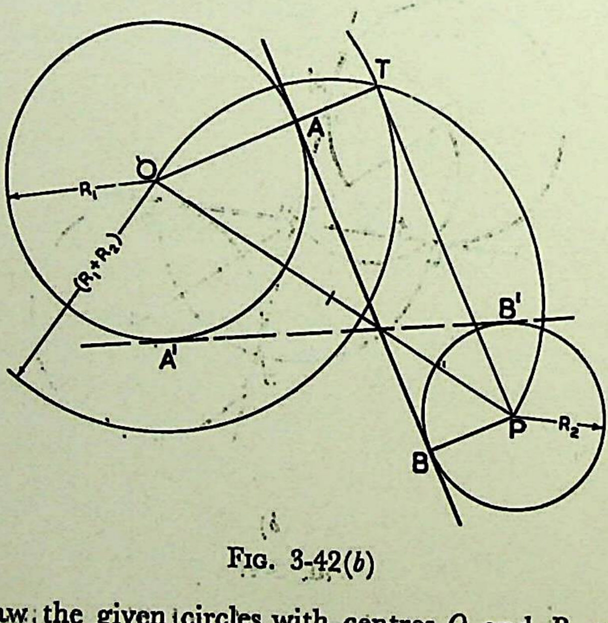


FIG. 3-42(*b*)

Draw the given circles with centres O and P, and radii R_1 and R_2 respectively, of which R_1 is greater than R_2.

(a) *External tangents* [fig. 3-42(*a*)]:

Draw a line joining centres O and P. With centre O and radius equal to $(R_1 - R_2)$, draw a circle.

From P, draw a tangent PT to this circle (Prob. 37). Draw a line OT and produce it, to cut the outer circle at A. Through P, draw a line PB parallel to OA, on the same side of OP and cutting the circle at B. Draw a line through A and B. Then this line is the required tangent.

The other similar tangent will pass through A' and B'.

(b) *Internal tangents* [fig. 3-42(*b*)]:

Draw a line joining the centres O and P. With centre O and radius equal to $(R_1 + R_2)$, draw a circle. From P, draw a line PT tangent to this circle. Draw a line OT cutting the circle at A. Through P, draw a line PB parallel to OA, on the other side of OP and cutting the circle at B. Draw a line through A and B. Then this line is the required tangent. The second tangent will pass through A' and B'.

LENGTHS OF ARCS:

Problem 41:

To determine the length of a given arc (fig. 3-43).

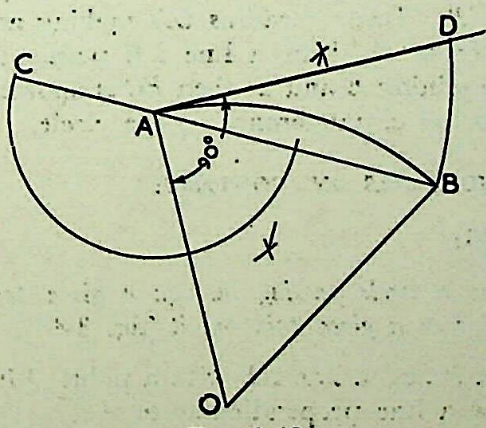


FIG. 3-43

Let AB be the given arc drawn with centre O.

At A, draw a tangent to the arc. Draw the chord AB and produce it beyond A to a point C such that $AC = \frac{1}{2} AB$. With centre C and radius equal to CB, describe an arc cutting the tangent at D. Then the length AD is approximately equal to the length of the arc AB. This method is satisfactory for arcs which subtend at the centre, angles smaller than 60°.

Problem 42:

To determine the length of the circumference of a given circle (fig. 3-44).

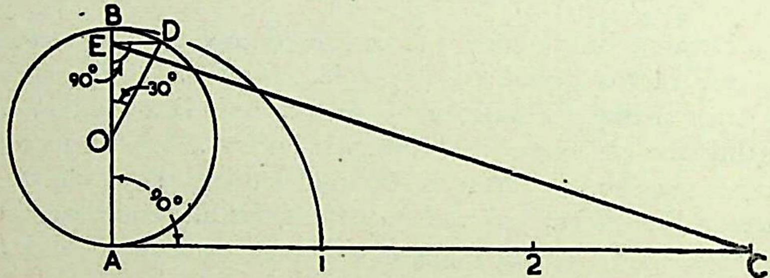


FIG. 3-44

Let the circle with centre O be given.

Draw a diameter AB. At A, draw a tangent AC equal to 3 times AB. Draw a radius OD making an angle of 30° with OB. From D, draw a line DE perpendicular to OB. Draw a line joining E and C; then EC is approximately equal in length to the circumference of the circle.

CIRCLES AND LINES IN CONTACT:

Problem 43:

To draw a circle passing through a given point and tangent to a given line at a given point on it (fig. 3-45).

A point P and a line AB with a point Q in it are given. At Q, draw a line perpendicular to AB.

Draw a line joining P and Q. Draw a perpendicular bisector of PQ to intersect the perpendicular from Q at O.

With centre O and radius OP or OQ, draw the required circle.

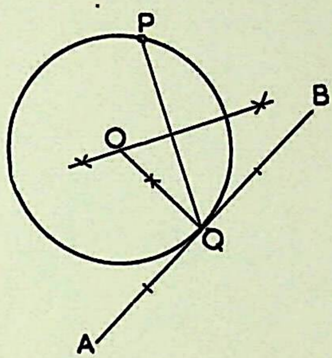


FIG. 3-45

Problem 44:

To draw a circle passing through a given point and touching a given circle at a given point on it (fig. 3-46).

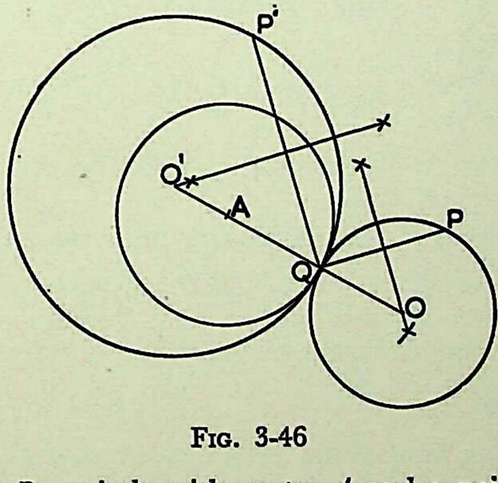


FIG. 3-46

A point P, a circle with centre A and a point Q on the circle are given.

Draw a line joining P and Q. Draw a perpendicular bisector of PQ, to intersect the line through A and Q at O. With centre O and radius OP, draw the required circle.

The required circle includes the given circle when the point is in a position such as P'.

Problem 45:

To draw a circle to touch a given line and a given circle at a given point on it (fig. 3-47).

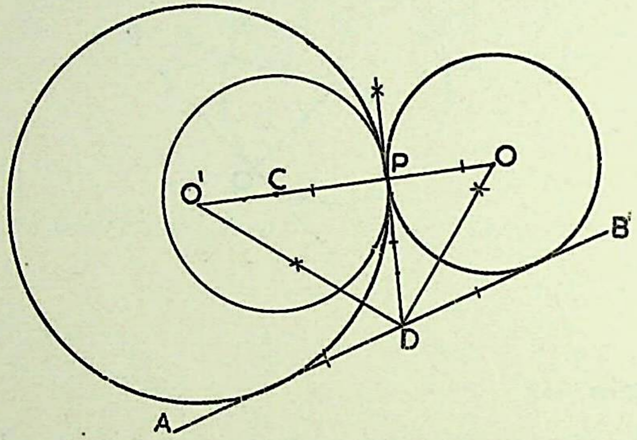


FIG. 3-47

A line AB, a circle with centre C and a point P on the circle are given.

From P, draw a tangent to the circle intersecting AB in D.

(a) Draw a bisector of $\angle PDB$, to intersect a line through C and P at O.

With centre O and radius OP, draw the required circle.

(b) Draw a bisector of $\angle PDA$ to meet a line through C and P at O'. Then O' is the centre of another circle which will include the given circle within it.

Problem 46:

To draw a circle to touch a given circle and a given line at a given point on it (fig. 3-48).

A circle with centre C and a line AB with a point P in it are given.

Through C, draw a line perpendicular to AB and cutting the circle in E or F.

(a) Draw a line joining P and F and intersecting the circle in G. At P, draw a perpendicular to AB intersecting

the line through C and G at O. With centre O and radius OP, draw the required circle.

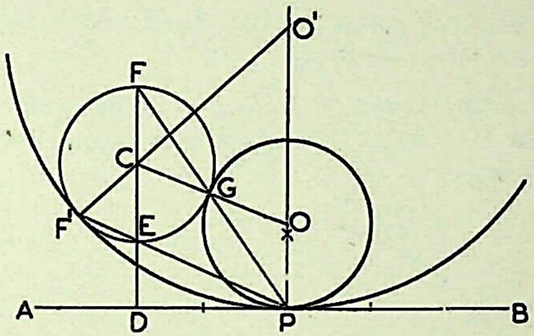


Fig. 3-48

(b) Draw a line through P and E and obtain centre O' for another circle in the same manner. It will include the given circle within it.

Problem 47:

To draw a circle touching two given circles, one of them at a given point on it [figs. 3-49(*a*) and 3-49(*b*)].

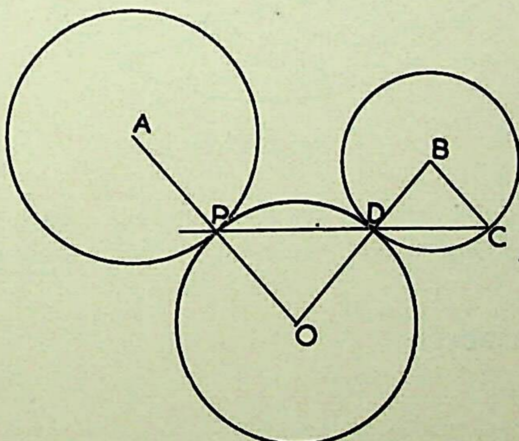


Fig. 3-49(*a*)

Circles with centres A and B, and a point P on the circle A are given [fig. 3-49(*a*)].

Draw a line joining A and P. Through B, draw a line parallel to AP and intersecting the circle in C.

Draw a line PC and produce it (if necessary) to cut the circle (with centre B) in D.

Draw a line through D and B to intersect AP or AP-produced, at O. With centre O and radius OP, draw the required circle.

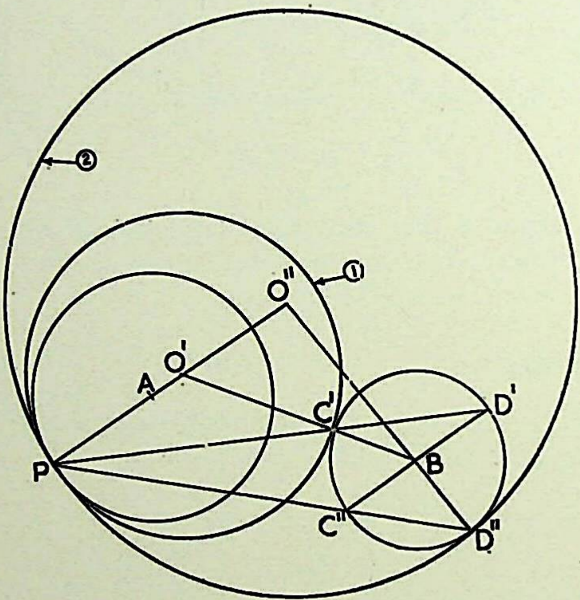


FIG. 3-49(b)

Fig. 3-49(b) shows circle-(1) which includes one of the given circles, and circle-(2) which includes both of them.

INSCRIBED CIRCLES:

Problem 48:

To inscribe a circle in a given triangle (fig. 3-50).

Let ABC be the triangle. Bisect any two angles by lines intersecting each other at O. Draw a perpendicular from O to any one side of the triangle, meeting it at P.

With centre O and radius OP, describe the required circle.

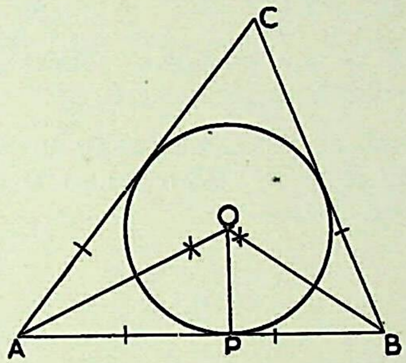


FIG. 3-50

Problem 49:

To draw a circle touching three lines inclined to each other but not forming a triangle (fig. 3-51).

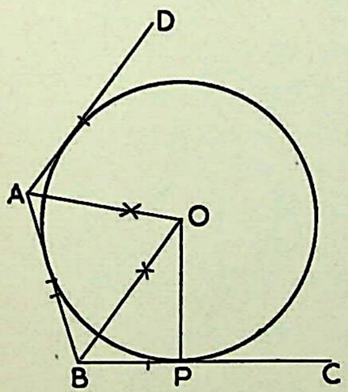


FIG. 3-51

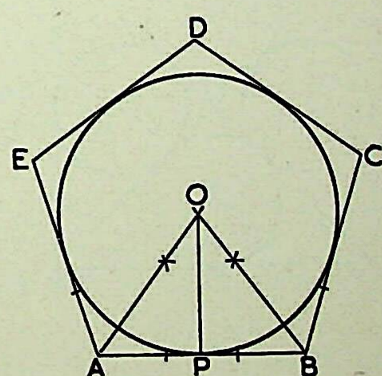


FIG. 3-52

Let AB, BC and AD be the given lines.

Draw bisectors of the two angles intersecting each other at O.

From O, draw a perpendicular to any one line intersecting it at P.

With centre O and radius OP, draw the required circle.

Problem 50:

To inscribe a circle in a regular polygon of any number of sides, say a pentagon (fig. 3-52).

Let *ABCDE* be the pentagon. Bisect any two angles by lines intersecting each other at *O*.

From *O*, draw a perpendicular to any one side of the pentagon cutting it at *P*. With centre *O* and radius *OP*, draw the required circle.

Problem 51:

To draw in a regular polygon, the same number of equal circles as the sides of the polygon, each circle touching one side of the polygon and two of the other circles (fig. 3-53).

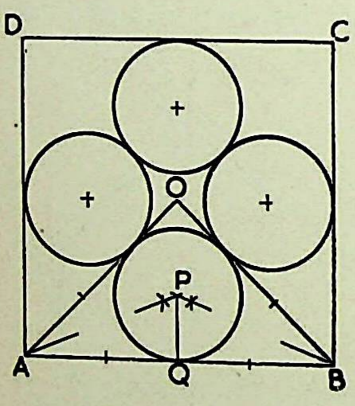


FIG. 3-53

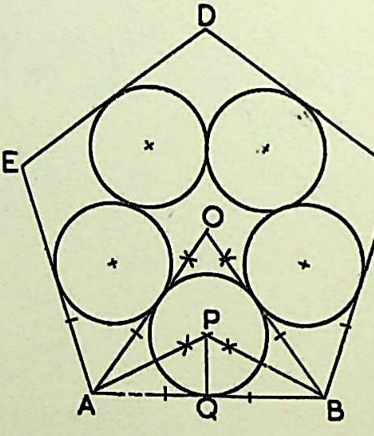


FIG. 3-54

Let *ABCD* be the given square. Draw bisectors of all the angles of the square. They will meet at *O*, thus dividing the square into four equal triangles.

In each triangle inscribe a circle (Prob. 48). Each circle will touch a side of the square and two other circles as required.

Fig. 3-54 shows five equal circles inscribed in a regular pentagon in a similar manner.

Problem 52:

To draw in a regular polygon, the same number of equal circles as the sides of the polygon, each circle touching two adjacent sides of the polygon and two of the other circles (fig. 3-55).

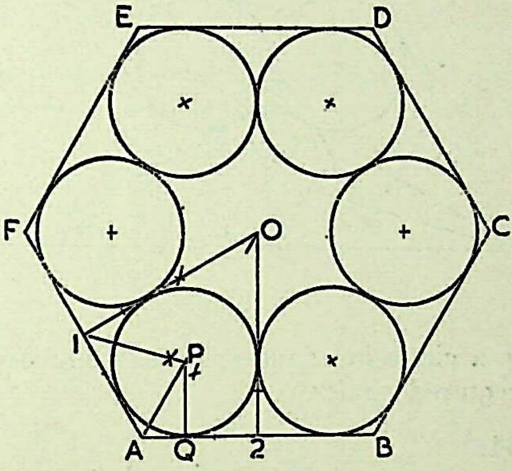


FIG. 3-55

Let *ABCDEF* be the given hexagon.

Draw the perpendicular bisectors of all the sides of the hexagon. They will meet at *O* and will divide the hexagon into six equal quadrilaterals.

Inscribe a circle in each quadrilateral as shown in case of *A* 1 *O* 2 and as explained below.

Bisect any two adjacent angles with bisectors intersecting at *P*.

From *P*, draw a perpendicular to any one side of the quadrilateral, meeting it at *Q*. With centre *P* and radius *PQ*, draw one of the required circles.

Draw other circles in the same manner.

Problem 53:

To draw in a given regular hexagon, three equal circles, each touching one side and two other circles (fig. 3-56).

Draw the given hexagon.

Draw perpendicular bisectors of its two alternate sides, to intersect each other at *O* and to meet the middle side produced on both sides at 1 and 2.

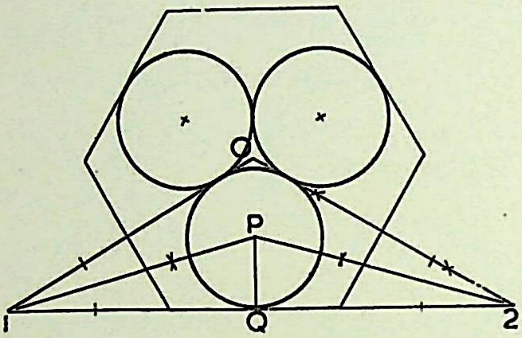


FIG. 3-56

Inscribe a circle in triangle *O* 1 2. Similarly, draw the other two required circles.

Problem 54:

To draw in a given circle, any number of equal circles, say four, each touching the given circle and two of the other circles (fig. 3-57).

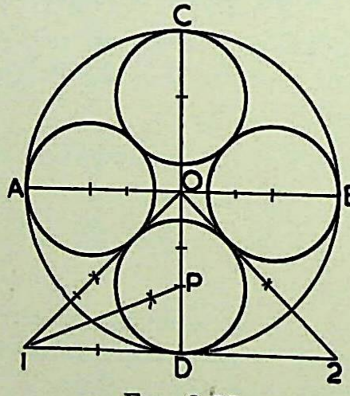


FIG. 3-57

Divide the given circle into four equal parts by diameters *AB* and *CD*.

Draw a tangent to the circle at *D*. Draw lines bisecting $\angle AOD$ and $\angle BOD$ and meeting the tangent at 1 and 2. Inscribe a circle in the triangle *O* 1 2.

Draw the other circles in the same manner. The centres for the remaining circles may also be determined by drawing a circle with centre O and radius OP to cut the diameters at the required points.

Problem 55:

To draw outside a given regular polygon, the same number of equal circles as the sides of the polygon, each circle touching one side and two of the other circles (fig. 3-58).

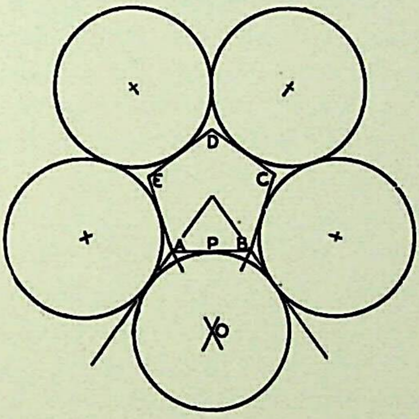


FIG. 3-58

Let $ABCDE$ be the given pentagon. Draw bisectors of two adjacent angles, say $\angle A$ and $\angle B$ and produce them outside the pentagon.

Draw a circle touching the extended bisectors and the side AB (Prob. 49). Obtain the other four required circles in the same manner.

Problem 56:

To draw outside a given circle any number of equal circles, say six, each touching the given circle and two other circles (fig. 3-59).

Draw the given circle and describe a regular hexagon about it.

Draw the required six equal circles outside the hexagon as shown in the previous problem.

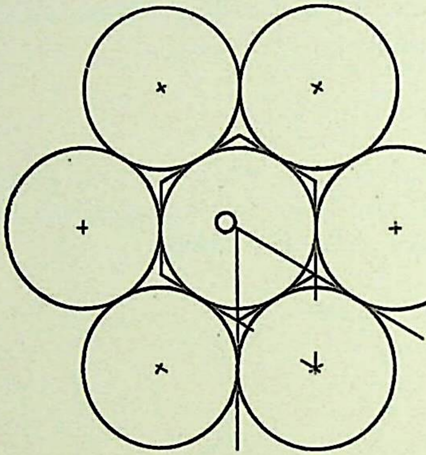


Fig. 3-59

EXERCISES III

(1) Draw a line 12·5 cm (5″) long and quadrisect it.

(2) Draw a line *AB* 8 cm (4″) long and divide it into five parts, one of them 2 cm (1″) long and the remaining each 1·5 cm ($\frac{3}{4}$″) long, by the method of bisection.

(3) With centre *O* and radius equal to 5 cm (2″), draw two arcs of any lengths on opposite sides of *O*. Bisect the two arcs and produce the bisectors till they meet.

(4) Draw a line *AB* 7·5 cm (3″) long. At *B*, erect a perpendicular *BC* 10 cm (4″) long. Draw a line joining *A* and *C*, and measure its length. Construct a square on each line as a side.

(5) Draw a line *PQ* 10 cm (4″) long. At any point *O* in it near its centre, erect a perpendicular *OA* 6·5 cm ($2\frac{1}{2}$″) long. Through *A*, draw a line parallel to *PQ*.

(6) Mark any point *O*. Draw a line *AB*, such that its shortest distance from *O* is 5 cm (2″).

(7) Construct a rectangle of sides 6·5 cm ($2\frac{1}{2}$″) and 4 cm ($1\frac{1}{2}$″).

(8) Draw a line *AB* 7·5 cm (3″) long. Mark a point *C*, 6·5 cm ($2\frac{1}{2}$″) from *A* and 9 cm ($3\frac{1}{2}$″) from *B*. Join *C* with *A* and *B*. Through the points *A*, *B* and *C*, draw lines (i) perpendicular and (ii) parallel to their opposite lines.

(9) Construct a square of 7·5 cm (3″) side. Draw the diagonals intersecting at *O*. From *O*, draw lines perpendicular to the sides of the square.

(10) Draw a circle of 5 cm (2″) radius. Divide it (i) into 8 equal parts by continued bisection and (ii) into 12 equal parts by bisection of a line and trisection of a right angle methods.

(11) Draw two lines *AB* and *AC* making an angle of 75°. Draw a circle of 2·5 cm (1″) radius touching them.

(12) Construct a right angle *PQR*. Describe a circle of 2 cm ($\frac{3}{4}$″) radius touching the sides *PQ* and *QR*.

(13) Draw a line *AB* of any length. Mark a point *O* at a distance of 2·5 cm (1″) from *AB*. With *O* as centre, draw a circle of 4 cm ($1\frac{1}{2}$″) diameter. Describe another circle (i) of 2 cm ($\frac{3}{4}$″) radius, touching the circle and *AB*; (ii) of 3·5 cm ($1\frac{3}{8}$″) radius, touching *AB* and the circle, and including the circle within it.

(14) Draw two circles of 2 cm ($\frac{3}{4}$″) and 3 cm ($1\frac{1}{4}$″) radii respectively with centres 6·5 cm ($2\frac{1}{2}$″) apart. (i) Describe a third circle of 5 cm (2″) radius touching the two circles and (*a*) outside them; (*b*) including 2 cm ($\frac{3}{4}$″) circle; (*c*) including 3 cm ($1\frac{1}{4}$″) circle. (ii) Describe a circle of 7·5 cm (3″) radius, touching both circles and including both of them within it.

(15) Mark points *A* and *B*, 5 cm (2″) apart. Mark a third point 7·5 cm (3″) from both *A* and *B*. Describe a circle passing through the three points.

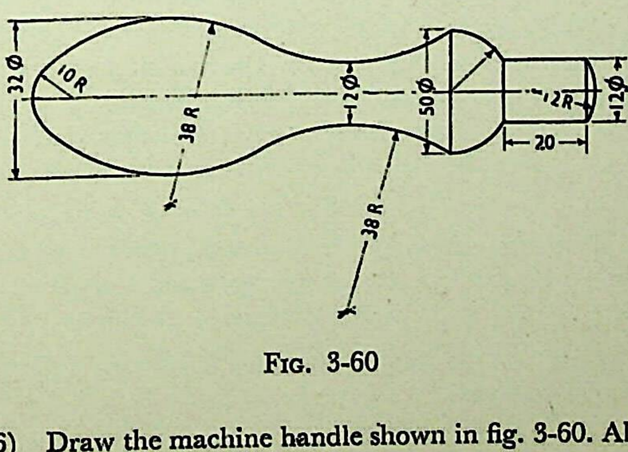


FIG. 3-60

(16) Draw the machine handle shown in fig. 3-60. All dimensions are in millimetres.

(17) The distance between the centres of two circles of 6·5 cm ($2\frac{1}{2}''$) and 9 cm ($3\frac{1}{2}''$) diameters, is 12 cm ($4\frac{3}{4}''$). Draw an internal and an external common tangent to the two circles.

(18) Draw a circle with centre *O* and radius equal to 3 cm ($1\frac{1}{4}''$). From a point *P*, 7·5 cm (3″) from *O*, draw a line joining *P* and *O*, and produce it to cut the circle at *Q*. From *P* and *Q*, draw tangents to the circle.

(19) Two shafts carry pulleys of 90 cm (3′) and 135 cm ($4\frac{1}{2}'$) diameters respectively. The distance between their centres is 270 cm (9′). Draw the arrangement showing the two pulleys connected by (i) direct belt (ii) crossed belt. Take 1 mm = 2 cm ($1'' = 1\frac{1}{2}'$).

(20) An arc *AB*, drawn with 5 cm (2″) radius subtends an angle of 45° at the centre. Determine approximately the length of *AB*.

(21) Determine the length of the circumference of a 7·5 cm (3″) diameter circle.

(22) A point *P* is 2·5 cm (1″) from a line *AB*. *Q* is a point in *AB* and is 5 cm (2″) from *P*. Draw a circle passing through *P* and touching *AB* at *Q*.

(23) The centre *O* of a circle of 3 cm ($1\frac{1}{4}''$) diameter is 2·5 cm (1″) from a line *AB*. Draw a circle (i) to touch the given circle and the line *AB* at a point *P*, 5 cm (2″) from *O*; (ii) to touch *AB* and the given circle at a point *Q*, 2 cm ($\frac{3}{4}''$) from *AB*.

(24) Two circles of 4 cm ($1\frac{1}{2}''$) and 5 cm (2″) diameters have their centres 6 cm ($2\frac{3}{8}''$) apart. Draw a circle to touch both circles and (i) to include the bigger circle, the point of contact on it being 7·5 cm (3″) from the centre of the other circle; (ii) to include both the circles, the point of contact being the same as in (i).

(25) Construct an equilateral triangle *ABC* of 4 cm ($1\frac{1}{2}''$) side. Construct a square, a regular pentagon and a regular hexagon on its sides *AB*, *BC* and *CA* respectively.

(26) Construct a regular pentagon of 3 cm ($1\frac{1}{4}''$) side by three different methods.

(27) On a line *AB* 4 cm ($1\frac{1}{2}''$) long, construct a regular heptagon by two different methods.

(28) Construct a regular octagon of 4 cm ($1\frac{1}{2}''$) side. Inscribe another octagon with its corners on the mid-points of the sides of the first octagon.

(29) Construct the following regular polygons in circles of 10 cm (4″) diameter, using a different method in each case: (i) Pentagon (ii) Heptagon.

(30) Draw the following regular figures, the distance between their opposite sides being 7·5 cm (3″): (i) Square (ii) Hexagon (iii) Octagon.

(31) Construct a regular octagon in a square of 7·5 cm (3″) side.

(32) Describe a regular pentagon about a circle of 10 cm (4″) diameter.

(33) Construct a triangle having sides 2·5 cm (1″), 3 cm ($1\frac{1}{4}''$) and 4 cm ($1\frac{1}{2}''$) long. Draw three circles, each touching one of the sides and the other two sides produced.

(34) Inscribe a circle in a triangle having sides, 5 cm (2″), 6·5 cm ($2\frac{1}{2}''$) and 7·5 cm (3″) long.

(35) Construct a regular heptagon of 2·5 cm (1″) side and inscribe a circle in it.

(36) Construct a regular hexagon of 4 cm ($1\frac{1}{2}''$) side and draw in it, six equal circles, each touching one side of the hexagon and two other circles.

(37) Construct a square of 5 cm (2″) side and draw in it, four equal circles, each touching two adjacent sides and two other circles.

(38) In a regular octagon of 4 cm ($1\frac{1}{2}''$) side, draw four equal circles, each touching one side of the octagon and two other circles.

(39) Draw a circle of 12·5 cm (5″) diameter and draw in it, five equal circles, each touching the given circle and two other circles.

(40) Construct a square of 2·5 cm (1″) side. Draw outside it four equal circles, each touching a side of the square and two other circles.

(41) Outside a circle of 2·5 cm (1″) diameter, draw five equal circles, each touching the given circle and two other circles.

CHAPTER IV

SCALES

Drawings of small objects can be prepared of the same size as the objects they represent. A 15 cm (6″) long pencil may be shown by a drawing of 15 cm (6″) length. Drawings drawn of the same size as the objects, are called full-size drawings. Ordinary full-size scales are used for such drawings.

Reducing and increasing scales: It may not be always possible to prepare full-size drawings. They are, therefore, drawn proportionately smaller or larger. When drawings are drawn smaller than the actual size of the objects (as in case of buildings, bridges, large machines etc.) the scale used is said to be a *Reducing scale*. Drawings of small machine parts, mathematical instruments, watches etc. are made larger than their real size. These are said to be drawn on an *Increasing scale*.

Representative fraction: The ratio of the drawing to the object is called the *Representative Fraction* (i.e., R.F.). When a 1″ long line in a drawing represents 1 yard length of the object, the R.F. is equal to $\frac{1 \text{ inch}}{1 \text{ yard}} = \frac{1}{36}$ and the scale of the drawing will be, '$\frac{1}{36}$ full size.' The *R*.F. of a drawing is greater than unity when it is drawn on increasing scale. For example, when a 2 mm long edge of an object is shown in a drawing by a line 1 cm long, the R.F. is $\frac{1 \text{ cm}}{2 \text{ mm}} = 5$. Such a drawing is said to be drawn on scale 'five times full-size'.

Types of scales: Scales may be classified into three main types, according to their use by different engineers:

(i) Mechanical Engineers generally use full-size, half full-size, one-fourth full-size and one-eighth full-size, i.e. $1'' = 1''$, $\frac{1}{2}'' = 1''$, $\frac{1}{4}'' = 1''$ and $\frac{1}{8}'' = 1''$ scales. Sometimes, $\frac{3}{4}$ and $\frac{3}{8}$ full-size scales are also used. For small machine

parts, double full-size ($2'' = 1''$) or three times full-size ($3'' = 1''$) scales are used. All these scales are 12″ long and each unit is sub-divided.

(ii) Architects have to use small R.F. for preparing drawings of buildings etc. The scales commonly used by them are $\frac{1}{4}'' = 1'$ and $\frac{1}{8}'' = 1'$. For details, they use $1'' = 1'$ and $\frac{1}{2}'' = 1'$ scales. Only the first unit of these scales is sub-divided.

(iii) Civil Engineers, who have to prepare survey maps, use very small R.F. for their drawings. Their scales are sub-divided throughout their lengths, in decimals, i.e., each inch is sub-divided into 10, 20, 30, 40, 50, 60 or 80 sub-divisions.

In metric system, the scales commonly available are of R.F. 1:1, 1:2, 1:5, 1:15, 1:25, 1:3000, 1:4000 etc. All these scales are 30 cm long and sub-divided throughout their lengths.

Scales on drawings: To prepare a drawing of any size or R.F., a scale is essential. When the required scale is not readily available, it may be constructed. In actual practice, the scale of a drawing is mentioned on the drawing sheet. It may be stated in words as 'Scale $1'' = 1'$' or 'Scale $\frac{1}{12}$ full-size' when the scale is such as is readily available. When an unusual scale is employed, it is constructed on the drawing sheet.

To construct a scale, the following information is essential:

(i) The R.F. of the scale. (ii) The units which it must represent; for example, feet and inches or miles and furlongs etc. (iii) The maximum length which it must show.

Generally, it is not possible to draw as long a scale as to measure the longest length on the drawing. The scale is therefore, made 15 cm (6″) to 30 cm (12″) in length, longer lengths being measured by marking them off in parts.

Plain scales: A plain scale consists of a line divided into suitable number of equal parts or units, the first of which is sub-divided into smaller parts. Plain scales represent either two units or a unit and its fraction.

In every scale,

(i) The zero should be placed at the end of the first main division, i.e., between the unit and its sub-divisions.

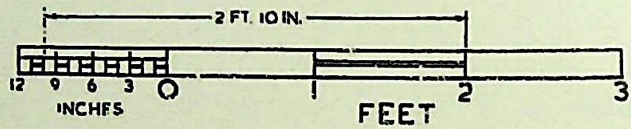
(ii) From the zero mark, the units should be numbered to the right and its sub-divisions to the left.

(iii) The names of the units and the sub-divisions should be stated clearly below or at the respective ends.

(iv) The name of the scale, (e.g. scale, 1 inch = 1 foot) or its R.F. should be mentioned below the scale.

Problem 1:

To construct a scale of '$1\frac{1}{2}$ inches = 1 foot' to show inches and long enough to measure upto 4 feet (fig. 4-1).



R.F. = $\frac{1}{8}$

FIG. 4-1

Draw a line, ($1\frac{1}{2}'' \times 4$) or 6 inches long.

Divide it into four equal parts (Prob. 7, Chap. III) each part representing one foot.

Mark 0 at the end of the first division and 1, 2 and 3 feet at the ends of subsequent divisions to its right.

Divide the first division into 12 equal parts, each representing 1". Mark inches to the left of 0, as shown.

The divisions are distinguished clearly by showing the scale as a rectangle of small width instead of only as a line. The division lines representing feet are drawn throughout the width of the scale. The inch divisions are made conspicuous by varying the lengths of lines as shown. The unit divisions are distinguished by drawing thick lines in the centre of alternate divisions.

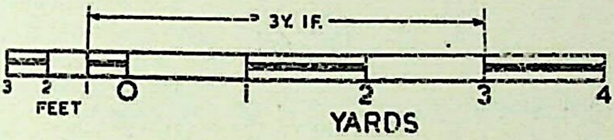
To set off any distance, say 2'-10", place one leg of the divider on 2' mark, and the other on 10" mark. The distance between the ends of the two legs will represent 2'-10".

Problem 2:

To construct a scale of R.F. $= \frac{1}{60}$ *to read yards and feet, and long enough to measure upto 5 yards* (fig. 4-2).

Length of the scale = R.F. × max. length

$$= \frac{1}{60} \times 5 = \frac{1}{12} \text{ yd} = 3 \text{ inches.}$$



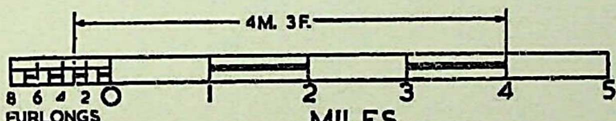
R. F. $= \frac{1}{60}$

FIG. 4-2

Draw a line 3 inches long and divide it into 5 equal parts. Divide the first part into 3 equal divisions. Mark the scale as shown.

Problem 3:

Construct a scale of R.F. $= \frac{1}{84480}$ *to show miles and furlongs and long enough to measure upto 6 miles* (fig. 4-3).



R.F. $= \frac{1}{84480}$

FIG. 4-3

$$\text{Length of the scale} = \frac{1}{84480} \times 6 = \frac{1}{14080} \text{ miles}$$

$$= 4\tfrac{1}{2}''.$$

Draw a line $4\frac{1}{2}''$ long and divide it into 6 equal parts. Divide the first part into 8 equal divisions and complete the scale as shown.

The distance 4m 3f is shown measured in the figure.

Problem 4:

To draw a scale of 1:60 to show metres and decimetres and long enough to measure upto 6 metres (fig. 4-4).

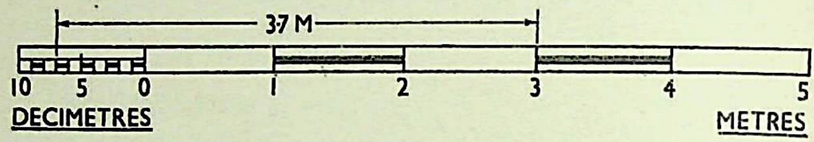


FIG. 4-4

$$\text{Length of the scale} = \frac{1}{60} \times 6 = \frac{1}{10} \text{ metre}$$
$$= 10 \text{ cm.}$$

Draw a line 10 cm long and divide it into 6 equal parts. Divide the first part into 10 equal divisions and complete the scale as shown. The length 3·7 metres is shown on the scale.

Diagonal scales: A diagonal scale is used when very minute distances such as $\frac{1''}{100}$ etc. are to be accurately measured or when measurements are required in three units; for example, yard, foot and inch, or foot, inch and $\frac{1}{10}$ inch.

Small divisions of short lines are obtained by the principle of diagonal division, explained below.

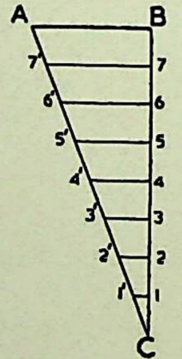


FIG. 4-5

To obtain divisions of a given short line *AB* in multiples of $\frac{1}{8}$ its length, e.g. $\frac{1}{8}$ *AB*, $\frac{1}{4}$ *AB*, $\frac{3}{4}$ *AB* etc. (fig. 4-5).

At one end, say B, draw a line perpendicular to AB and along it, step-off eight equal divisions of any length, starting from B and ending at C.

Number the division-points, 1, 2, 3 etc. as shown. Join AC. Through the points 1, 2 etc., draw lines parallel to AB and cutting AC at $1'$, $2'$ etc.

It is evident that triangles $C\,1\,1'$, $C\,2\,2'$ .... CBA are all similar.

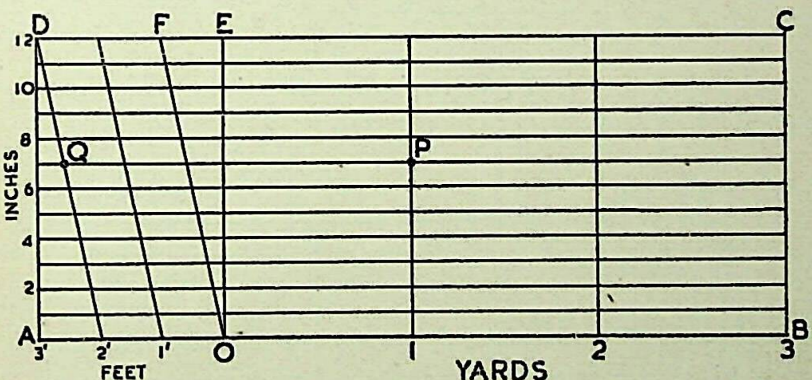
Since $C4 = \frac{1}{2}CB$, $4'4 = \frac{1}{2}AB$.

Similarly, $1'1 = \frac{1}{8}AB$, $2'2 = \frac{1}{4}AB$ etc.

Thus, each horizontal line becomes progressively shorter in length by $\frac{1}{8}AB$.

Problem 5:

To construct a diagonal scale of R.F. $= \frac{1}{32}$ *showing yards, feet and inches and to measure upto 4 yards* (fig. 4-6).



R.F. $= \frac{1}{32}$

FIG. 4-6

Length of the scale $= \frac{1}{32} \times 4 = \frac{1}{8}$yd $= 4\frac{1}{2}''$.

Draw a line AB $4\frac{1}{2}''$ long.

Divide it into 4 equal parts to show yards. Divide the first part $A0$ into 3 equal divisions showing feet.

At A, erect a perpendicular and step-off along it, 12 equal divisions of any length, ending at D. Complete the rectangle $ABCD$.

Erect perpendiculars at yard-divisions 0, 1 and 2. Draw horizontal lines through the division-points on AD. Join D with the end of the first small division-point from A along $A0$ viz. $2'$, and through the remaining points, draw lines parallel to $D2'$.

In $\triangle OEF$, EF represents 1 foot.

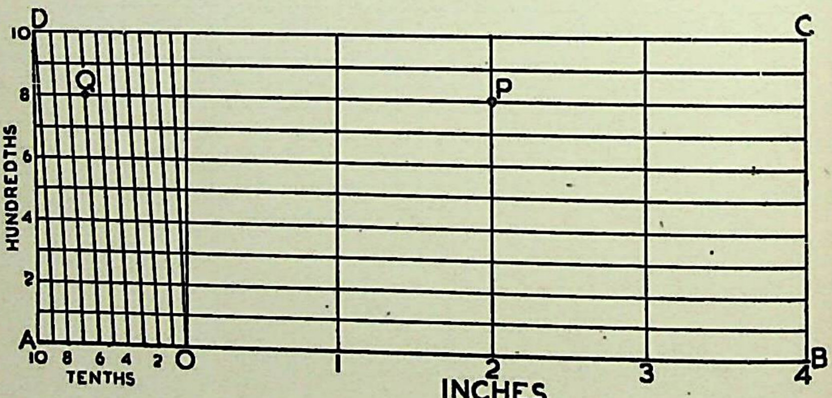
Each horizontal line below it, diminishes in length by $\frac{1}{12}EF$. Thus, the next line below EF is equal to $\frac{11}{12}$ EF and represents $\frac{11}{12} \times 1$ foot $= 11$ inches.

Any length between 1″ and 4 yards can be measured from this scale.

To show a distance of 1 yard, 2 feet and 7 inches, place one leg of the divider at P, where the horizontal through 7″ meets the vertical from 1 yard and the other leg at Q where the diagonal through $2'$ meets the same horizontal.

Problem 6:

To draw a scale of full-size, showing $\frac{1}{100}$ inch and to measure upto 5 inches (fig. 4-7).



R.F. = 1

Fig. 4-7

Draw a line AB 5″ long and divide it into five equal parts. Each part will show one inch.

Sub-divide the first part into 10 equal divisions. Each division will measure $\frac{1}{10}$ inch.

At A, draw a perpendicular to AB and on it, step-off ten equal divisions of any length, ending at D. Draw the rectangle $ABCD$ and complete the scale as in the previous problem.

The line PQ shows 2·68 inches.

Problem 7:

To construct a diagonal scale of R.F. $= \dfrac{1}{4000}$ *to show metres and long enough to measure upto 500 metres* (fig. 4-8).

$$\text{Length of the scale} = \frac{1}{4000} \times 500 = \tfrac{1}{8} \text{ metre}$$

$$= 12{\cdot}5 \text{ cm.}$$

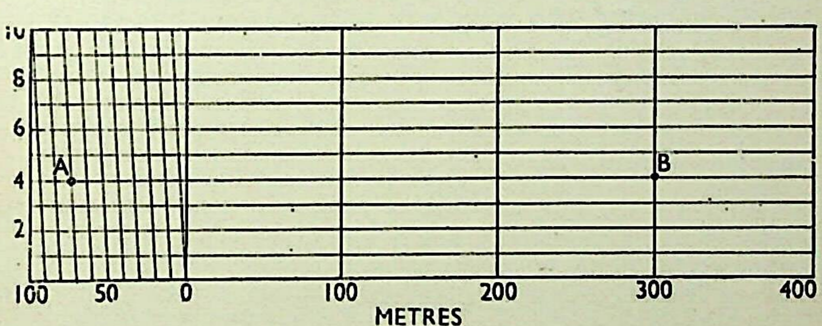


Fig. 4-8

Draw a line 12·5 cm long and divide it into 5 equal parts. Each part will show 100 metres.

Divide the first part into ten equal divisions. Each division will show 10 metres.

At the left-hand end, erect a perpendicular and on it, step-off 10 equal divisions of any length. Draw the rectangle and complete the scale as shown in Problem 5.

The distance between points A and B shows 374 metres.

Comparative scales: Scales having same representative fraction but graduated to read different units are called *comparative scales.* A drawing drawn with a scale reading inch units can be read in metric units by means of a metric comparative scale, constructed with the same representative fraction. Comparative scales may be plain scales or diagonal scales and may be constructed separately or one above the other.

Problem 8:

A drawing is drawn in inch units to a scale $\frac{3}{8}$ full size. Draw the scale showing $\frac{1}{8}$ inch divisions and to measure upto 15 inches. Construct a comparative scale showing centimetres and millimetres, and to read upto 40 centimetres.

(i) Inch scale [fig. 4-9(*i*)].

Length of the scale $= 15 \times \frac{3}{8} = \frac{45}{8} = 5\frac{5}{8}$ inches.

Construct the diagonal scale as shown in fig. 4-9(*i*).

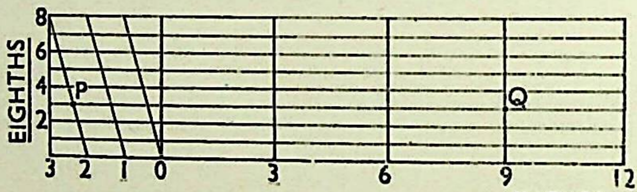


FIG. 4-9 (*i*)

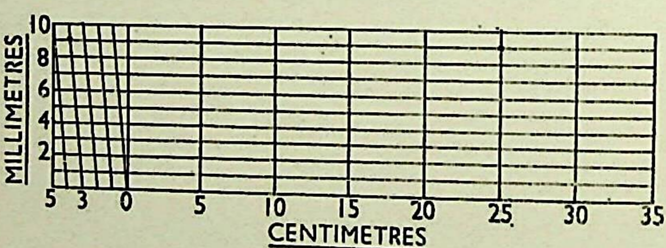


FIG. 4-9(*ii*)

(ii) Comparative scale [fig. 4-9(*ii*)]:

Length of the scale $= 40 \times \frac{3}{8} = 15$ cm.

Construct the diagonal scale as shown in fig. 4-9(*ii*).

The line *PQ* on the inch scale shows a length equal to $11\frac{3}{8}''$. Its equivalent, when measured on the comparative scale is 28·9 cm.

Problem 9:

Draw comparative scales of R.F. $= \dfrac{1}{485000}$ *to read upto 80 kilometres and 80 versts. 1 verst = 1·067 Km* (fig. 4-10).

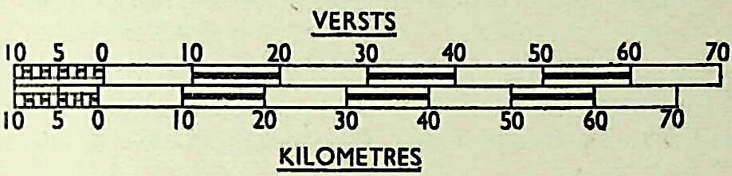


FIG. 4-10

$$\text{Length of kilometre scale} = \frac{1}{485000} \times 80 \times 1000 \times 100$$
$$= 16{\cdot}5 \text{ cm.}$$

$$\text{Length of verst scale} = \frac{1}{485000} \times 80 \times 1{\cdot}067 \times 1000 \times 100$$
$$= 17{\cdot}6 \text{ cm.}$$

Draw the two scales one above the other as shown in the figure.

Problem 10:

On a road map, a scale of miles is shown. On measuring from this scale, a distance of 25 miles is shown by a line 10 cm long. Construct this scale to read miles and to measure upto 40 miles. Construct a comparative scale, attached to this scale, to read kilometres upto 60 kilometres. 1 mile = 1·609 Km (fig. 4-11).

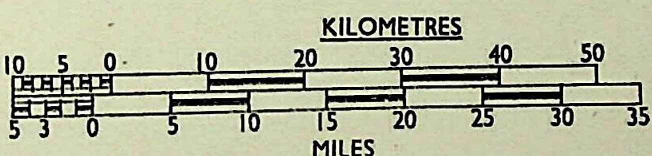


FIG. 4-11

(i) Scale of miles:

$$\text{Length of the scale} = \frac{10 \times 40}{25} = 16 \text{ cm.}$$

Draw a line 16 cm long and construct a plain scale to show miles.

(ii) Scale of kilometres:

$$\text{R.F.} = \frac{10}{25 \times 1{\cdot}609 \times 1000 \times 100} = \frac{1}{402250}$$

$$\text{Length of the scale} = \frac{1}{402250} \times 60 \times 1000 \times 100$$

$$= 14{\cdot}9 \text{ cm.}$$

Construct the plain scale 14·9 cm long, above the scale of miles and attached to it, to read kilometres.

Vernier scales: Vernier scales, like diagonal scales, are used to read to a very small unit with great accuracy. A vernier scale consists of two parts — a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions. As it would be difficult to sub-divide the minor divisions in the ordinary way, it is done with the help of the vernier. The graduations on the vernier are derived from those on the primary scale.

Fig. 4-12 shows a part of a plain scale in which the length *A*0 represents 10 cm. If we divide *A*0 into ten equal parts, each part will represent 1 cm. It would not be easy to divide each of these parts into ten equal divisions to get measurements in millimetres.

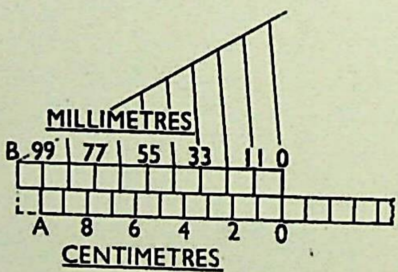


FIG. 4-12

Now, if we take a length *B*0 equal to 10 + 1 = 11 such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent $\frac{11}{10} = 1{\cdot}1$ cm or 11 mm. The difference between one part of *A*0 and one division of *B*0 will be equal to 1·1 − 1·0 = 0·1 cm or 1 mm. Similarly, the difference between two parts of each will be

0·2 cm or 2 mm. The upper scale $B0$ is the vernier. The combination of the plain scale and the vernier is the vernier scale.

In general, if a line representing n units is divided into n equal parts, each part will show $\frac{n}{n} = 1$ unit. But, if a line equal to $n + 1$ of these units is taken and then divided into n equal parts, each of these parts will be equal to $\frac{n+1}{n} = 1 + \frac{1}{n}$ units. The difference between one such part and one former part will be equal to $\frac{n+1}{n} - \frac{n}{n} = \frac{1}{n}$ unit. Similarly, the difference between two parts from each will be $\frac{2}{n}$ unit.

Problem 11:

Draw a vernier scale of R.F. = $\frac{1}{25}$ to read centimetres upto 4 metres and on it, show lengths representing 2·39 m and 0·91 m (fig. 4-13).

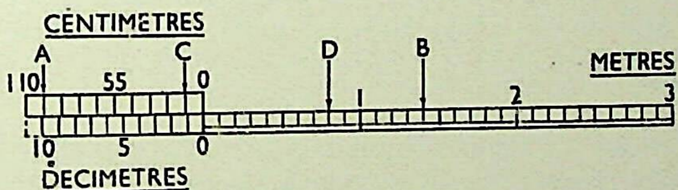


FIG. 4-13

Length of the scale = $\frac{1}{25} \times 4 \times 100 = 16$ cm.

Draw a line 16 cm long and divide it into 4 equal parts to show metres. Divide each of these parts into 10 equal parts to show decimetres.

To construct a vernier, take 11 parts of dm length and divide it into 10 equal parts. Each of these parts will show a length of 1·1 dm or 11 cm.

To measure a length representing 2·39 m, place one leg of the divider at A on 99 cm mark and the other leg at B on 1·4 m mark. The length AB will show 2·39 metres (0·99 + 1·4 = 2·39).

Similarly, the length CD shows 0·91 metre $(0{\cdot}8 + 0{\cdot}11 = 0{\cdot}91)$.

The necessity of dividing the plain scale into minor divisions throughout its length is quite evident from the above measurements.

Problem 12:

Construct a full-size vernier scale of inches and show on it lengths 3·67″, 1·54″ and 0·48″ (fig. 4-14).

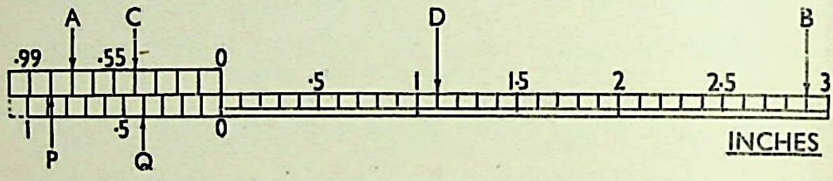


FIG. 4-14

Draw a plain full-size scale 4″ long and divide it fully to show 0·1″ lengths.

Construct a vernier of length equal to $10 + 1 = 11$ parts and divide it into 10 equal parts. Each of these parts will show $\dfrac{11 \times 0{\cdot}1}{10} = 0{\cdot}11''$.

The line AB shows a length of 3·67″ $(0{\cdot}77'' + 2{\cdot}9'' = 3{\cdot}67'')$. Similarly, lines CD and PQ show lengths of 1·54″ $(0{\cdot}44'' + 1{\cdot}1'' = 1{\cdot}54'')$ and 0·48″ $(0{\cdot}88'' - 0{\cdot}4'' = 0{\cdot}48'')$ respectively.

Problem 13:

Construct a vernier scale of R.F. $= \frac{1}{80}$ *to read inches and to measure upto 15 yards* (fig. 4-15).

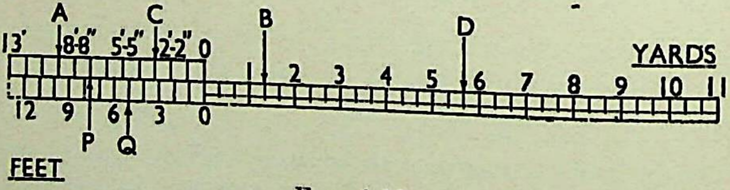


FIG. 4-15

Length of the scale $= \frac{1}{80} \times 15 = \frac{3}{16}$ yd $= 6\frac{3}{4}''$.

Draw the plain scale $6\frac{3}{4}''$ long and divide it fully to show yards and feet.

To construct the vernier, take a length of $12 + 1 = 13$ feet-divisions and divide it into 12 equal parts. Each part will represent $\frac{13}{12}$ ft or $1'$-$1''$.

Lines *AB*, *CD* and *PQ* show respectively lengths representing 4 yd 1 ft 9 in ($9'$-$9'' + 4'$), 6 yd 2 ft 3 in ($3'$-$3'' + 17'$) and 0 yd 2 ft 7 in ($7'$-$7'' - 5'$)

Scale of chords: The scale of chords is used to set out or measure angles when a protractor is not available. It is based on the lengths of chords of different angles measured on the same arc and is constructed as shown below.

Draw a line *AB* of any length (fig. 4-16). At *B*, erect a perpendicular. With *B* as centre, describe an arc *AC* cutting the perpendicular at a point *C*. Then, the arc *AC* (or the chord *AC*) subtends an angle of 90° at the centre *B*.

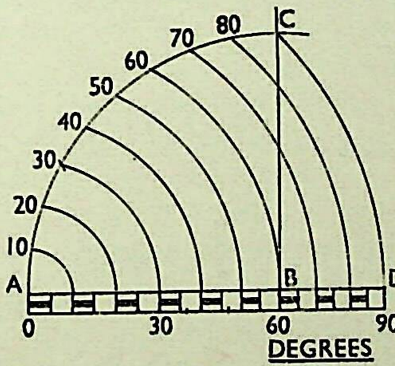


Fig. 4-16

Divide *AC* into nine equal parts. This may be done (i) by dividing the arc *AC* into three equal parts by drawing arcs with centres *A* and *C* and radius *AB*, and then (ii) by dividing each of these parts into three equal parts by trial and error method. Each of the nine equal parts subtends an angle of 10° at the centre *B*.

Transfer each division point from the arc to the straight line *AB*-produced, by taking *A* as centre and radii equal to

chords *A*-10, *A*-20 etc. Complete the scale by drawing a rectangle below *AD*. The divisions obtained are unequal, decreasing gradually from *A* to *D*. It is quite evident that the distance from *A* to a division point on the scale is equal to the length of the chord of the angle subtended by it at the centre *B*. It may be noted that the chord *A*-60 is equal to the radius *AB*.

The scale may be fully divided, i.e., each division divided into ten equal parts to show degrees. In the figure, degrees are shown in multiples of 5.

Problem 14:

Construct angles of 47° and 125° by means of the scale of chords (fig. 4-17).

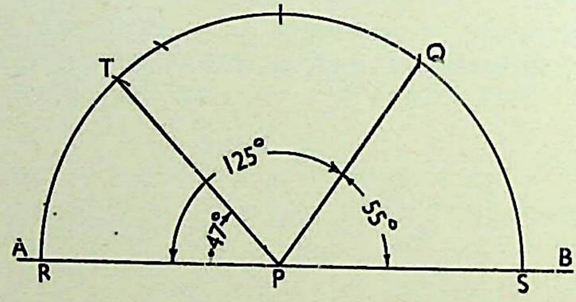


FIG. 4-17

Draw any line *AB*. With any point *P* on it as centre and radius equal to 0-60 (from the scale of chords), draw an arc cutting *AP* at a point *R*. With *R* as centre and radius equal to 0-47 (chord of 47°) cut the arc at a point *T*. Join *P* with *T*. Then $\angle RPT = 47°$.

As the scale of chords gives angles upto only 90°, angle of 125° may be set-off in two parts viz. 60° + 65° or 90° + 35° as shown in the figure.

It may also be obtained by setting-off a chord *SQ* of 55° (180° − 125°) from the right.

Problem 15:

To measure the given angle PQR by means of the scale of chords (fig. 4-18).

With Q as centre and radius equal to 0-60, draw an arc cutting PQ at A and RQ at B. Take the chord length AB and apply it to the scale of chords which shows the angle to be of 37°.

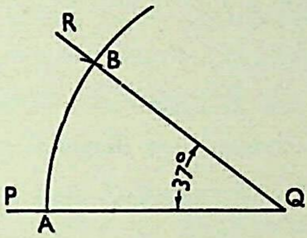


FIG. 4-18

EXERCISES IV

(1) Construct the following scales and show below each, its R.F. and the units which its divisions represent:

- (a) Scale of $1\frac{1}{4}'' = 1$ foot, to measure upto 5 feet and showing feet and inches.
- (b) Scale of $\frac{3}{4}'' = 1$ yard, to measure upto 10 yards and showing yards and feet.
- (c) Scale of $1\frac{3}{8}'' = 1$ mile, to measure upto 4 miles and showing miles and furlongs.

(2) Construct a scale of $1''=1$ foot to read upto 6 feet and show on it, 4′-7″ length.

(3) A line $1\frac{1}{4}''$ long represents a length of 16′-8″. Extend the line to measure upto 60 feet and show on it, units of 10 feet and one foot. Show on it, lengtns of 47 feet and 13 feet.

(4) Construct a scale of $\frac{3}{8}'' = 1''$ and with it, draw a triangle having sides $8\frac{5}{8}''$, $11\frac{1}{4}''$ and $15\frac{3}{8}''$ long.

(5) The R.F. of a scale showing miles, furlongs and chains is $\frac{1}{50688}$. Draw a scale to read upto 5 miles and show on it, the length representing 3m 5f 3ch.

(6) Draw a 4″ long diagonal scale of $1'' = 1''$ and show on it, lengths of 2·14″ and 3·79″.

(7) The distance between two points on a map is $5\frac{3}{4}''$. The points are actually 20 miles apart. Construct a diagonal scale of the map, showing miles and furlongs and to read upto 25 miles.

(8) Construct a scale of 1·5 cm = 1 dm to read upto 1 metre and show on it a length of 0·66 metre.

(9) Draw a diagonal scale of R.F. $= \frac{1}{6250}$ to read upto 1 kilometre and show on it a distance of 653 metres.

(10) Construct comparative diagonal scales of metres and yards having R.F. $= \frac{1}{2700}$, to show upto 400 metres. 1 metre = 1·0936 yards.

(11) Define simple and comparative scales. What is the R.F. of a scale which measures 2·5 inches to a mile? Draw a comparative scale of kilometres to read upto 10 Km. 1 Mile = 1·609 Km.

(12) On a map showing a scale of kilometres, 60 Km are found to equal 4·5 inches. What is the R.F.? Construct a comparative scale of English miles. 1 Km = 1093·6 yards.

(13) On a Russian map, a scale of versts is shown. On measuring it with a metric scale, 120 versts are found to measure 10 cm. Construct comparative scales for the two units to measure upto 150 versts and 150 Km respectively. 1 Verst = 1·067 Km.

(14) Prepare a scale of knots comparative to a scale of 1 cm = 5 Km. Assume suitable lengths. 1 Knot = 1·85 Km.

(15) Draw a full-size vernier scale to read $\frac{1}{8}''$ and $\frac{1}{64}''$ lengths and mark on it lengths of $5\frac{7}{32}''$, $2\frac{51}{64}''$ and $\frac{39}{64}''$.

(16) Construct a scale of R.F. $= \frac{1}{2 \cdot 5}$ to show decimetres and centimetres and by a vernier to read millimetres, to measure upto 4 decimetres.

(17) Construct a vernier scale to show yards, the R.F. being $\frac{1}{3300}$. Show the distance representing 2 furlongs 99 yards.

(18) Construct a scale of chords showing 5° divisions and with its aid set-off angles of 25°, 40°, 55° and 130°.

(19) Draw a triangle having sides 8 cm, 9 cm and 10 cm long respectively and measure its angles with the aid of a scale of chords.

CHAPTER V

LOCI OF POINTS

A locus (plural loci) is the path of a point which moves in space.

The locus of a point *P* moving in a plane about another point *O* in such a way that its distance from it is constant, is a circle of radius equal to *OP* (fig. 5-1).

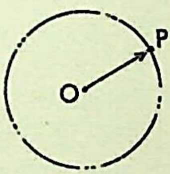


FIG. 5-1

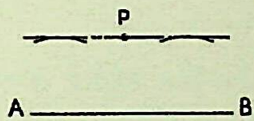


FIG. 5-2(*i*)

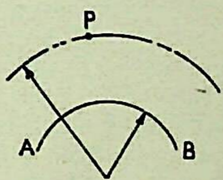


FIG. 5-2(*ii*)

The locus of a point *P* moving in a plane in such a way that its distance from a fixed line *AB* is constant, is a line through *P*, parallel to the fixed line [fig. 5-2(*i*)].

When the fixed line is an arc of a circle, the locus will be another arc drawn through *P* with the same centre [fig. 5-2(*ii*)].

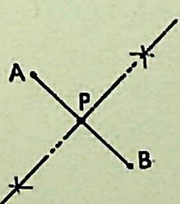


FIG. 5-3

The locus of a point equidistant from two fixed points *A* and *B* in the same plane, is the perpendicular bisector of the line joining the two points (fig. 5-3).

The locus of a point equidistant from two fixed non-parallel straight lines *AB* and *CD* will be a straight line bisecting the angle between them (fig. 5-4).

Problem 1:

To draw the locus of a point equidistant from a fixed straight line and a fixed point (fig. 5-5).

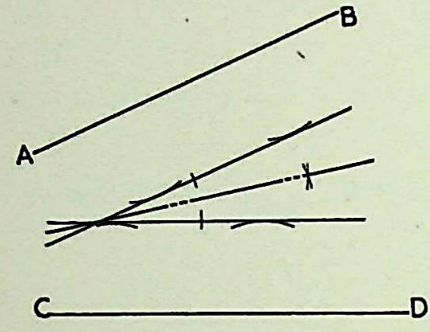


Fig. 5-4

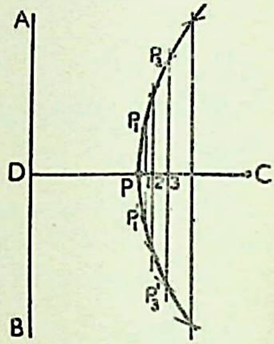


Fig. 5-5

Let AB be the given line and C the given point. From C, draw a line CD perpendicular to AB. The mid-point P of CD is equidistant from AB and C, and hence, it lies on the locus.

To obtain more points, mark a number of points 1, 2 etc. on PC and through them, draw lines parallel to AB.

With centre C and radius $D1$, draw an arc cutting the line through 1 at points P_1 and P_1'. Similarly, obtain more points and through them, draw a smooth curve which will be the required locus.

Problem 2:

To draw the locus of a point equidistant from a fixed circle and a fixed point (fig. 5-6).

A circle with centre O and a point C are given.

Draw a line joining O and C and cutting the circle at a point A. The mid-point P of the line AC will lie on the locus. Mark a number of points 1, 2 etc. on PC and through them, draw arcs with O as centre.

With centre C and radius equal to $A1$, draw an arc cutting the arc through 1 at points P_1 and P_1'.

Similarly, obtain more points and draw the required curve through them.

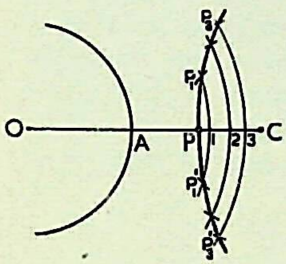


Fig. 5-6

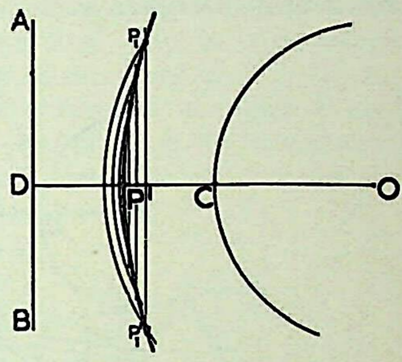


Fig. 5-7

Problem 3:

To draw the locus of a point equidistant from a fixed straight line and a fixed circle (fig. 5-7).

A line AB and a circle with centre O are given.

From O, draw a line OD perpendicular to AB and cutting the circle at C. Find the mid-point P of the line DC. It will lie on the curve. Mark any point 1 on PC and through it, draw a line parallel to AB. With centre O and radius equal to $(OC + D1)$, draw an arc cutting the line through 1 at P_1 and P_1'. Similarly, obtain more points and draw the curve through them.

Problem 4:

To draw the locus of a point equidistant from two given circles (fig. 5-8).

Circles with centres A and B are given.

Draw a line joining A and B and cutting the circles at points C and D.

Find the mid-point P of the line CD. Mark any point 1 on PD and through it, draw an arc with centre A. With centre B and radius equal to $(BD + C1)$, draw an arc, cutting the arc through 1, at points P_1 and P_1'. Similarly, locate more points and draw the curve through them.

The curves obtained in the above four problems are also the loci of centres of circles which will touch the given line, point or circles as the case may be.

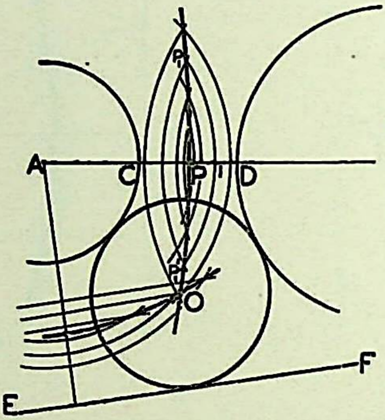


FIG. 5-8

Problem 5:

To draw a circle touching two given circles and a given straight line (fig. 5-8).

The circles with centres *A* and *B* and the line *EF* are given.

Draw the locus of a point equidistant from one of the circles, say the smaller circle, and the line *EF*.

The point of intersection of this curve, with the locus of the point equidistant from the two given circles, viz. *O* is the centre of the required circle.

Simple mechanisms: In simple mechanisms, it is often necessary to know the paths of points on their moving parts. These are determined by assuming a number of different positions of the moving parts and then locating the corresponding positions of the points.

The slider-crank mechanism shown diagrammatically in fig. 5-9 is one of the simplest mechanisms. The end *A* of the connecting rod *AB* is connected to the crank *OA* which rotates about *O*. The other end *B* is attached to a slider

which slides along a straight line. The locus of A will be a circle and that of the end B will be a straight line. The locus of any other point, say P, on the connecting rod will be neither a circle nor a straight line and may be determined (i) by assuming various positions of the crank-end A, (ii) by obtaining the corresponding positions of the end B and finally (iii) by locating the positions of P on the lines joining the first two positions. A smooth curve drawn through the points thus located will be the locus of P.

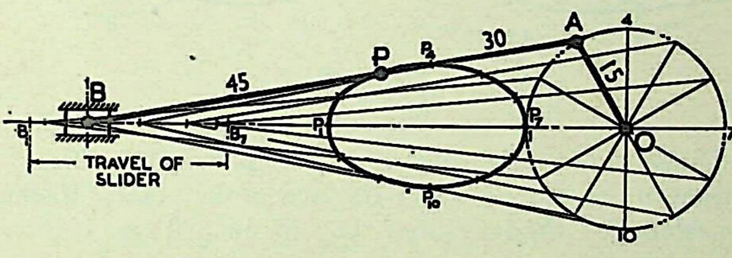


FIG. 5-9

Problem 6:

In a slider-crank mechanism, the connecting rod AB is 75 cm (2′-6″) long and the crank OA is 15 cm (6″) long. The end B moves along a straight line passing through O. Trace the locus of a point P, 30 cm (1′-0″) from A along the rod, for one revolution of OA (fig. 5-9).

Divide the circle (path of A) into 12 equal parts. With centre 1 and radius AB, cut the path of B at a point B_1. Draw a line joining 1 and B_1. Again, with centre 1 and radius PA, cut the line $1B_1$ at a point P_1. Obtain other points in similar manner and draw a smooth curve through these points. Then this curve is the locus of the point P.

Note that B_1B_7 is the travel of the slider and is equal to twice the length of the crank. But the movement of the slider is not uniform with that of the crank-end A.

Problem 7:

In the off-set crank mechanism shown in fig. 5-10, the slider-end B moves in guides along the line CD, 22·5 cm (9″) below the axis

O of the crank-shaft. Plot the locus of a point P, 45 cm (1′-6″) from A along AB and of a point Q along the extension of the rod, 30 cm (1′-0″) beyond B. Determine also the travel of the slider.

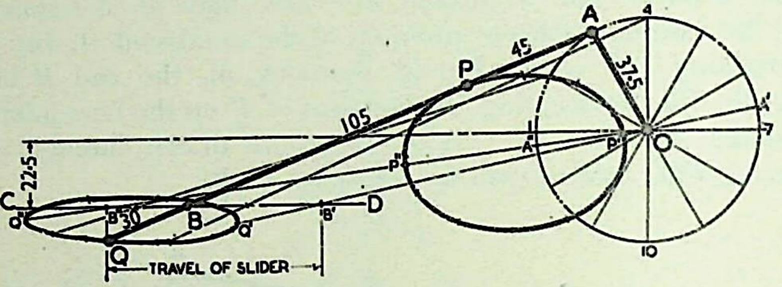


FIG. 5-10

Divide the circle into 12 equal parts and obtain the positions of the end B on its path, and of the point P as shown in problem 6. Produce lines $1B$, $2B$ etc. 30 cm (1′-0″) further and mark positions Q_1, Q_2 etc.

In addition to the above 12 points, it is also necessary to determine the limiting positions of the end B. They will occur when the connecting rod and the crank are in a straight line. These are found by drawing arcs with centre O and radii (i) $(AB - AO)$ and (ii) $(AB + AO)$, and cutting the path of B at points B' and B''. The travel of the slider is shown by the distance $B''B'$. The corresponding positions of P and Q for the limiting positions, viz. P', P'' and Q', Q'' are obtained as already explained.

Problem 8:

In the mechanism shown in fig. 5-11, the connecting rod is constrained to pass through the guide at D. Trace the locus of the end B and of a point P for a complete revolution of the crank. $AB = 180$ *cm (6′-0″),* $AO = 37{\cdot}5$ *cm (1′-3″) and* $AP = 75$ *cm (2′-6″).*

Divide the circle into 12 equal parts. Draw a line from the point 1, passing through D and obtain a point B_1 such that $1B_1 = AB$. Similarly, locate other positions of the end B and draw a curve through them. The locus of P is drawn in the same manner as explained in the previous problem.

The limiting positions B' and B'' (of the end B) are found by drawing a line through O and D and making OB' equal to $(AB - AO)$ and OB'' equal to $(AB + AO)$.

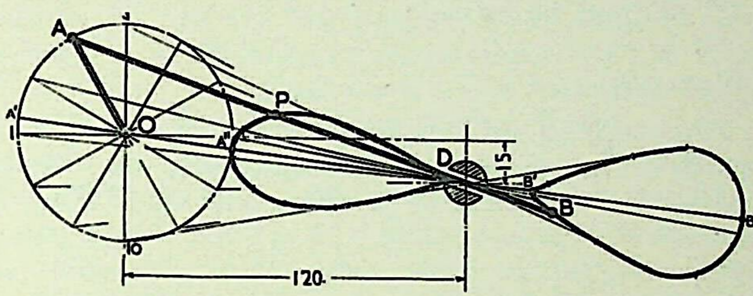


FIG. 5-11

Note that $A'B' = A''B'' = AB$.

Problem 9:

Two equal cranks AB and CD (fig. 5-12) *connected by the link BD, rotate in opposite directions. Draw the locus of a point P on BD and of Q along extension of the rod beyond B for one revolution of AB.* $AB = CD = 45$ *cm* $(1'\text{-}6'')$; $AC = BD = 150$ *cm* $(5'\text{-}0'')$; $PD = 30$ *cm* $(1'\text{-}0'')$; $BQ = 30$ *cm* $(1'\text{-}0'')$.

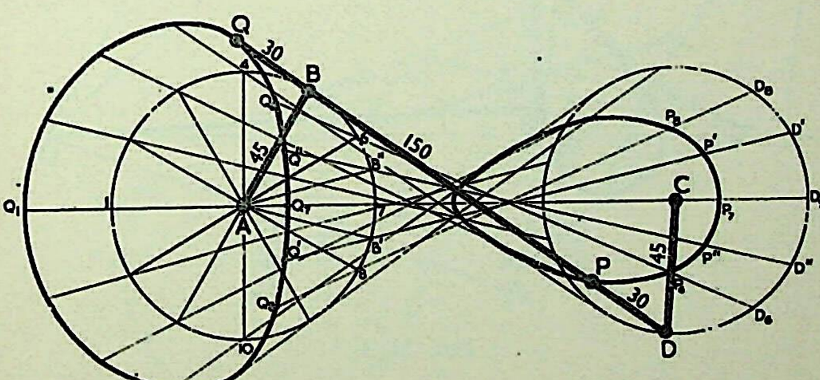


FIG. 5-12

Divide one of the circles (say path of B), into 12 equal parts. Determine the position of D on its path (the other circle), for every position of B and find the corresponding positions of P and Q for these positions as shown. It will

be found that there is a wide gap between the points for positions 6, 7 and 8 of the end B. A few more positions such as AB' and AB'' etc. may be taken and points P', P'', Q', Q'' etc. may be located.

Problem 10:

Two cranks AO and BQ (fig. 5-13) *oscillate about O and Q respectively. Trace the locus of the mid-point P of the connecting link AB.* $AO = 45$ *cm* (1′-6″); $BQ = 67{\cdot}5$ *cm* (2′-3″); $AB = 37{\cdot}5$ *cm* (1′-3″).

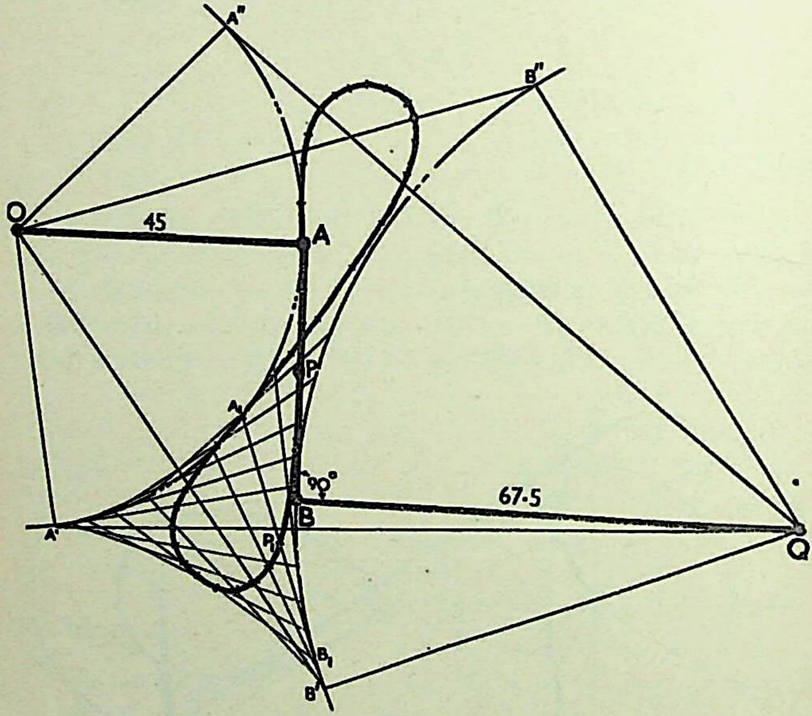


FIG. 5-13

The limiting positions of the ends A and B will be, when the link and each of the cranks are in straight lines and are found as shown below:

With centre Q and radius $(BQ + AB)$, draw arcs cutting the path of A at points A' and A''.

With centre O and radius $(AO + AB)$, draw arcs cutting the path of B at points B' and B''.

Assume A to be moving downwards. Then B will move towards B'. After B has reached B', if A moves further towards A', B will begin its return-motion. A will go upto A' and then move backwards. The movement will be repeated in a similar manner at B'' and A'' also.

To draw the locus of P, mark a number of points on the path of A. With centre A_1 and radius AB, draw an arc cutting the path of B at B_1. Mark point P_1 such that $A_1P_1 = AP$. Similarly, locate other points during the complete oscillation of the crank OA, from A' to A''. It is not necessary to draw the cranks in various positions.

Problem 11:

Two cranks AB and CD (fig. 5-14) *are connected by a link BD. AB rotates about A, while CD oscillates about C. Trace the locus of the mid-point of the link BD during one complete revolution of the crank AB. AB = 45 cm (1′-6″), CD = 75 cm (2′-6″), BD = 105 cm (3′-6″). Distance between A and C is equal to 90 cm (3′-0″).*

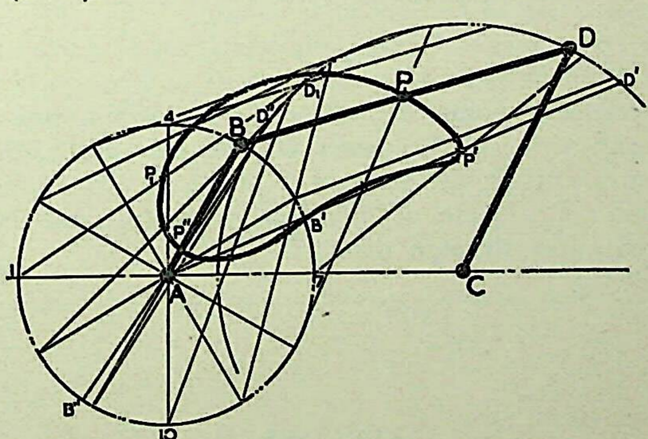


FIG. 5-14

Divide the circle into 12 equal parts. With centre 1 and radius BD, cut the path of D at D_1 Locate P_1, the mid-point of $1D_1$. Similarly, find other points. In addition,

find points P' and P'' for limiting positions, when $AD' = (BD + AB)$ and $AD'' = (BD - AB)$.

Problem 12:

The end A of a rod AB (fig. 5-15) *rotates about O, while the end B slides along a straight line. A crank CQ oscillates about Q. Draw the locus of the mid-point P of the connecting link CD for one revolution of the crank OA. AB = 150 cm (5′-0″), CD = 75 cm (2′-6″), OA = 45 cm (1′-6″) and CQ = 120 cm (4′-0″).*

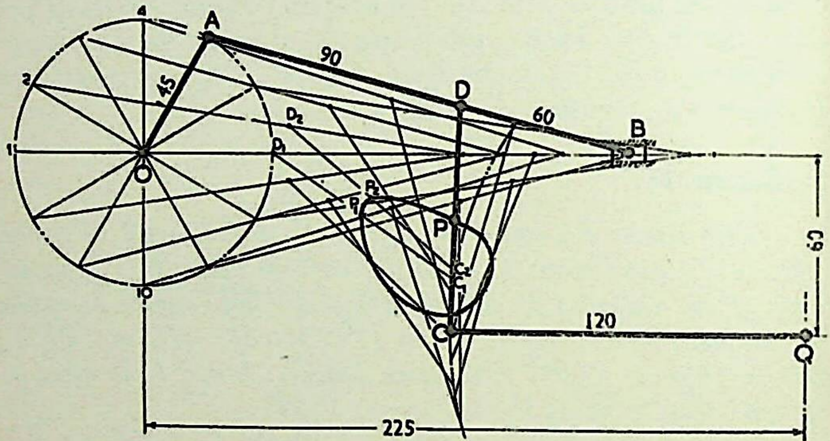


FIG. 5-15

Determine points D_1, D_2 etc. for various positions of the crank OA. With centres D_1, D_2 etc. and radius CD, draw arcs cutting the path of C at C_1, C_2 etc. Locate the mid-points P_1, P_2 etc. of lines D_1C_1, D_2C_2 etc. and draw the required curve through them.

EXERCISES V

(1) P, Q and R are the centres of three circles of diameters 7·5 cm (3″), 4·5 cm ($1\frac{3}{4}$″) and 3 cm ($1\frac{3}{16}$″) respectively. $PQ = 9·5$ cm ($3\frac{3}{4}$″), $QR = 5$ cm (2″) and $PR = 7·5$ cm (3″). Draw a circle touching the three circles.

(2) The end P of a line PQ, 10 cm (4") long (fig. 5-16), slides vertically downwards. The end Q moves along the line AB towards A and then back to B. Plot the locus of the point O on PQ and 4 cm ($1\frac{1}{2}$") from P.

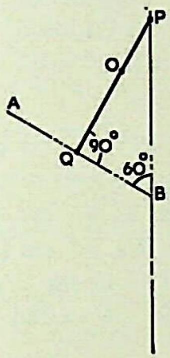


FIG. 5-16

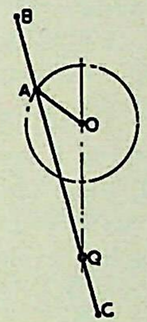


FIG. 5-17

(3) In a slider-crank mechanism, the crank OA is 45 cm (1'-6") long, and the connecting rod AB, 105 cm (3'-6") long. Plot the locus of (i) the mid-point P of AB, and (ii) a point 60 cm (2'-0") from A on BA extended, for one revolution of the crank.

(4) The rod BC (fig. 5-17) is attached to the crank OA at A. OA rotates about O and the rod BC is constrained to pass through the point Q. Draw the loci of the ends B and C, for one complete revolution of OA. $BC = 120$ cm (4'-0"), $OA = 22{\cdot}5$ cm (0'-9"), $AB = 30$ cm (1'-0") and $OQ = 52{\cdot}5$ cm (1'-9").

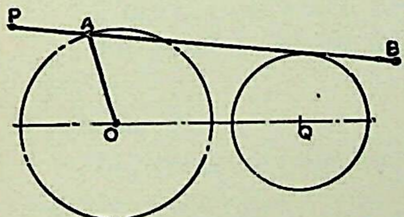


FIG. 5-18

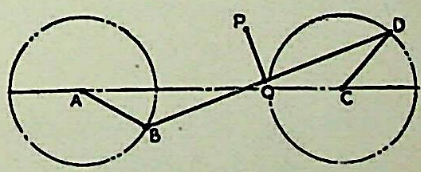


FIG. 5-19

(5) The crank OA (fig. 5-18) rotates about O and the connecting rod AB slides in the same plane, on the curved surface of a shaft (with centre Q) of 45 cm (1'-6") diameter. Trace the locus of (i) the end B and (ii) the point P beyond AB and 30 cm

(1′-0″) from A for one revolution of OA. OA = 37·5 cm (1′-3″), AB = 120 cm (4′-0″) and OQ = 70 cm (2′-4″).

(6) Two equal cranks AB and CD (fig. 5-19) rotate in opposite directions about A and C, and are connected by the rod BD. Plot the locus of the end P of the link PQ, attached at right angles to BD at its mid-point Q for one complete revolution of the cranks. AB = 30 cm (1′-0″); BD = AC = 105 cm (3′-6″); PQ = 22·5 cm (0′-9″).

(7) Two cranks AB and CD (fig. 5-20) are connected by a link BD. The end B moves round the circumference of the circle with centre A, while the end D oscillates on an arc about C as centre. Plot the locus of the point P on BD, 45 cm (1′-6″) from B, for one complete revolution of AB. AB = 45 cm (1′-6″), CD = 105 cm (3′-6″), BD = 135 cm (4′-6″) and AC = 165 cm (5′-6″).

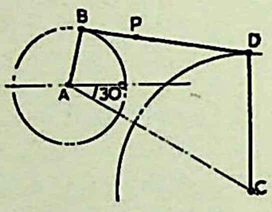


FIG. 5-20

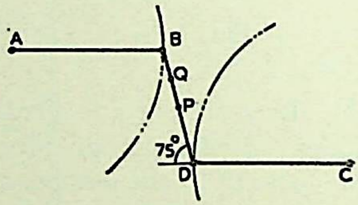


FIG. 5-21

(8) Two equal links AB and CD (fig. 5-21) connected by a rod BD, oscillate about their ends A and C. Plot the locus of (i) the mid-point P of BD and (ii) the point Q on BD. AB = CD = 120 cm (4′-0″); BD = 90 cm (3′-0″); BQ = 22·5 cm (0′-9″).

CHAPTER **VI**

CURVES USED IN ENGINEERING PRACTICE

Conic sections: The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called *conics*. For example:

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse*.

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola*.

(iii) When the section plane cuts both the parts of the double cone on one side of the axis, the section is a *hyperbola*.

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant.

The fixed point is called the *focus*, and the fixed line, the *directrix*.

The ratio $\dfrac{\text{distance from the focus}}{\text{distance from the directrix}}$ is called *eccentricity*.

It is always less than 1 for ellipse, equal to 1 for parabola and always greater than 1 for hyperbola.

The line passing through the focus and perpendicular to the directrix is called the *axis*.

The point at which the conic cuts its axis is called the *vertex*.

Problem 1:

To construct an ellipse when the distance of the focus from the directrix is equal to 5 cm (2") and eccentricity is $\frac{2}{3}$ (fig. 6-1).

Draw any line AB as directrix.

At any point C in it, draw the axis. Mark a focus F on the axis such that $CF = 5$ cm (2"). Divide CF into 5 equal divisions and mark the vertex V on the third division from C. Thus, $\dfrac{VF}{VC} = \dfrac{2}{3}$.

A scale may now be constructed on the axis (as explained below), which will directly give the distances in the required ratio.

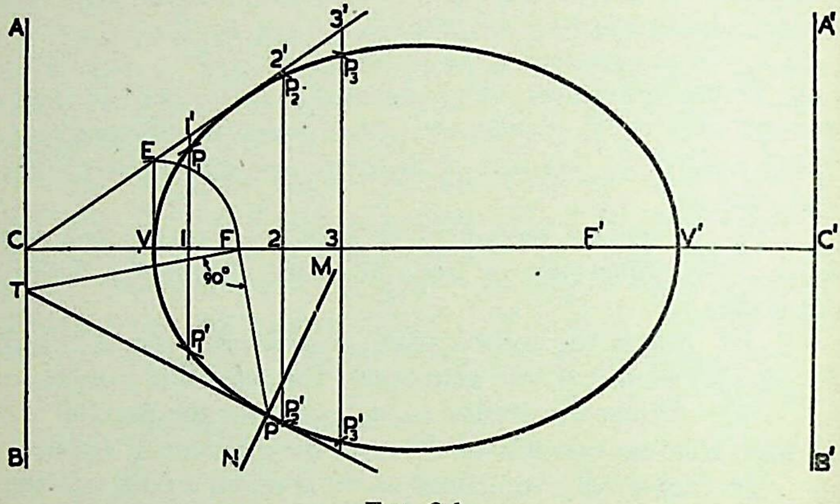


FIG. 6-1

At V, draw a perpendicular VE equal to VF.

Draw a line joining C and E.

Thus, in triangle CVE, $\frac{VE}{VC} = \frac{VF}{VC} = \frac{2}{3}$.

Mark any point 1 on the axis and through it, draw a perpendicular to meet CE-produced, at 1′.

With centre F and radius equal to 1-1′, draw arcs to intersect the perpendicular through 1 at points P_1 and P_1'.

These are the points on the ellipse, because the distance of P_1 from AB is equal to $C1$, P_1F = 1-1′ and $\frac{1\text{-}1'}{C1} = \frac{VF}{VC} = \frac{2}{3}$.

Similarly, mark points 2, 3 etc. on the axis and obtain points P_2 and P_2', P_3 and P_3' etc.

Draw the ellipse through these points. It is a closed curve and has two foci and two directrices.

Problem 2:

To construct a parabola, when the distance of the focus from the directrix is 5 cm (2″) (fig. 6-2).

Draw the directrix AB and the axis CD.
Mark focus F on CD, 5 cm (2″) from C.
Bisect CF in V the vertex (because eccentricity = 1).

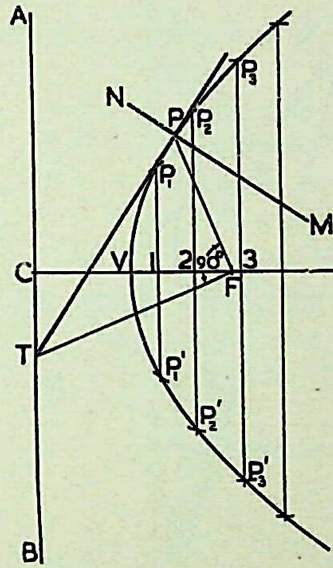


FIG. 6-2

Mark a number of points 1, 2, 3 etc. on the axis and through them, draw perpendiculars to it.

With centre F and radius equal to $C1$, draw arcs cutting the perpendicular through 1 at P_1 and P_1'.

Similarly, locate points P_2 and P_2', P_3 and P_3' etc. on both the sides of the axis.

Draw a smooth curve through these points; this curve is the required parabola. It is an open curve.

Problem 3:

To construct a hyperbola, when the distance of the focus from the directrix = 6·5 cm ($2\frac{1}{2}''$) and eccentricity = $\frac{3}{2}$ (fig. 6-3).

Draw the directrix AB and the axis CD.

Mark the focus F on CD and 6·5 cm ($2\frac{1}{2}''$) from C.

Divide CF into 5 equal divisions and mark V the vertex, on the second division from C. Thus, $\dfrac{VF}{VC} = \dfrac{3}{2}$.

To construct the scale for the ratio $\frac{3}{2}$, draw a line VE perpendicular to CD such that $VE = VF$. Join C and E.

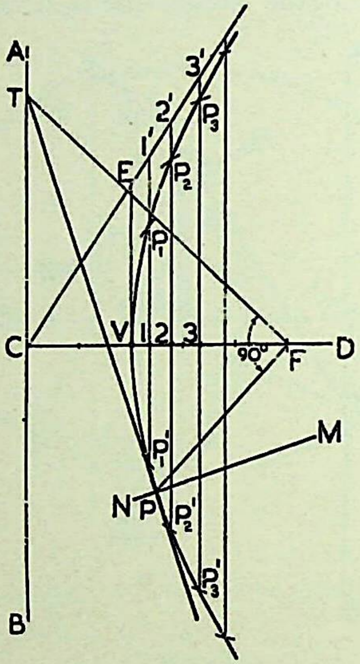


FIG. 6-3

Thus, in triangle CVE, $\frac{VE}{VC} = \frac{VF}{VC} = \frac{3}{2}$.

Mark any point 1 on the axis and through it, draw a perpendicular to meet CE-produced at 1′.

With centre F and radius equal to 1-1′, draw arcs intersecting the perpendicular through 1 at P_1 and P_1'.

Similarly, mark a number of points 2, 3 etc. and obtain points P_2 and P_2', P_3 and P_3' etc.

Draw the hyperbola through these points.

TANGENTS AND NORMALS TO CONICS:

The rule for drawing tangents and normals:

When a tangent at any point on the curve is produced to meet the directrix, the line joining the focus with this meeting point, will be at right angles to the line joining the focus with the point of contact.

The normal to the curve at any point is perpendicular to the tangent at that point.

Problem 4:

To draw a tangent at any point P on the conic (figs. 6-1, 6-2 and 6-3).

Join P and F. From F, draw a line perpendicular to PF to meet AB at T. Draw a line through T and P; this line is the tangent to the curve. Through P, draw a line NM perpendicular to TP; NM is the normal to the curve.

CONSTRUCTION OF CONICS BY OTHER METHODS:

Ellipse: Ellipse is also defined as a curve traced out by a point, moving in the same plane as and in such a way that the sum of its distances from two fixed points is always the same.

The fixed points are called the *foci* (plural of focus).

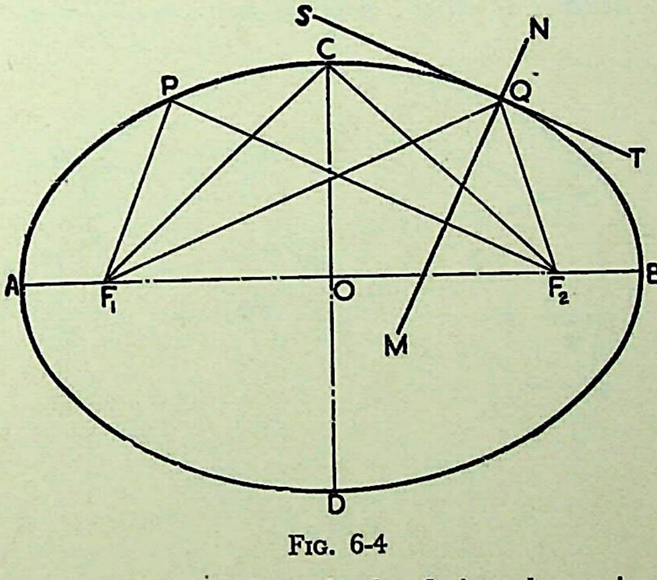


FIG. 6-4

The line passing through the foci and terminated by the curve, is called the *major axis*.

The line bisecting the major axis at right angles and terminated by the curve, is called the *minor* axis.

In fig. 6-4, AB is the major axis, CD the minor axis

and F_1 and F_2 are the foci. The foci are equidistant from the centre O.

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

But $(AF_1 + AF_2) = AB$

$\therefore \quad (PF_1 + PF_2) = AB$, the major axis.

Therefore, the sum of the distances of any point on the curve from the two foci, is equal to the major axis.

Again, $(CF_1 + CF_2) = AB$.

But $CF_1 = CF_2 \quad \therefore \; CF_1 = CF_2 = \frac{1}{2}AB$.

Hence, the distance of the ends of the minor axis from the foci is equal to half the major axis.

Problem 5:

To construct an ellipse, given the major and minor axes.

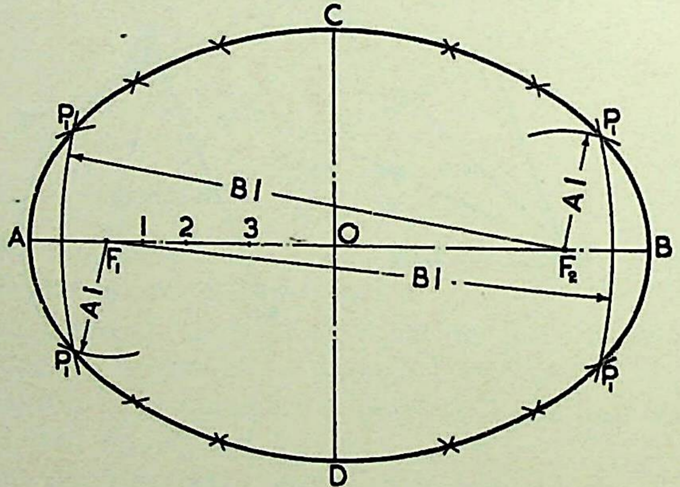


FIG. 6-5

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

Method (i): 'Arcs of circles' method (fig. 6-5).

Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O.

With centre C and radius equal to half AB (i.e. AO), draw arcs cutting AB at F_1 and F_2, the foci of the ellipse.

Mark a number of points 1, 2, 3 etc. on AB. With centres F_1 and F_2, and radius equal to $A1$, draw arcs on both sides of AB.

With same centres and radius equal to $B1$, draw arcs intersecting the previous arcs at four points marked P_1.

Similarly, with radii $A2$ and $B2$, $A3$ and $B3$ etc. obtain more points.

Draw a smooth curve through these points. This curve is the required ellipse.

Method (ii): 'Concentric circles' method (fig. 6-6).

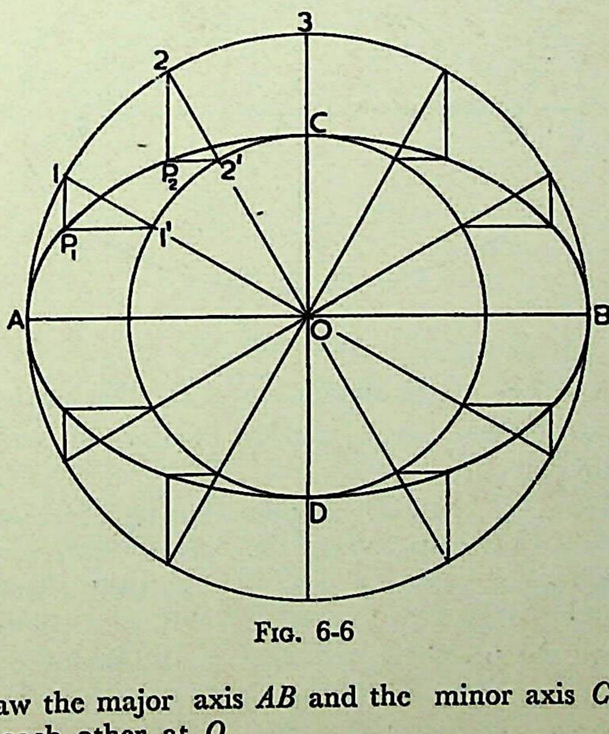


FIG. 6-6

Draw the major axis AB and the minor axis CD intersecting each other at O.

With centre O and diameters AB and CD respectively, draw two circles.

Divide the major-axis-circle into a number of equal divisions, say 12 and mark points 1, 2 etc. as shown.

Draw lines joining these points with the centre O and cutting the minor-axis-circle, at points $1'$, $2'$ etc. Through the point 1 on the major-axis-circle, draw a line parallel to CD, the minor axis. Through point $1'$ on the minor-axis-circle, draw a line parallel to AB, the major axis. The point P_1, where these two lines intersect is on the required ellipse.

Repeat the construction through all the points. Draw the ellipse through A, P_1, P_2.... etc.

Method (iii): 'Loop of the thread' method (fig. 6-7).

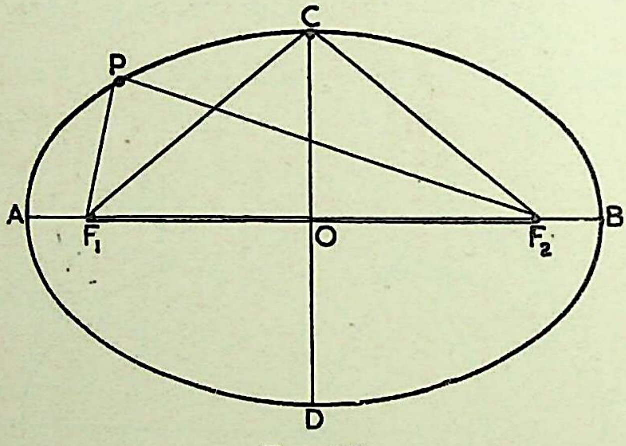


FIG. 6-7

This is a practical application of the first method.

Draw the two axes AB and CD intersecting at O. Locate the foci F_1 and F_2.

Insert a pin at each focus-point and tie a piece of thread in the form of a loop around the pins, in such a way that the pencil point when placed in the loop (keeping the thread tight), is just on the end C of the minor axis.

Move the pencil around the foci, maintaining an even tension in the thread throughout and obtain the ellipse.

It is evident that $PF_1 + PF_2 = CF_1 + CF_2$ etc.

Method (iv): 'Oblong' method [fig. 6-8(*i*)].

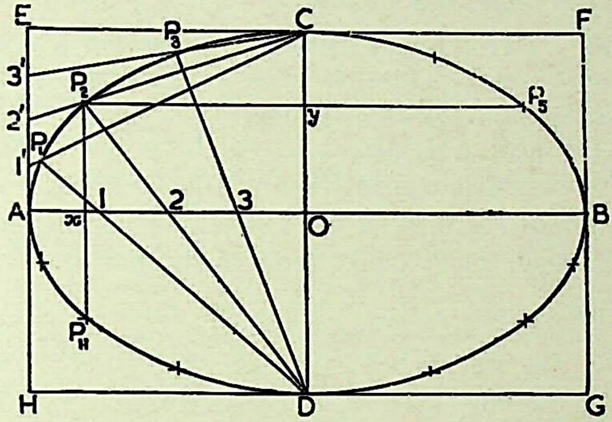


FIG. 6-8(*i*)

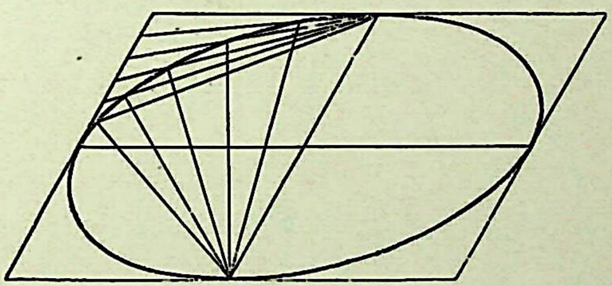


FIG. 6-8(*ii*)

Draw the two axes *AB* and *CD* intersecting at *O*. Construct the oblong *EFGH* having its sides equal to the two axes.

Divide the semi-major axis *AO* into a number of equal parts, say 4, and *AE* into the same number of equal parts, numbering them from *A* as shown.

Draw lines joining 1′, 2′ and 3′ with *C*. From *D*, draw lines through 1, 2 and 3 intersecting *C*1′, *C*2′ and *C*3′ at points P_1, P_2 and P_3 respectively.

Draw the curve through *A*, P_1...... *C*. It will be one quarter of the ellipse.

Complete the curve by the same construction in each of the three remaining quadrants.

As the curve is symmetrical about the two axes, points in the remaining quadrants may be located by drawing perpendiculars and horizontals from P_1, P_2 etc. and making each of them of equal length on both the sides of the two axes For example, $P_2x = xP_{11}$ and $P_2y = yP_5$.

An ellipse can be inscribed within a parallelogram by using the above method as shown in fig. 6-8(*ii*).

Method (v): '*Trammel*' *method* (fig. 6-9).

Draw the two axes *AB* and *CD* intersecting at *O*.

Along the edge of a strip of paper which may be used as a trammel, mark *PQ* equal to half the minor axis and *PR* equal to half the major axis.

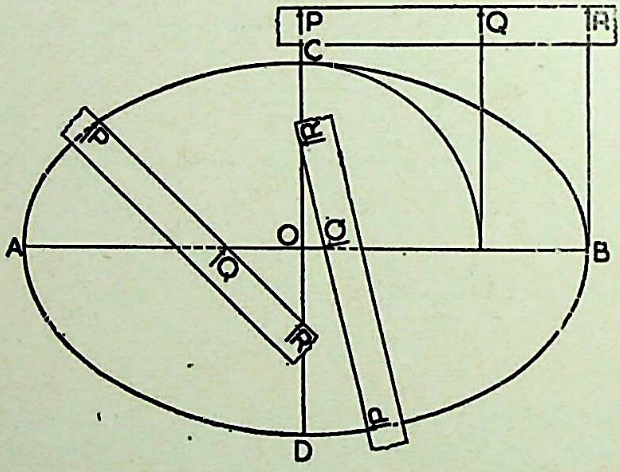


FIG. 6-9

Place the trammel so that *R* is on the minor axis *CD* and *Q* on the major axis *AB*. Then *P* will be on the required ellipse. By moving the trammel to new positions, always keeping *R* on *CD* and *Q* on *AB*, obtain other points.

Draw the ellipse through these points.

NORMAL AND TANGENT TO AN ELLIPSE:

The normal to an ellipse at any point on it bisects the angle made by lines joining that point with the foci.

The tangent to the ellipse at any point is perpendicular to the normal at that point.

Problem 6:

To draw a normal and a tangent to the ellipse, at a point Q on it (fig. 6-4).

Join Q with the foci F_1 and F_2. Draw a line NM bisecting $\angle F_1QF_2$; NM is the normal to the ellipse. Draw a line ST through Q and perpendicular to NM; ST is the tangent to the ellipse at the point Q.

CURVE PARALLEL TO AN ELLIPSE:

Problem 7:

To draw a curve parallel to an ellipse and at distance R from it (fig. 6-10).

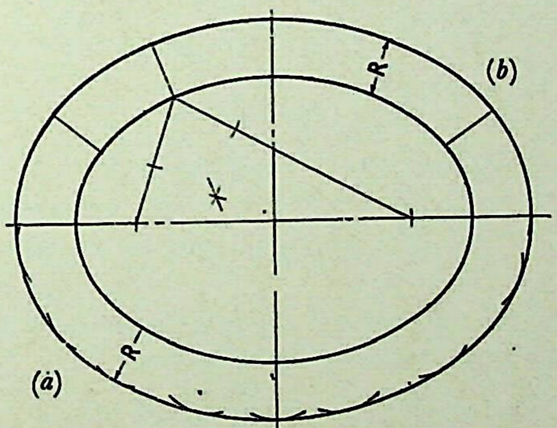


FIG. 6-10

This may be drawn by two methods: (a) A large number of arcs of radius, equal to the required distance R, with centres on the ellipse, may be described. The curve drawn touching these arcs will be parallel to the ellipse.

(b) It may also be obtained by drawing a number of normals to the ellipse, making them equal to the required distance R and then drawing a smooth curve through their ends.

PARABOLA:

Problem 8:

To construct a parabola given the base and the axis

I. Rectangle method [fig. 6-11(*i*)].

Draw the base AB. At its mid-point E, draw the axis EF at right angles to AB. Construct a rectangle $ABCD$, making side BC equal to EF.

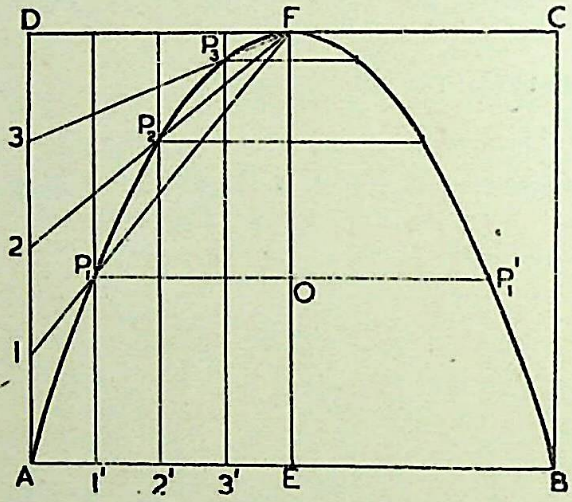


FIG. 6-11(*i*)

Divide AE and AD into the same number of equal parts and name them as shown (starting from A).

Draw lines joining F with points 1, 2 and 3.

Through 1′, 2′ and 3′, draw perpendiculars to AB intersecting $F1$, $F2$ and $F3$ at points P_1, P_2 and P_3 respectively.

Draw a curve through A, P_1, P_2 etc. It will be a half parabola.

Repeat the same construction in the other half of the rectangle to complete the parabola.

Or, locate the points by drawing lines through the points P_1 etc. parallel to the base and making each of them of equal length on both the sides of EF, e.g. $P_1O = OP_1'$.

AB and *EF* are called the base and the axis respectively of the parabola.

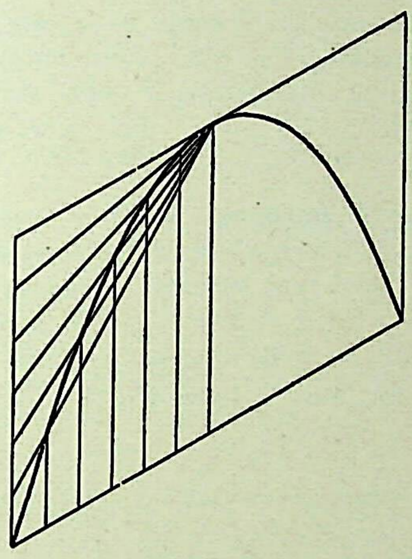


FIG. 6-11(*ii*)

Fig. 6-11(*ii*) shows a parabola drawn in a parallelogram by this method.

II. Tangent method [fig. 6-11(*iii*)]:

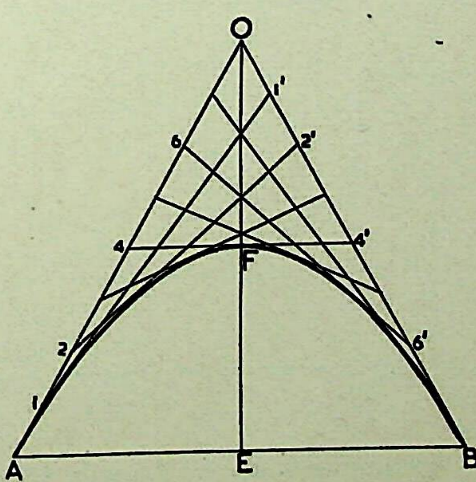


FIG. 6-11(*iii*)

Draw the base *AB* and the axis *EF*. (These are taken different from those in method I). Produce *EF* to *O* so that $EF = FO$. Join *O* with *A* and *B*. Divide lines *OA* and *OB* into the same number of equal parts, say 8. Mark the division-points as shown in the figure. Draw lines joining 1 with 1′, 2 with 2′ etc. Draw a curve starting from *A* and tangent to lines 1 1′, 2 2′ etc. This curve is the required parabola.

Rectangular hyperbola: It is a curve traced out by a point moving in such a way that the product of its distances from two fixed lines at right angles to each other is a constant.

The fixed lines are called *asymptotes*. This curve graphically represents the Boyle's Law, viz. $P \times V =$ a constant.

Problem 9:

To draw a rectangular hyperbola, given the position of a point P on it [fig. 6-12(*i*)].

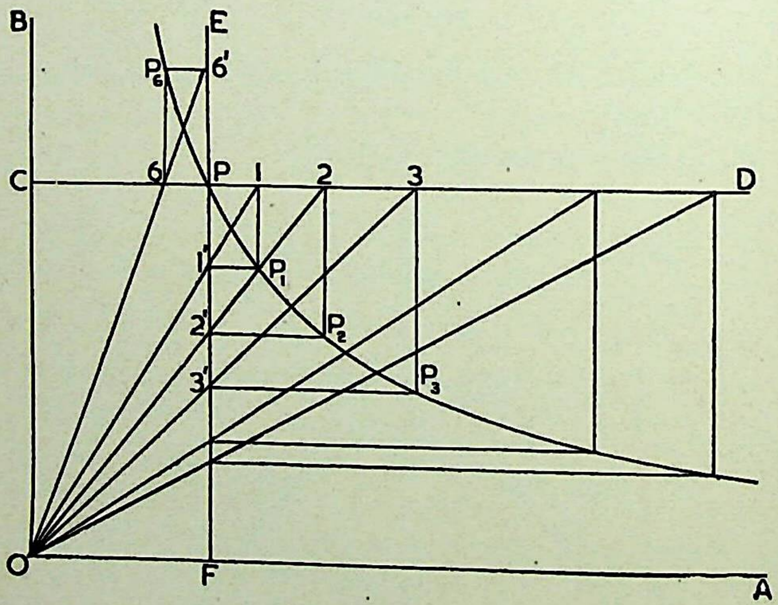


FIG. 6-12(*i*)

Draw lines *OA* and *OB* at right angles to each other. Mark the position of the point *P*.

Through P, draw lines CD and EF parallel to OA and OB respectively. Along PD, mark a number of points 1, 2, 3 etc. not necessarily equidistant. Join $O1$, $O2$ etc. cutting PF at points $1'$, $2'$ etc. Through point 1, draw a line parallel to OB and through $1'$, draw a line parallel to OA, intersecting each other at a point P_1.

Obtain other points in similar manner.

For locating the point, say P_6, to the left of P, the line $O6$ should be extended to meet PE at $6'$.

Draw the hyperbola through the points P_6, P, P_1 etc.

A hyperbola, through a given point situated between two lines making any angle between them, can similarly be drawn, as shown in fig. 6-12(*ii*).

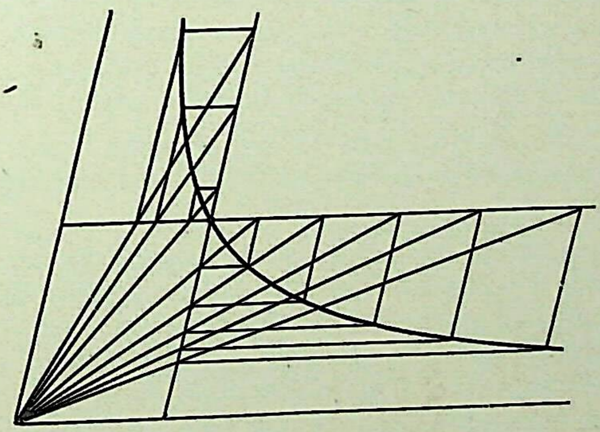


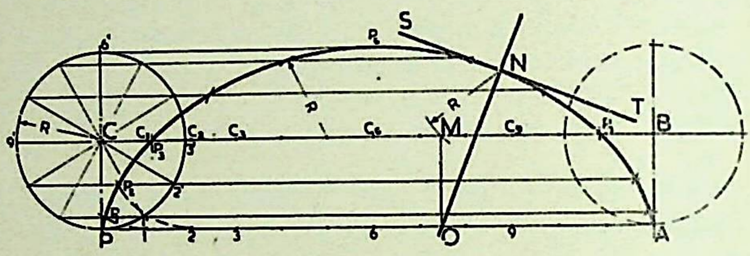
FIG. 6-12(*ii*)

Cycloidal curves: These curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle. The rolling circle is called *generating circle* and the fixed straight line or circle is termed *directing line* or *directing circle*.

Cycloid: Cycloid is a curve generated by a point on the circumference of a circle which rolls along a straight line.

Problem 10:

To construct a cycloid, given the diameter of the generating circle (fig. 6-13).



Cycloid
FIG. 6-13

With centre *C* and given radius *R*, draw a circle. Let *P* be the generating point.

Draw a line *PA* tangential to and equal to the circumference of the circle. Divide the circle and the line *PA* into the same number of equal parts, say 12, and mark the division-points as shown. Through *C*, draw a line *CB* parallel and equal to *PA*.

Draw perpendiculars at points 1, 2 etc. cutting *CB* at C_1, C_2 etc.

Assume that the circle starts rolling to the right. When point 1′ coincides with 1, centre *C* will move to C_1. In this position of the circle, the generating point *P* will have moved to position P_1 on the circle, at a distance equal to *P*1′ from point 1. It is evident that P_1 lies on the horizontal line through 1′ and at a distance *R* from C_1. Similarly, P_2 will lie on the horizontal line through 2′ and at the distance *R* from C_2.

Construction: Through the points 1′, 2′ etc., draw lines parallel to *PA*.

With centres C_1, C_2 etc. and radius equal to *R*, draw arcs cutting the lines through 1′, 2′ etc. at points P_1, P_2 etc. respectively.

Draw a smooth curve through points *P*, P_1, P_2...*A*; this curve is the required cycloid.

NORMAL AND TANGENT:

The rule for drawing a normal to all cycloidal curves:

The normal at any point on a cycloidal curve will pass through the corresponding point of contact between the generating circle and the directing line or circle.

The tangent at any point is perpendicular to the normal at that point.

Problem 11:

To draw a normal and a tangent to a cycloid at a given point N on it (fig. 6-13).

With centre N and radius equal to R, draw an arc cutting CB at M.

Through M, draw a line MO perpendicular to the directing line PA and cutting it at O.

O is the point of contact and M is the position of the centre of the generating circle, when the generating point P is at N.

Draw a line through N and O; this line is the required normal.

Through N, draw a line ST at right angles to NO; ST is the tangent to the cycloid.

Trochoid: Trochoid is a curve generated by a point fixed to a circle, within or outside its circumference, as the circle rolls along a straight line.

When the point is within the circle, the curve is called an *inferior trochoid* and when outside the circle, it is termed a *superior trochoid.*

Problem 12:

To draw a trochoid, given the rolling circle and the generating point (figs. 6-14, 6-15 and 6-16).

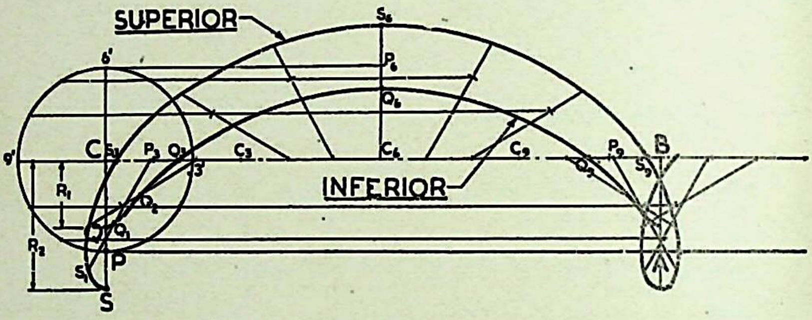
(a) *Inferior trochoid:* Let Q be the point within the circle and at a distance R_1 from the centre C.

Draw the circle and mark a point Q on the line CP and at a distance R_1 from C.

Draw a tangent PA equal to the circumference of the circle and a line CB equal and parallel to PA.

Divide the circle and the line CB into 12 equal parts.

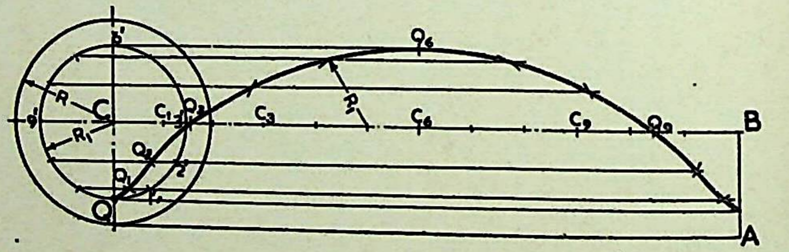
Method I (fig. 6-14): Determine the positions P_1, P_2 etc. for the cycloid as shown in Problem 10. Join C_1P_1, C_2P_2 etc. With centres C_1, C_2 etc. and radius equal to R_1, draw arcs cutting C_1P_1, C_2P_2 etc. at points Q_1, Q_2 etc. respectively.



Trochoids
FIG. 6-14

Draw a curve through these points; this curve is the inferior trochoid.

Method II (fig. 6-15): With centre C and radius equal to R_1, draw a circle and divide it into 12 equal parts.



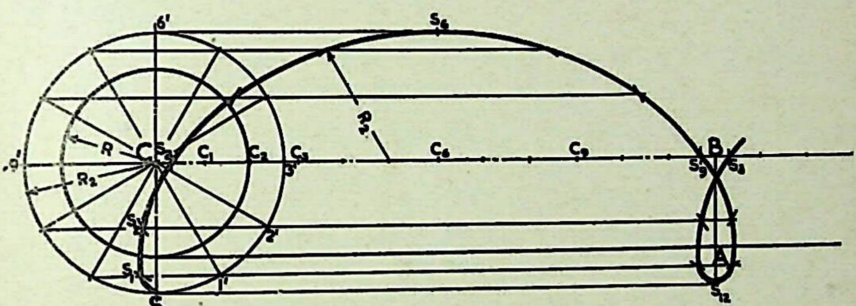
Inferior trochoid
FIG. 6-15

Through the division-points, draw lines parallel to PA. With centres C_1, C_2 etc. and radius equal to R_1, draw arcs to cut the lines through $1'$, $2'$ etc. at points Q_1, Q_2 etc. Draw the trochoid through these points.

(b) *Superior trochoid:* Let S be the point outside the circle and at a distance R_2 from the centre.

Method I (fig. 6-14): Adopt the same method as method I used for inferior trochoid. Point S will lie on line CP-produced, at distance R_2 from C. Points S_1, S_2 etc. are obtained by cutting the lines C_1P_1-produced, C_2P_2-produced etc. with arcs drawn with centres C_1, C_2 etc. and radius equal to R_2. S, S_1, S_2 etc. are the points on the superior trochoid.

Method II (fig. 6-16): Same as method II for inferior trochoid. Note that the radius of the circle is equal to R_2.



Superior trochoid

FIG. 6-16

Note the loop that is formed when the circle rolls for more than one revolution.

Epicycloid and Hypocycloid: The curve generated by a point on the circumference of a circle, which rolls along another circle *outside* it, is called an *epicycloid*.

When the circle rolls along another circle *inside* it, the curve is called a *hypocycloid*.

Problem 13:

To draw an epicycloid and a hypocycloid, given the generating and directing circles of radii r and R respectively.

Epicycloid (fig. 6-17): With centre O and radius R, draw the directing circle (only a part of it may be drawn). Draw a radius OP and produce it to C, so that $CP = r$.

With C as centre, draw the generating circle. Let P be the generating point.

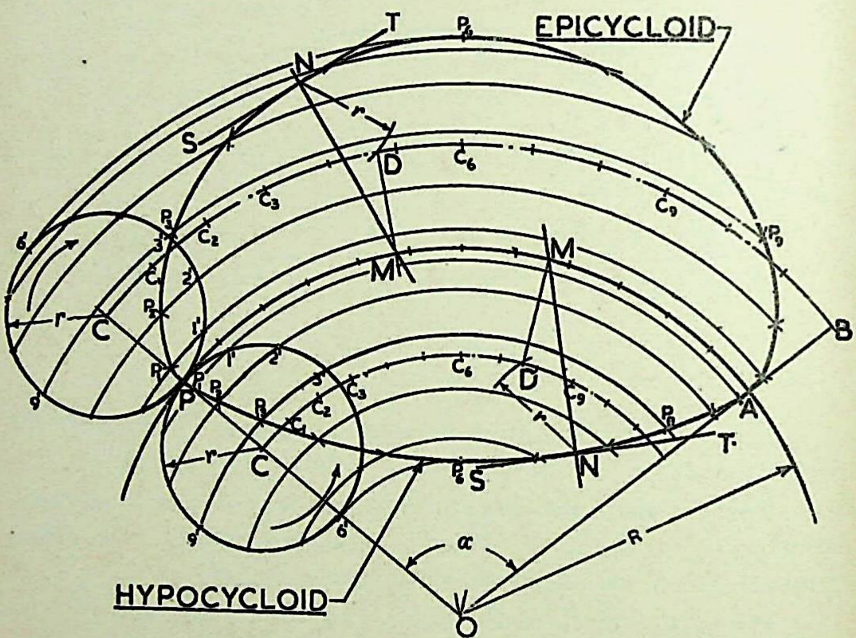
In one revolution of the generating circle, the point P will move to a point A, so that the arc PA = the circumference of the generating circle.

The position of A may be located by calculating the angle subtended by the arc PA at centre O, by the formula,

$$\frac{\angle POA}{360^\circ} = \frac{\text{arc } PA}{\text{circum. of directing circle}} = \frac{2\pi r}{2\pi R} = \frac{r}{R}$$

$$\therefore \qquad \angle POA = 360^\circ \times \frac{r}{R}.$$

Set off this angle and obtain the position of A.



Epicycloid and hypocycloid

FIG. 6-17

With centre O and radius equal to OC, draw an arc intersecting OA-produced at B. This arc CB is the locus of the centre C. Divide CB and the generating circle into twelve equal parts.

With centre O, describe arcs through points 1′, 2′, 3′ etc. With centres C_1, C_2 etc. and radius equal to r, draw arcs cutting the arcs through 1′, 2′ etc. at points P_1, P_2 etc.

Draw the required epicycloid through the points P, P_1, $P_2 \ldots A$.

Hypocycloid: The method for drawing the hypocycloid is same as for epicycloid. Note that the centre C of the generating circle is inside the directing circle, as shown in fig. 6-17.

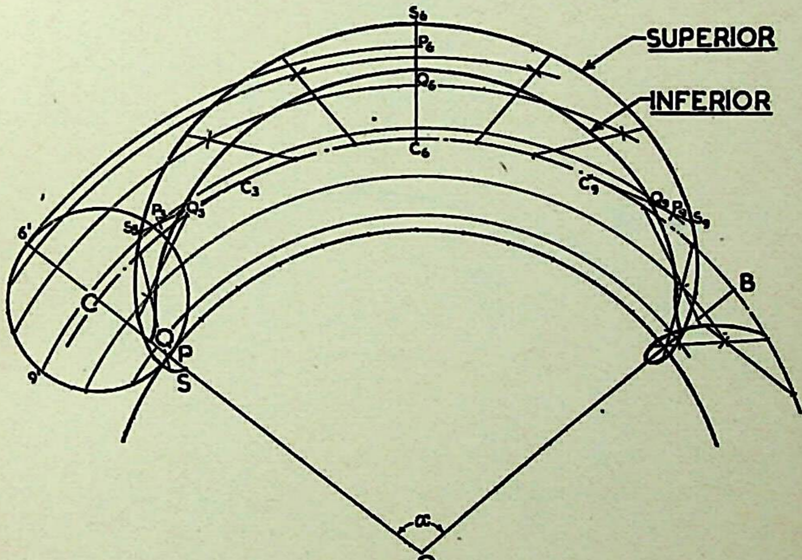
Note: When the diameter of the rolling circle is half the diameter of the directing circle, the hypocycloid is a straight line and is a diameter of the directing circle.

NORMAL AND TANGENT:

Problem 14:

To draw a normal and a tangent to an epicycloid and a hypocycloid at a point N in each of them (fig. 6-17).

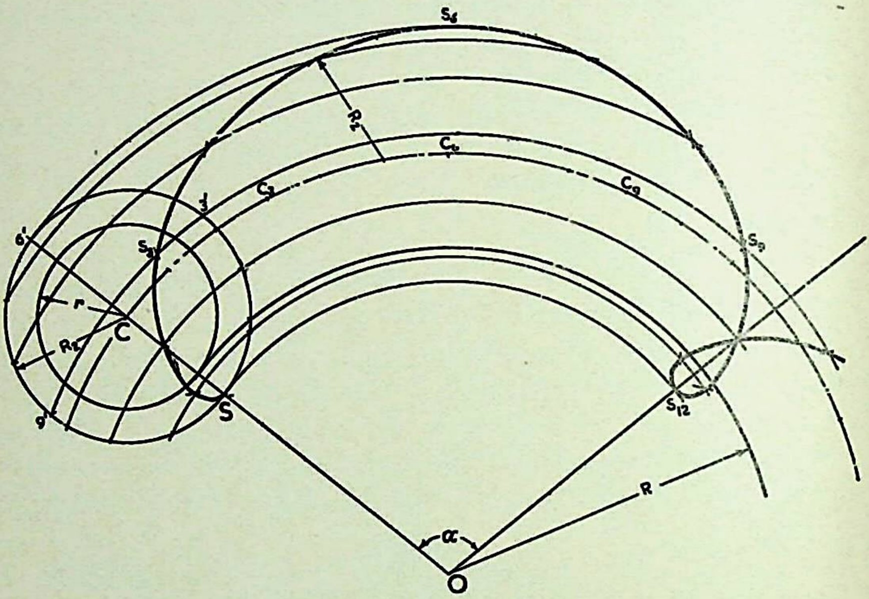
With centre N and radius equal to r, draw an arc cutting the lacus of the centre C at a point D. Draw a line through O and D, cutting the directing circle at M. Draw a line through N and M; this line is the normal. Draw a line ST through N and at right angles to NM; ST is the tangent.



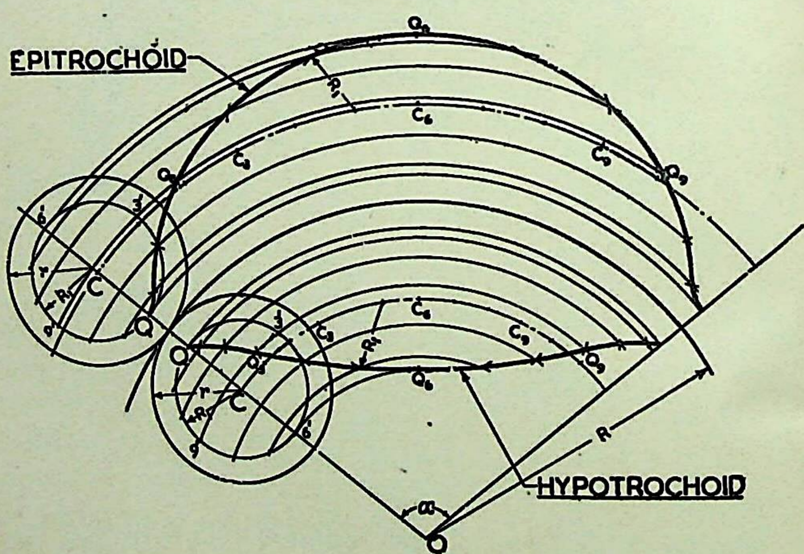
Epitrochoids
Fig. 6-18

Epitrochoid and hypotrochoid: Epitrochoid is a curve generated by a point fixed to a circle (within or outside its circumference, but in the same plane) rolling on the

outside of another circle. When the circle rolls inside another circle, the curve is called a hypotrochoid.

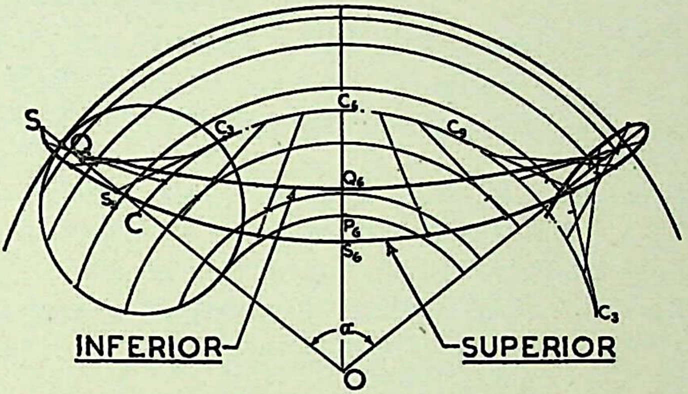


Superior epitrochoid
FIG. 6-19



Inferior epitrochoid and hypotrochoid
FIG. 6-20

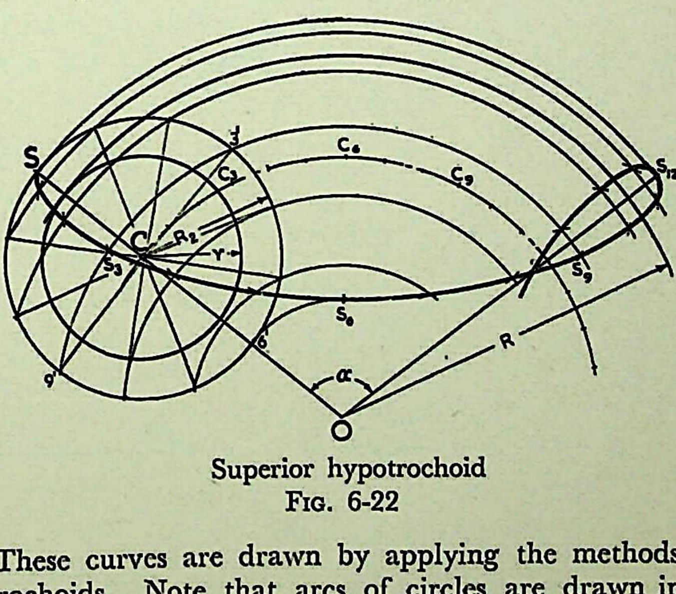
The curve is termed inferior or superior, according to the position of the point being inside or outside the rolling circle.



Hypotrochoids
FIG. 6-21

Problem 15:

To draw an epitrochoid and a hypotrochoid, given the rolling and directing circles and the generating points.



Superior hypotrochoid
FIG. 6-22

These curves are drawn by applying the methods used for trochoids. Note that arcs of circles are drawn instead of horizontal lines.

Epitrochoids:	Method I:	Superior and inferior — see fig. 6-18
	Method II:	Superior — see fig. 6-19 Inferior — see fig. 6-20
Hypotrochoids:	Method I:	Superior and inferior — see fig. 6-21
	Method II:	Superior — see fig. 6-22 Inferior — see fig. 6-20

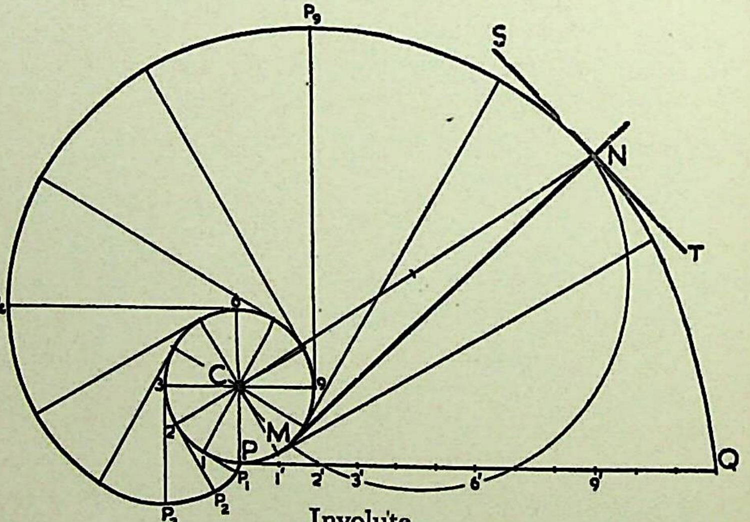
Note: When the diameter of the rolling circle is half the diameter of the directing circle, the hypotrochoid will be an ellipse.

Involute: The involute of a circle is a curve traced out by an end of a piece of thread unwound from the circle, the thread being kept tight.

It may also be defined as a curve traced out by a point in a straight line which rolls along a circle.

Problem 16:

To draw an involute of a given circle (fig. 6-23).



Involute
FIG. 6-23

With centre *C*, draw the given circle. Let *P* be the starting point, i.e. the end of the thread.

Suppose the thread to be partly unwound, say upto a point 1. P will move to a position P_1 such that $1P_1$ is tangent to the circle and is equal to the arc $1P$. P_1 will be a point on the involute.

Construction: Draw a line PQ, tangent to the circle and equal to the circumference of the circle.

Divide PQ and the circle into 12 equal parts.

Draw tangents at points 1, 2, 3 etc. and mark on them points P_1, P_2, P_3 etc. such that $1P_1 = P1'$, $2P_2 = P2'$, $3P_3 = P3'$ etc.

Draw the involute through the points P, P_1, P_2.... etc.

NORMAL AND TANGENT:

The normal to an involute is tangent to the circle.

Problem 17:

To draw a normal and a tangent to the involute at a point N on it (fig. 6-23).

Draw a line joining C and N. With CN as diameter describe a semi-circle cutting the circle at M.

Draw a line through N and M; this line is the normal. Draw a line ST, perpendicular to NM and passing through N. ST is the tangent to the involute.

Problem 18 (fig. 6-24):

To trace the paths of the ends of a straight line AP, 10 cm (4") long, when it rolls, without slipping, on a semi-circle having its diameter AB, 7·5 cm (3") long. (Assume the line AP to be tangent to the semi-circle in the starting position).

Draw the semi-circle and divide it into six equal parts. Draw the line AP and mark points 1, 2 etc. such that $A1 =$ arc $A1'$, $A2 =$ arc $A2'$ etc. The last division $5P$ will be of a shorter length. On the semi-circle mark a point P' such that $5'P' = 5P$.

At points $1'$, $2'$ etc. draw tangents and on them, mark points P_1, P_2 etc. such that $1'P_1 = 1P$, $2'P_2 = 2P$.....and $5'P_5 = 5'P' = 5P$; similarly, mark points A_1, A_2 etc. such that $A_1 1' = A1$, $A_2 2' = A2$....and $A'P' = AP$. Draw the

required curves through points P, P_1. . . .and P', and through A, A_1. . . .and A'.

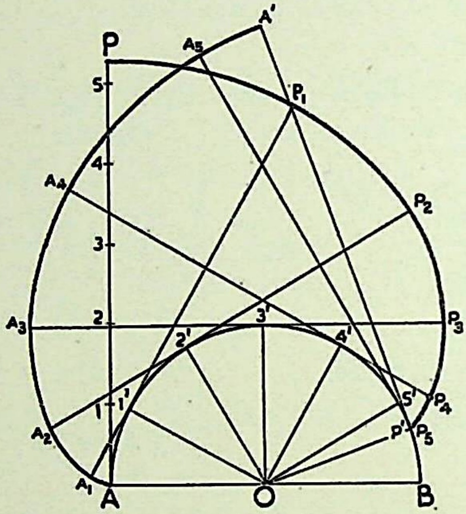


FIG. 6-24

If AP is an inelastic string with the end A attached to the semi-circle, the end P will trace out the same curve PP', when the string is wound round the semi-circle.

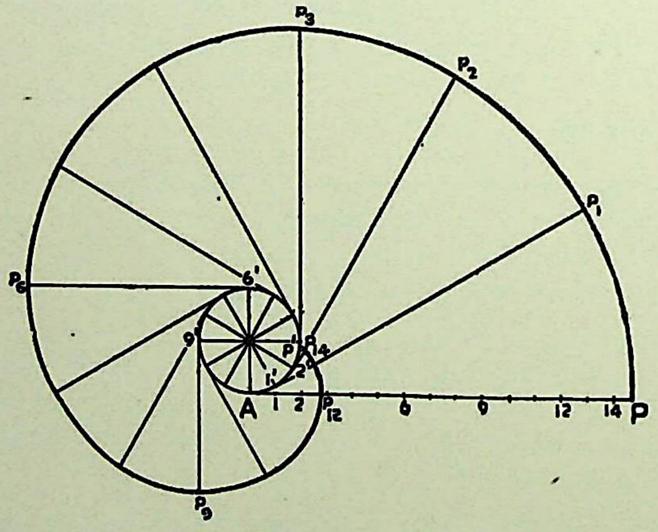


FIG. 6-25

Fig. 6-25 shows the curve traced out by the end of a thread which is longer than the circumference of the circle

on which it is wound. Note that the tangent $1'P_1 = 1P$, $2'P_2 = 2P$ etc. and lastly $2'P' = 2'P_{14} = 14P$.

Evolute:

APB is a given curve (fig. 6-26). *O* is the centre of a circle drawn through three points *C*, *P* and *D* on this curve. If the points *C* and *D* are moved to converge towards *P*, until they are indefinitely close together, then in the limit, the circle becomes the *circle of curvature* of the curve *APB* at *P*. The centre *O* of the circle of curvature lies on the normal to the curve at *P*. This centre is called the *centre of curvature*. The radius of the circle is called the *radius of curvature* at *P*. The locus of the centre of curvature of a curve is called the *evolute* of the curve. A curve has only one evolute.

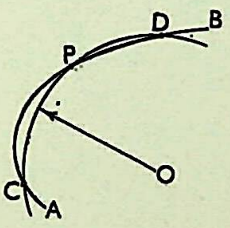


FIG. 6-26

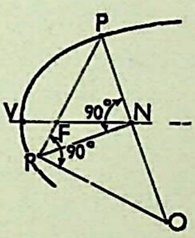


FIG. 6-27

Problem 19:

To determine the centre of curvature at a given point on a conic (fig. 6-27).

Let *P* be the given point on the conic and *F*, the focus.

Join *P* and *F*. At *P*, draw a normal *PN*, cutting the axis at *N*. Draw a line *NR* perpendicular to *PN* and cutting *PF* or *PF*-produced at *R*. Draw a line *RO* perpendicular to *PR* and cutting *PN*-produced at *O*. Then *O* is the centre of curvature of the conic at the point *P*.

The above construction does not hold good when the given point coincides with the vertex. As the point *P* approaches the vertex, the points *R*, *N* and *O* move nearer to one another, so that when *P* is at the vertex, the three points coincide on the axis.

In a parabola, PF will be equal to FR; hence, when P is at V the vertex, the centre of curvature O is on the axis so that $OF = VF$.

In an ellipse or a hyperbola, the ratio of the distances of P from the foci is equal to the ratio of focal distances of N, i.e. $\frac{PF}{PF_1} = \frac{NF}{NF_1}$. Hence, when P coincides with V the vertex, the ratio becomes $\frac{VF}{VF_1} = \frac{OF}{OF_1}$. Similarly, when P coincides with the other vertex V_1, the ratio becomes $\frac{V_1F_1}{V_1F} = \frac{O_1F_1}{O_1F}$.

Problem 20:

To determine the centre of curvature O, when the point P is at the vertex V of a conic (fig. 6-28).

(i) Parabola [fig. 6-28(*i*)]: Mark the centre of curvature O on the axis such that $OF = VF$.

(ii) Ellipse: Method I [fig. 6-28(*ii*)]: Draw a line F_1G inclined to the axis and equal to VF_1. Produce F_1G to H so that $GH = VF$. Join H with F. Draw a line GO parallel to HF and intersecting the axis at O. Then O is the required centre of curvature.

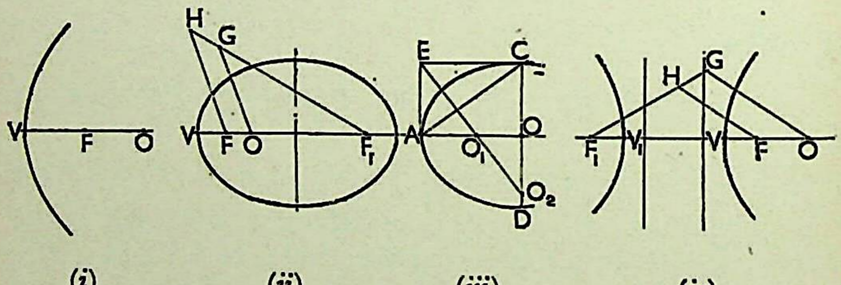


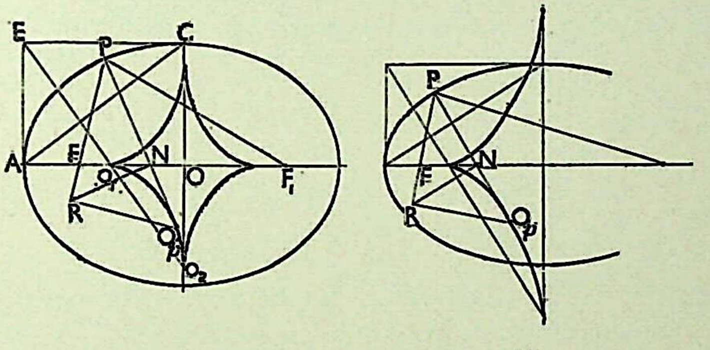
Fig. 6-28

Method II [fig. 6-28(*iii*)]: Draw a rectangle $AOCE$ in which $AO = \frac{1}{2}$ major axis and $CO = \frac{1}{2}$ minor axis. Join A with C. Through E, draw a line perpendicular to AC and cutting the major axis at O_1 and the minor axis at O_2. Then O_1 and O_2 are the centres of curvature when the point P is at A and C respectively.

(iii) Hyperbola [fig. 6-28(*iv*)]: Draw a line F_1G inclined to the axis and equal to FV_1. On F_1G, mark a point H such that $HG = VF$. Join H with F. Draw a line GO parallel to HF and cutting the axis at O. Then O is the centre of curvature at the vertex V.

Problem 21:

To draw the evolute of an ellipse (fig. 6-29).



(*i*) (*ii*)

FIG. 6-29

The ellipse with major axis AB and minor axis CD is given. Mark a number of points on the ellipse. Determine the centres of curvatures at these points (as shown at the point P) and draw a smooth curve through them. This is the evolute of the ellipse. The evolute may sometimes go outside the ellipse as shown in fig. 6-29(*ii*). The centres of curvature at points A and C are shown by method II.

Problem 22:

To draw the evolute of a parabola (fig. 6-30).

Mark a number of points on the parabola and determine the centres of curvature at these points (as shown at the point P). Draw the evolute through these centres. Note that $PF = FR$.

Problem 23:

To draw the evolute of a hyperbola (fig. 6-31).

Mark a number of points on the hyperbola and determine the centres of curvature at these points (as shown at the point P). Draw the evolute through these centres. To

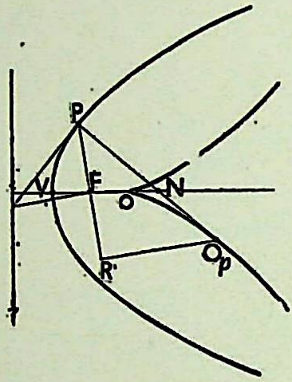


Fig. 6-30

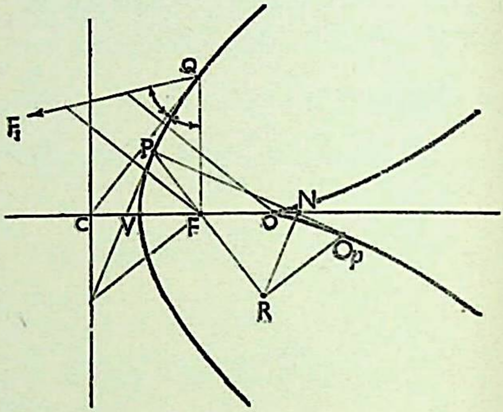


Fig. 6-31

obtain the centre of curvature at the vertex, the position of the other focus F_1 must be found. It is determined by making use of the following rule:

The tangent at any point on the curve bisects the angle made by lines joining that point with the two foci, i.e. $\angle F_1QC = \angle FQC$.

Problem 24:

To draw the evolute of an involute of a circle (fig. 6-23).

In the involute of a circle, the normal NM at any point N is tangent to the circle at the point of contact M. M is the centre of curvature at the point N. Hence, the evolute of the involute is the circle itself.

Problem 25:

To draw the evolute of a cycloid (fig. 6-32).

Mark a point P on the cycloid and draw the normal PN to it (Prob. 11). Produce PN to O_p so that $NO_p = PN$. O_p is the centre of curvature at the point P. Similarly, mark a number of points on the cycloid and determine centres of curvature at these points. The curve drawn through these centres is the evolute of the cycloid. It is an equal cycloid.

Problem 26:

To draw the evolute of an epicycloid (fig. 6-33).

Mark a point P on the epicycloid and draw the normal PN to it (Prob. 14). Draw the diameter PQ of the rolling circle. Join Q with O, the centre of the directing circle. Produce PN to cut QO at Op, which is the centre of curvature at the point P. Mark a number of points on the epicycloid and similarly, obtain centres of curvature at these points. The curve drawn through these centres is the evolute of the epicycloid.

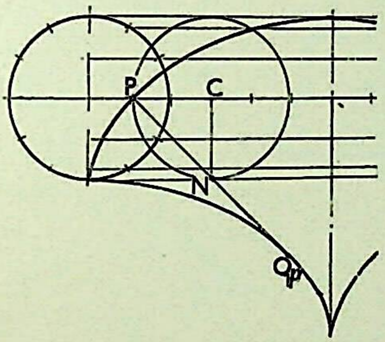


FIG. 6-32

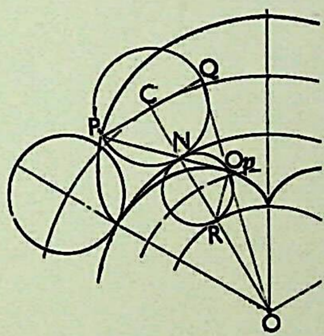


FIG. 6-33

Through Op, draw a line, perpendicular to POp and intersecting the line joining C (the centre of the rolling circle) with O at a point R. The evolute is the epicycloid of the circle of diameter NR, rolling along the circle of radius OR.

Problem 27:

To draw the evolute of a hypocycloid (fig. 6-34).

Mark a point P on the hypocycloid and draw the normal PN to it (Prob. 14). Draw the diameter PQ of the rolling circle. Join Q with C, the centre of the directing circle. Produce PN to cut OQ-produced at Op, which is the centre of curvature at the point P. Mark a number of points on the hypocycloid and similarly, obtain centres of curvature at these points. The curve drawn through these centres is the evolute of the hypocycloid.

Through Op, draw a line, perpendicular to POp and intersecting OC-produced at a point R. The evolute is the hypocycloid of the circle of diameter NR rolling along the circle of radius OR.

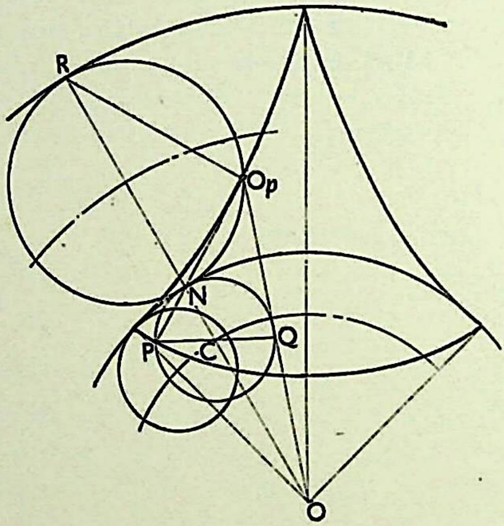


FIG. 6-34

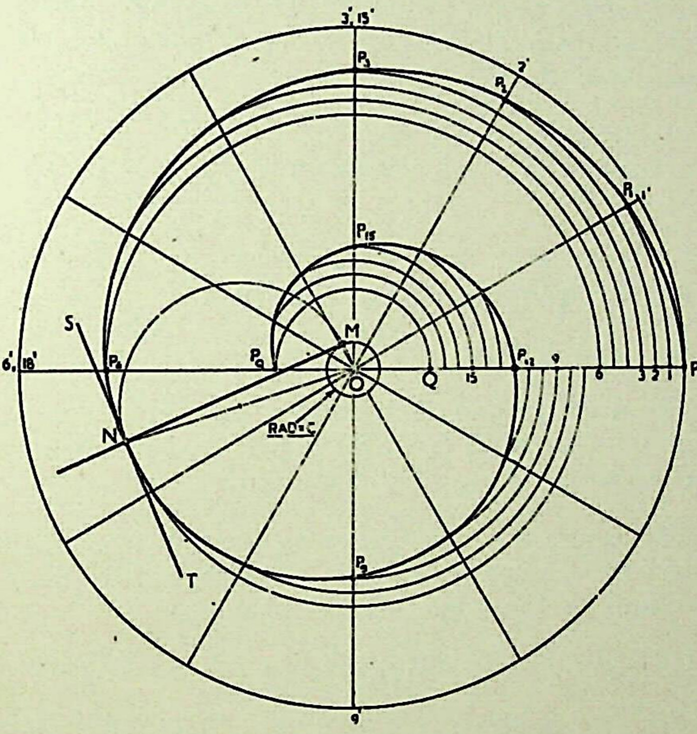
Spiral: If a line rotates in a plane about one of its ends and if at the same time, a point moves along the line continuously in one direction, the curve traced out by the moving point is called a *spiral*. The point about which the line rotates is called a *pole*.

The line joining any point on the curve with the pole is called the *radius vector* and the angle between this line and the line in its initial position is called the *vectorial angle*. Each complete revolution of the curve is termed the *convolution*. A spiral may make any number of convolutions before reaching the pole.

Archemedian spiral: It is a curve traced out by a point moving in such a way that its movement towards or away from the pole is uniform with the increase of the vectorial angle from the starting line.

Problem 28:

To construct an Archemedian spiral of $1\frac{1}{2}$ convolutions, given the greatest and the shortest radii (fig. 6-35).



Archemedian spiral

FIG. 6-35

Let O be the pole, OP the greatest radius and OQ the shortest radius.

With centre O and radius equal to OP, draw a circle.

OP revolves around O for $1\frac{1}{2}$ revolutions. During this period, P moves towards O, the distance equal to $(OP - OQ)$ i.e. PQ.

Divide the angular movement of OP, viz. $1\frac{1}{2}$ revolutions i.e. 540°, and the line PQ into the same number of equal parts, say 18 (one revolution divided into 12 equal parts).

When the line OP moves through one division, i.e. 30°, the point P will move towards O by a distance equal to one division of PQ to a point P_1.

To obtain points systematically, draw arcs with centre O and radii $O1$, $O2$, $O3$ etc. intersecting lines $O1'$, $O2'$, $O3'$ etc. at points P_1, P_2, P_3 etc. respectively.

In one revolution P will reach the 12th division along PQ and in the next half revolution it will be at the point P_Q on the line 0-18′.

Draw a curve through points P, P_1, $P_2 \ldots P_Q$; this curve is the required Archemedian spiral.

NORMAL AND TANGENT:

The normal to an Archemedian spiral at any point is a tangent to the circle, having the pole as its centre and a radius equal to the *constant of the curve.*

The constant of the curve is equal to the difference between the lengths of any two radii divided by the circular measure of the angle between them.

OP and OP_3 (fig. 6-35), are two radii making 90° angle between them; but $90° = \frac{\pi}{2} = 1{\cdot}57$, in circular measure.

Therefore, the constant of the curve, $C = \frac{OP - OP_3}{1{\cdot}57}$.

Problem 29:

To draw a normal to the Archemedian spiral at a point N on it (fig. 6-35).

With centre O and radius equal to C, draw a circle.

Draw a line NM, tangent to this circle.

Then NM is the normal to the spiral.

Draw a line ST through N and perpendicular to NM. ST is the tangent to the spiral.

Problem 30:

A link 22·5 cm (9″) long, swings on a pivot O from its vertical position of rest to the right, through an angle of 75° and returns to its initial position at uniform velocity. During that period, a point P moving at uniform speed along the centre line of the link from a point at a distance of 2·5 cm (1″) from O, reaches the end of the link. Draw the locus of the point P (fig. 6-36).

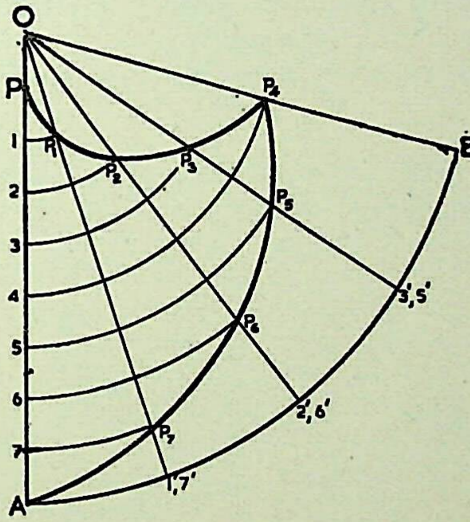


FIG. 6-36

Draw a vertical line OA, 22·5 cm (9″) long. With centre O and radius equal to OA, draw an arc. Draw a line OB making $\angle AOB$ equal to 75° and cutting the arc at B.

Mark a point P along OA and at a distance of 2·5 cm (1″) from O.

Divide the angular movement of the link and the line PA into the same number of equal parts, say 8.

The end A of the link moves to B and returns to its original position. Hence, the arc AB must be divided into four equal parts.

With centre O and radii $O1$, $O2$, $O3$ etc., draw arcs intersecting lines $O1'$, $O2'$, $O3'$ etc. at points P_1, P_2, P_3 etc. respectively.

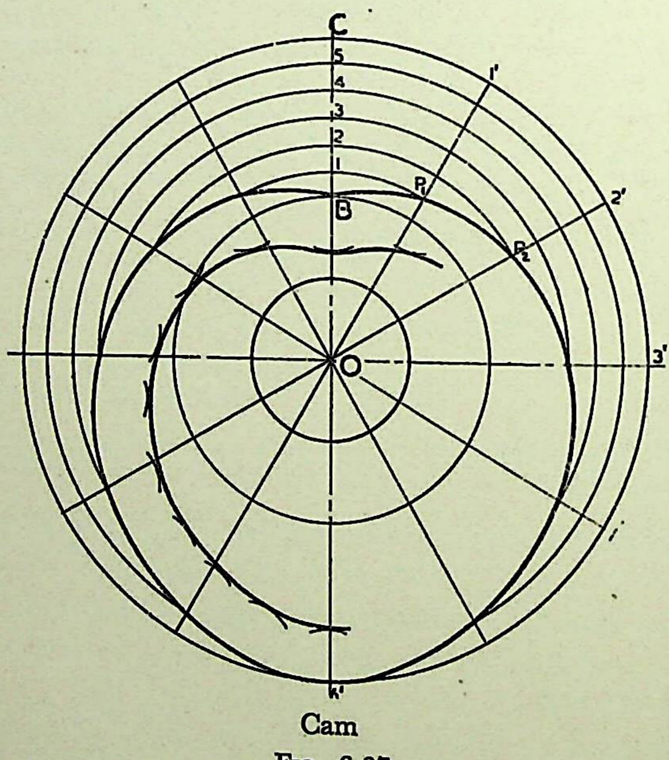
Draw a curve through P, P_1,....P_4 ....A; this curve is the locus of the point P.

Cam: A cam is a machine-part which, while rotating at uniform velocity, imparts reciprocating linear motion to another machine-part called a follower. The motion imparted may be either uniform or variable, depending upon the shape of the cam profile.

The shape of the cam to transmit uniform linear motion is determined by the application of the principle of Archemedian spiral, as shown in the following problem:

Problem 31:

Draw the shape of a cam to give a uniform rise and fall of 5 cm (2") to a point, during each revolution of the cam. Diameter of the shaft = 5 cm (2"); least radius of the cam = 5 cm (2") (fig. 6-37).



Cam

FIG. 6-37

Draw a circle (for the shaft) with centre O.

With the same centre and radius OB equal to 5 cm (2″) (the least radius to the cam), draw another circle.

Produce OB to C making BC equal to 5 cm (2″) (the rise of the point). The point is to be uniformly raised during $\frac{1}{2}$ revolution of the shaft. Therefore, divide BC and angle 180° into the same number of equal parts, say 6.

Obtain points P_1, P_2 etc. as in Archemedian spiral and draw the curve through them.

The point is to fall the same distance during the same period; hence, the curve will be exactly of the same shape as for the rise.

The followers are generally provided with rollers to give smooth working. In such cases, the profile of the cam is designed initially, to transmit motion to a point (the centre of the roller) and then, a parallel curve is drawn inside it at a distance equal to the radius of the roller. This is done by first drawing a number of arcs with points on the curve as centres and radius equal to the radius of the roller, and then drawing a smooth curve touching these arcs, as shown in fig. 6-37.

Logarithmic or Equiangular spiral:

In a *logarithmic spiral*, the ratio of the lengths of consecutive radius vectors enclosing equal angles is always constant. In other words, the values of vectorial angles are in arithmetical progression and the corresponding values of radius vectors are in geometrical progression. The logarithmic spiral is also known as *equiangular spiral* because of its property that the angle which the tangent at any point on the curve makes with the radius vector at that point is constant.

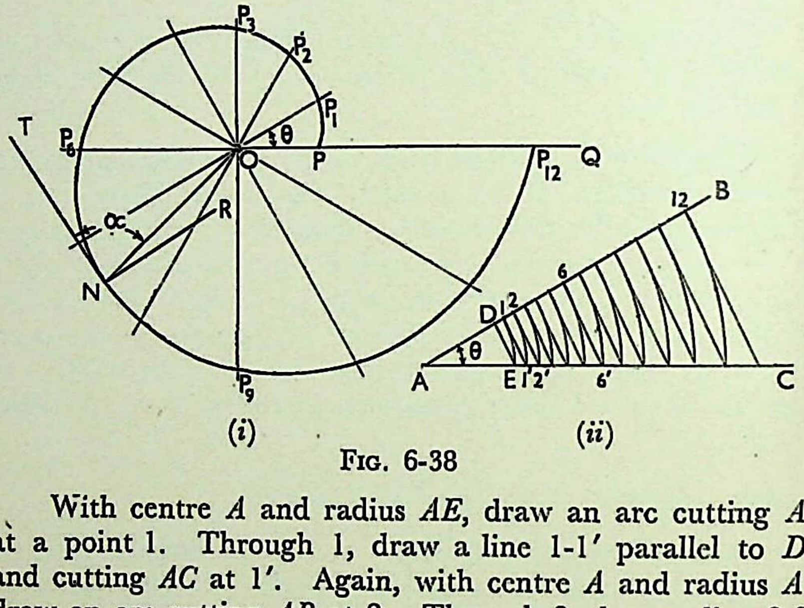
Problem 32:

To construct a logarithmic spiral of one convolution, given the length of the shortest radius and the ratio of the lengths of radius vectors enclosing an angle θ (fig. 6-38).

Let the shortest radius be 1 cm long, θ equal to 30° and the ratio $\frac{10}{9}$.

The lengths of radius vectors are determined from the scale which is constructed as shown below.

Draw lines AB and AC [fig. 6-38(*ii*)] making an angle of 30° between them. On AB, mark AD, 1 cm long. On AC, mark a point E such that $AE = \frac{10}{9} \times 1$ cm $= \frac{10}{9}$ cm. Join D with E.



(*i*) (*ii*)

FIG. 6-38

With centre A and radius AE, draw an arc cutting AB at a point 1. Through 1, draw a line 1-1′ parallel to DE and cutting AC at 1′. Again, with centre A and radius $A1'$ draw an arc cutting AB at 2 Through 2, draw a line 2-2′, parallel to DE and cutting AC at 2′ Repeat the construction and obtain points 3, 4....12. $A1$, $A2$ etc. are the lengths of consecutive radius vectors.

Draw a horizontal line OQ, and on it mark OP, 1 cm long [fig. 9-38(*i*)]. Through O, draw radial lines making 30° angles between two consecutive lines. These are the radius vectors.

Mark points $P_1, P_2 \ldots P_{12}$ on consecutive radius vectors such that $OP_1 = A1$, $OP_2 = A2 \ldots OP_{12} = A\text{-}12$.

Draw a smooth curve through $P_1, P_2 \ldots P_{12}$; this curve is the required logarithmic spiral.

The equation to the logarithmic spiral is $r = a^\theta$, where r is the radius vector, θ is the vectorial angle and a is a constant.

Hence, $\log r = \theta \log a$.

If $\theta = 0$, $\log r = 0 \quad \therefore \quad r = 1$.

When $\theta = 30° = \frac{\pi}{6}$ radians, $r = \frac{10}{9} \times 1 = \frac{10}{9}$

$$\therefore \quad \log \frac{10}{9} = \frac{\pi}{6} \log a$$

$$\text{i.e.} \quad \log a = \frac{6}{\pi} \log \frac{10}{9}$$

$$\therefore \quad a = 1{\cdot}22.$$

Problem 33:

To draw a normal and a tangent at a given point N on the logarithmic spiral in Prob. 32 [fig. 6-38(i)].

The value of the tangent to the constant angle is obtained by the formula

$$\tan \alpha = \frac{\log_e}{\log_a} \quad \text{where } e = 2{\cdot}718$$

$$\text{i.e. } \tan \alpha = \frac{\log 2{\cdot}718}{\frac{6}{\pi} \log \frac{10}{9}}$$

$$\therefore \quad \alpha = 78°\text{-}38'.$$

Join N with O. Draw a line NT making an angle of 78°-38′ with NO. NT is the required tangent. NR drawn perpendicular to NT is the normal to the spiral.

EXERCISES VI

(1) Draw a straight line AB of any length. Mark a point F, 6·5 cm ($2\frac{1}{2}''$) from AB. Trace the paths of a point P moving in such a way, that the ratio of its distance from the point F, to its distance from AB is (i) 1 (ii) $\frac{3}{2}$ (iii) $\frac{2}{3}$. Plot at least 8 points. Name each curve. Draw a normal and a tangent to each curve at a point on it, 5 cm ($2''$) from F.

(2) A fixed point is 7·5 cm ($3''$) from a fixed straight line. Draw the locus of a point P moving in such a way that its distance from the fixed straight line is (i) twice its distance from the fixed point; (ii) equal to its distance from the fixed point. Name the curves.

(3) The vertex of a hyperbola is 6·5 cm ($2\frac{1}{2}''$) from its focus. Draw the curve if the eccentricity is $\frac{5}{3}$. Draw a normal and a tangent at a point on the curve, 7·5 cm ($3''$) from the directrix.

(4) The major axis of an ellipse is 15 cm ($6''$) long and the minor axis is 10 cm ($4''$) long. Find the foci and draw the ellipse by 'arcs of circles' method. Draw a tangent to the ellipse at a point on it 2·5 cm ($1''$) above the major axis.

(5) The foci of an ellipse are 9 cm ($3\frac{1}{2}''$) apart and the minor axis is 6·5 cm ($2\frac{1}{2}''$) long. Determine the length of the major axis and draw half the ellipse by concentric-circles method and the other half by oblong method. Draw a curve parallel to the ellipse and 2·5 cm ($1''$) away from it.

(6) The major axis of an ellipse is 10 cm ($4''$) long and the foci are at a distance of 1·6 cm ($\frac{5}{8}''$) from its ends. Find the minor axis. Prepare a trammel and draw the ellipse using the same.

(7) Two fixed points A and B are 10 cm ($4''$) apart. Trace the complete path of a point P moving (in the same plane as that of A and B) in such a way that, the sum of its distances from A and B is always the same and equal to 12·5 cm ($5''$). Name the curve. Draw another curve parallel to and 2·5 cm ($1''$) away from this curve.

(8) Inscribe an ellipse in a parallelogram having sides 15 cm ($6''$) and 10 cm ($4''$) long and an included angle of 120°.

(9) Two points A and B are 10 cm ($4''$) apart. A point C is 7·5 cm ($3''$) from A and 6 cm ($2\frac{3}{8}''$) from B. Draw an ellipse passing through A, B and C.

(10) Draw a rectangle having its sides 12·5 cm ($5''$) and 7·5 cm ($3''$) long. Inscribe two parabolas in it with their axes bisecting each other.

(11) A ball thrown up in the air reaches a maximum height of 45 metres (150 ft) and travels a horizontal distance of 75 metres (250 ft). Trace the path of the ball, assuming it to be parabolic.

(12) A point P is 3 cm ($1\frac{1}{4}''$) and 5 cm ($2''$) respectively from two straight lines which are at right angles to each other. Draw a rectangular hyperbola from P within 1·2 cm ($\frac{1}{2}''$) distance from each line.

(13) Two straight lines OA and OB make an angle of 75° between them. P is a point 4 cm ($1\frac{1}{2}''$) from OA and 5 cm ($2''$) from OB. Draw a hyperbola through P, with OA and OB as asymptotes, marking at least ten points.

(14) Two points A and B are 5 cm (2″) apart. Draw the curve traced out by a point P moving in such a way that the difference between its distances from A and B is always constant and equal to 2 cm ($\frac{3}{4}$″).

(15) A circle of 5 cm (2″) diameter rolls along a straight line without slipping. Draw a curve traced out by a point P on the circumference, for one complete revolution of the circle. Name the curve. Draw a tangent to the curve at a point on it 4 cm ($1\frac{1}{2}$″) from the line.

(16) Two points Q and S lie on a straight line through the centre C of a circle of 5 cm (2″) diameter, rolling along a fixed straight line. Draw and name the curves traced out by the points Q and S during one revolution of the circle. $CQ =$ 2 cm ($\frac{3}{4}$″); $CS = 3{\cdot}5$ cm ($1\frac{3}{8}$″).

(17) A circle of 5 cm (2″) diameter rolls on the circumference of another circle of 17·5 cm (7″) diameter and outside it. Trace the locus of a point on the circumference of the rolling circle for one complete revolution. Name the curve. Draw a tangent and a normal to the curve at a point 12·5 cm (5″) from the centre of the directing circle.

(18) Construct a hypocycloid, rolling circle 5 cm (2″) diameter and directing circle 17·5 cm (7″) diameter. Draw a tangent to it at a point 5 cm (2″) from the centre of the directing circle.

(19) A circle of 11·5 cm ($4\frac{1}{2}$″) diameter rolls on another circle of 7·5 cm (3″) diameter with internal contact. Draw the locus of a point on the circumference of the rolling circle for its one complete revolution.

(20) A circle of 5 cm (2″) diameter rolls on another circle of 17·5 cm (7″) diameter. Draw and name the curves traced out by two points Q and S lying on a straight line through the centre C of the rolling circle and respectively 2 cm ($\frac{3}{4}$″) and 3·5 cm ($1\frac{3}{8}$″) from it, when it rolls (i) outside and (ii) inside the other circle.

(21) Show by means of a drawing that when the diameter of the directing circle is twice that of the generating circle, the hypocycloid is a straight line. Take the diameter of the generating circle equal to 5 cm (2″).

(22) A circle of 5 cm (2") diameter rolls on a horizontal line for a half revolution and then on a vertical line for another half revolution. Draw the curve traced out by a point P on the circumference of the circle.

(23) Draw an involute of a circle of 4 cm ($1\frac{1}{2}$") diameter. Also, draw a normal and a tangent to it at a point 10 cm (4") from the centre of the circle.

(24) AB is a rope 1·6 metres long, tied to a peg at B (fig. 6-39). Keeping it always tight, the rope is wound round the pole O. Draw the curve traced out by the end A. Scale $\frac{1}{10}$ full size.

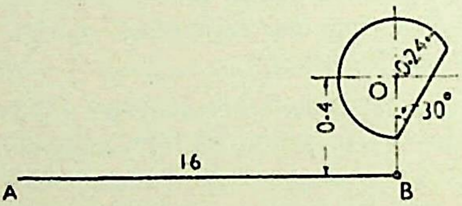


Fig. 6-39

(25) An inelastic string 14·5 cm ($5\frac{3}{4}$") long, has its one end attached to the circumference of a circular disc of 4 cm ($1\frac{1}{2}$") diameter. Draw the curve traced out by the other end of the string, when it is completely wound around the disc, keeping the string always tight.

(26) Draw a circle with diameter AB equal to 6·5 cm ($2\frac{1}{2}$"). Draw a line AC 15 cm (6") long and tangent to the circle. Trace the path of A, when the line AC rolls on the circle, without slipping.

(27) Draw an Archemedian spiral of two convolutions, the greatest and the least radii being 11·5 cm ($4\frac{1}{2}$") and 1·5 cm ($\frac{1}{2}$") respectively. Draw a tangent and a normal to the spiral at a point, 6·5 cm ($2\frac{1}{2}$") from the pole.

(28) A point P moves towards another point O, 7·5 cm (3") from it and reaches it while moving around it once, its movement towards O being uniform with its movement around it. Draw the curve traced out by the point P.

(29) A link OA, 10 cm (4") long rotates about O in an anti-clockwise direction. A point P on the link, 1·5 cm ($\frac{1}{2}$")

away from O, moves and reaches the end A, while the link has rotated through $\frac{3}{4}$ of a revolution. Assuming the movements of the link and the point to be uniform, trace the path of the point P.

(30) Draw a cam to give the following uniform motions to a point: Rise 4 cm ($1\frac{1}{2}''$) in 90°. In upper position for 75°. Fall the same distance in 120°. In lower position for the remaining period of the revolution. Diameter of shaft = 5 cm ($2''$). Least metal = 2 cm ($\frac{3}{4}''$).

(31) Draw the shape of the cam to give the same motions as in Ex. 30, to a roller of 2·5 cm ($1''$) diameter.

(32) A point is raised by a cam uniformly 2·5 cm ($1''$) in $\frac{3}{8}$ of a revolution; kept at the same height for $\frac{1}{8}$ of the revolution; then allowed to drop through 1 cm ($\frac{3}{8}''$) height and remain there for $\frac{1}{8}$ of the revolution; lowered uniformly to its original position in $\frac{1}{4}$ of the revolution and kept there for the rest of the revolution. Draw the shape of the cam. Diameter of shaft = 4 cm ($1\frac{1}{2}''$). Least metal = 2·5 cm ($1''$).

(33) A circle of 4·5 cm diameter rolls on the inside of a circle of 9 cm diameter. A point P lies within the rolling circle at a distance of 1·5 cm from its centre. Trace the path of the point P for one revolution of the circle.

(34) The distance between the foci of an ellipse is 10 cm and the minor axis is 6·5 cm long. Draw the evolute of the ellipse.

(35) Draw the evolutes of the two curves, the data of which is given in Ex. 2.

(36) Construct the evolute of the hyperbola whose data is given in Ex. 3.

(37) Draw the evolute of the cycloid obtained in Ex. 15.

(38) Draw the epicycloid and hypocycloid, when the generating circle and the directing circle are of 5 cm and 17·5 cm diameters respectively. Construct the evolutes of the two curves.

(39) In a logarithmic spiral, the shortest radius is 4 cm. The lengths of adjacent radius vectors enclosing 30° are in the ratio of 9:8. Construct one convolution of the spiral. Draw a tangent to the spiral at any point on it.

CHAPTER **VII**

ORTHOGRAPHIC PROJECTION

Practical Solid Geometry or *Descriptive Geometry* deals with the representation of points, lines, planes and solids on a flat surface (such as a sheet of paper), in such a manner that their relative positions and true forms can be accurately determined.

Projection: If straight lines are drawn from various points on the contour of an object to meet a plane, the object is said to be projected on that plane. The figure formed by joining, in correct sequence, the points at which these lines meet the plane, is called the *projection* of the object. The lines, from the object to the plane, are called *projectors*.

Orthographic projection: When the projectors are parallel to each other and also perpendicular to the plane, the projection is called an *orthographic projection*.

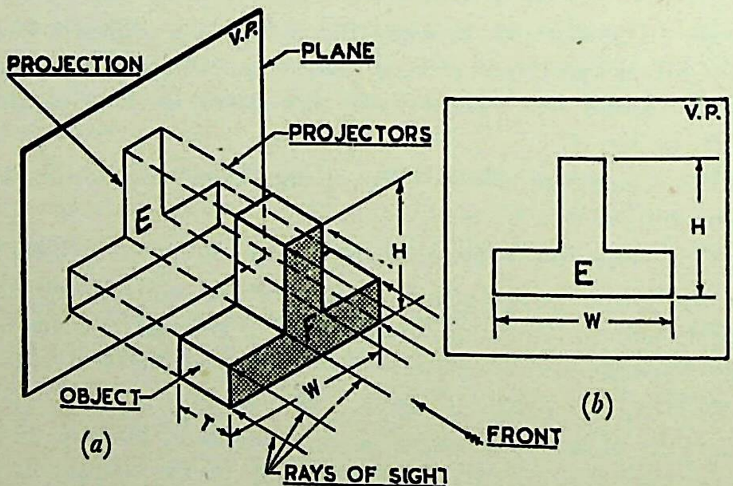


Fig. 7-1

Imagine that a person looks at the block [fig. 7-1(*a*)] from a theoretically infinite distance, so that the rays of sight from his eyes are parallel to one another and perpendicular

to the front surface *F*. The view of this block will be the shaded figure, showing the front surface of the object in its true shape and proportion.

If these rays of sight are extended further to meet perpendicularly a plane (marked V.P.) set up behind the block, and the points at which they meet the plane are joined in proper sequence, the resulting figure (marked *E*) will also be exactly similar to the front surface. This figure is the projection of the block. The lines from the block to the plane are the projectors. As the projectors are perpendicular to the plane on which the projection is obtained, it is the orthographic projection. The projection is shown separately in fig. 7-1(*b*). It shows only two dimensions of the block viz. the height *H* and the width *W*. It does not show the thickness. Thus, we find that only one projection is insufficient for complete description of the block.

Let us further assume that another plane marked H.P. [fig. 7-2(*a*)] is hinged at right angles to the first plane, so that the block is in front of the V.P. and above the H.P. The projection on the H.P. (figure *P*) shows the top surfaces of the block. If a person looks at the block from above, he will obtain the same view as the figure *P*. It shows the width *W* and the thickness *T* of the block; it however does not show the hei–ht of the block.

One of the planes is now rotated or turned around on the hinges so that it lies in extension of the other plane. This can be done in two ways: (i) by turning the V.P. in direction of arrows *A* or (ii) by turning the H.P. in direction of arrows *B*. The H.P. when turned and brought in line with the V.P. is shown by dashed lines. The two projections can now be drawn on a flat sheet of paper, in correct relationship with each other, as shown in fig. 7-2(*b*). When studied together, they supply all information regarding the shape and the size of the block. Any solid may thus be represented by means of orthographic projections or orthographic views.

Planes of projection: The two planes employed for the purpose of orthographic projections are called

Reference planes or *Principal planes of projection*. They intersect each other at right angles. The *vertical plane* of projection (in front of the observer) is usually denoted by the letters V.P. The other is the *horizontal plane* of projection known as the H.P. The line in which they intersect is termed the

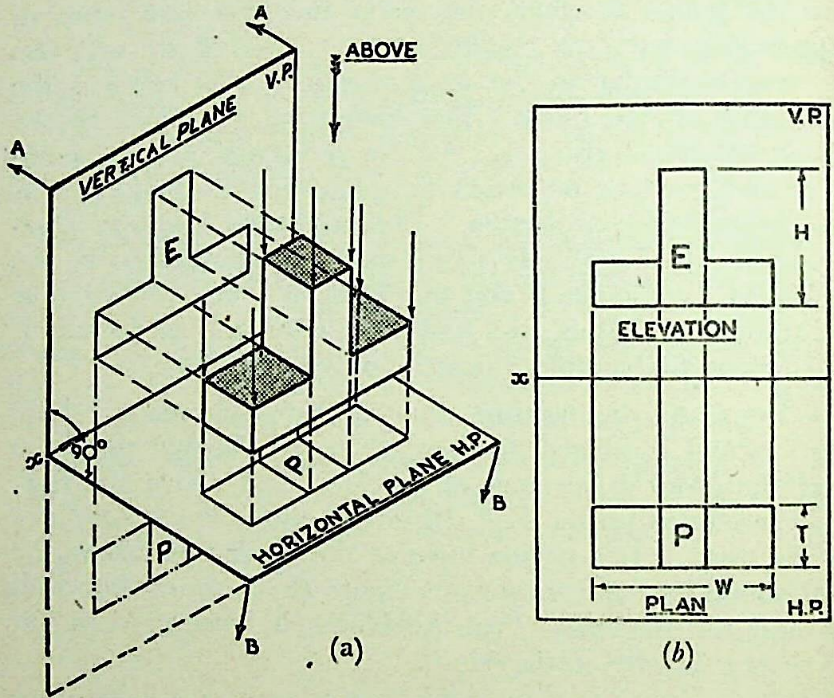


FIG. 7-2

ground line and is denoted by letters *xy*. The projection on the V.P. is the *front elevation* of the object and is commonly known as only *elevation;* while that on the H.P. is called the *plan*. The elevation is thus another name for *front view;* similarly, the plan is another name for *top view*.

Four quadrants: When the planes of projection are extended beyond the line of intersection, they form four quadrants or dihedral angles which may be numbered as in fig. 7-3. The object may be situated in any one of the quadrants, its position relative to the planes being described as 'above or below the H.P.' and 'in front of or behind the V.P.' The planes are assumed to be transparent. The

projections are obtained by drawing perpendiculars from the object to the planes, i.e., by looking from the front and from above. They are then shown on a flat surface by rotating one of the planes as already explained. It should be remembered that the first quadrant is always opened out while rotating the planes. The positions of the views with respect to the ground line will change according to the quadrant in which the object may be situated. This has been explained in detail in the next chapter.

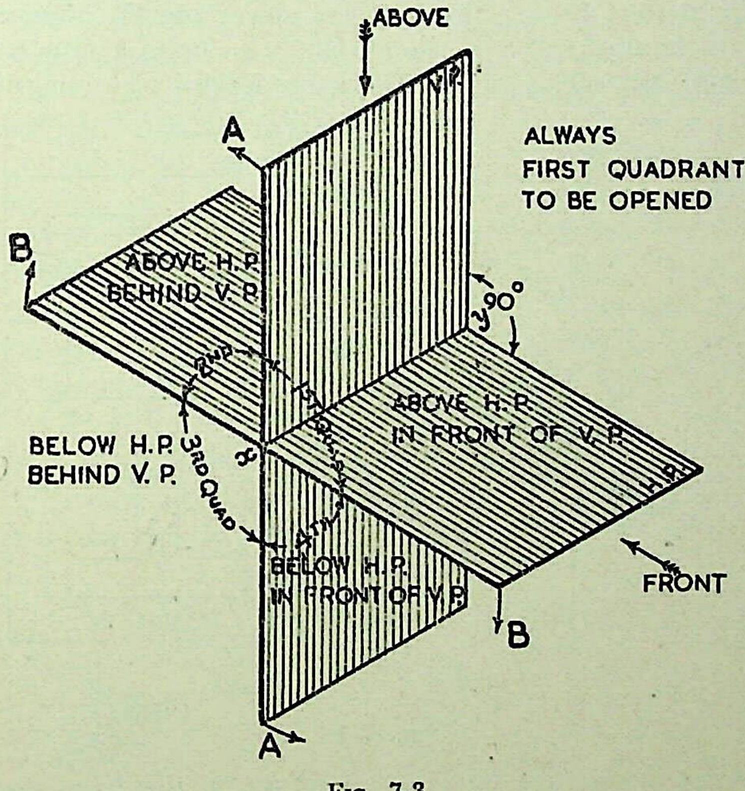
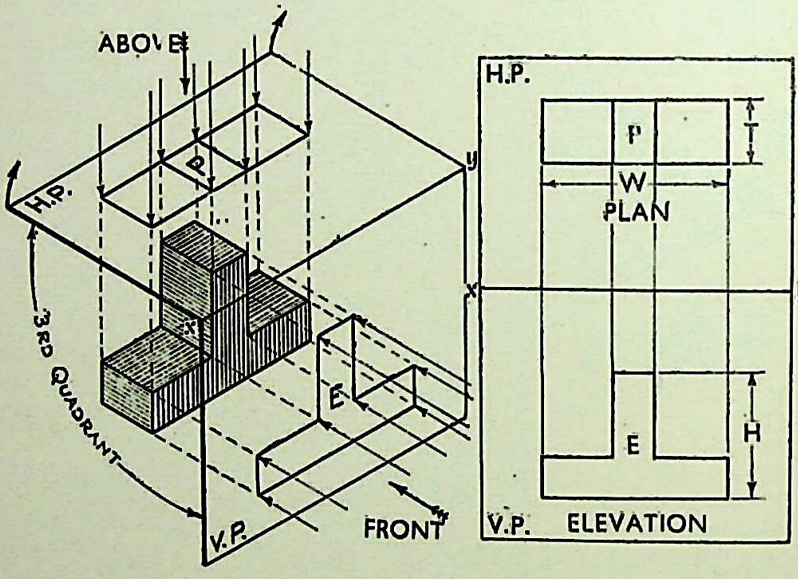


FIG. 7-3

First-angle projection: We have assumed the object to be situated in front of the V.P. and above the H.P. i.e. in the first quadrant and then projected it on these planes. This method of projection is known as *first-angle projection method*. The object lies between the observer and the plane of projection. In this method, when the views are drawn

in their relative positions, the plan comes below the elevation. In other words, the view seen from above is placed on the other side of (i.e. below) the elevation. Each projection shows the view of that surface (of the object) which is remote from the plane on which it is projected and which is nearest to the observer.

Third-angle projection: In this method of projection, the object is assumed to be situated in the third quadrant [fig. 7-4(*i*)]. The planes of projection are assumed to be transparent. They lie between the object and the observer. When the observer views the object from the front, the rays of sight intersect the V.P. The figure formed by joining the



(*i*) (*ii*)

Fig. 7-4

points of intersection in correct sequence is the front view or the elevation of the object. The top view or the plan is obtained in a similar manner by looking from above. When the two planes are brought in line with each other, the views will be seen as shown in fig. 7-4(*ii*). The plan in this case comes above the elevation. In other words, the view seen from above the object is placed on the same side of (i.e. above) the elevation. Each projection shows the view of that

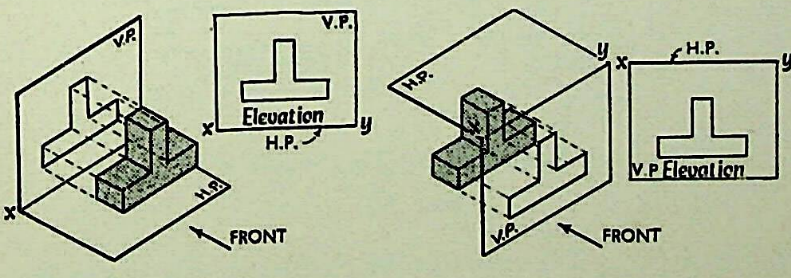
surface (of the object) which is nearest to the plane on which it is projected.

On comparison, it is quite evident that the views obtained by the two methods of projection are completely identical in shape, size and all other details. The difference lies in their relative positions only.

The method of first-angle projection is the British standard practice. The third-angle projection is the standard practice followed in America and in the continent of Europe. In our country, till recently, the first-angle projection method was in use. The Indian Standards Institution has recommended the adoption of third-angle projection method as a standard practice in our country and hence, it is fast coming in general use. Persons in engineering profession may come across drawings from countries following any one method. It is, therefore, necessary for them to be conversant with both the methods.

In this edition of the book, the third-angle projection method has been adequately introduced. Both the methods are treated side by side. However, when nothing is stated, the projections should be assumed to be by first-angle projection method.

Ground line: Studying the projections independently, it can be seen that while considering the elevation (fig. 7-5), which is the view as seen from the front, the H.P. coincides

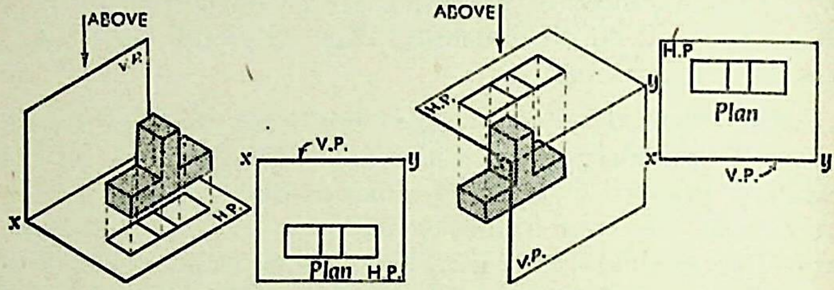


First-angle proj. Third-angle proj.

FIG. 7-5

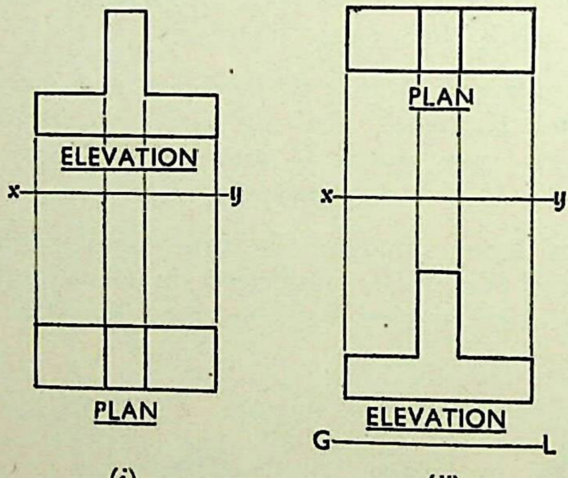
with the line xy. In other words, xy represents the H.P. Similarly, while considering the plan (fig. 7-6), which is the view obtained by looking from above, the same line xy re-

presents the V.P. Hence, when the two projections are drawn in correct relationship with each other (fig. 7-7), *xy* represents both the H.P. and the V.P. This line *xy* is called



First-angle projection Third-angle projection

FIG. 7-6



(*i*) First-angle projection (*ii*) Third-angle projection

FIG. 7-7

the *ground line*. The squares or rectangles for individual planes are thus unnecessary and may, therefore, be discarded.

Further, in first-angle projection method, the H.P. is always assumed to be so placed as to coincide with the ground on or above which the object is situated. Hence, in this method, the line *xy* is also the line for the ground.

In third-angle projection method, the H.P. is assumed to be placed above the object. The object may be situated on or above the ground. Hence, in this method, the line *xy* does not represent the ground. The line for the ground, denoted by letters *GL*, may be drawn parallel to *xy* and below the elevation [fig. 7-7(*ii*)].

In brief, when an object is situated on the ground, in first-angle projection method, the bottom of its elevation will coincide with xy; in third-angle projection method, it will coincide with GL, while xy will be above the elevation and parallel to GL.

Conventions employed: In this book, actual points, ends of lines, corners of solids etc. in space are denoted by capital letters *A*, *B*, *C* etc. Their plans are marked by corresponding small letters *a*, *b*, *c* etc. and their elevations by small letters with dashes *a′*, *b′*, *c′* etc. In pictorial views, the projectors from the points in space to the planes are shown by dashed lines. The lines from the projections to the ground line *xy* (which are also called projectors, though they are the projections of the projectors) are shown as dash and dot lines. In orthographic views, the projectors and other construction lines are shown continuous, but thinner than the lines for actual projections.

CHAPTER **VIII**

PROJECTIONS OF POINTS

A point may be situated, in space, in any one of the four quadrants formed by the two principal planes of projection or may lie in any one or both of them. Its projections are obtained by extending projectors perpendicular to the planes One of the planes is then rotated so that the first quadrant is opened out. The projections are shown on a flat surface in their respective positions either above or below or in *xy*.

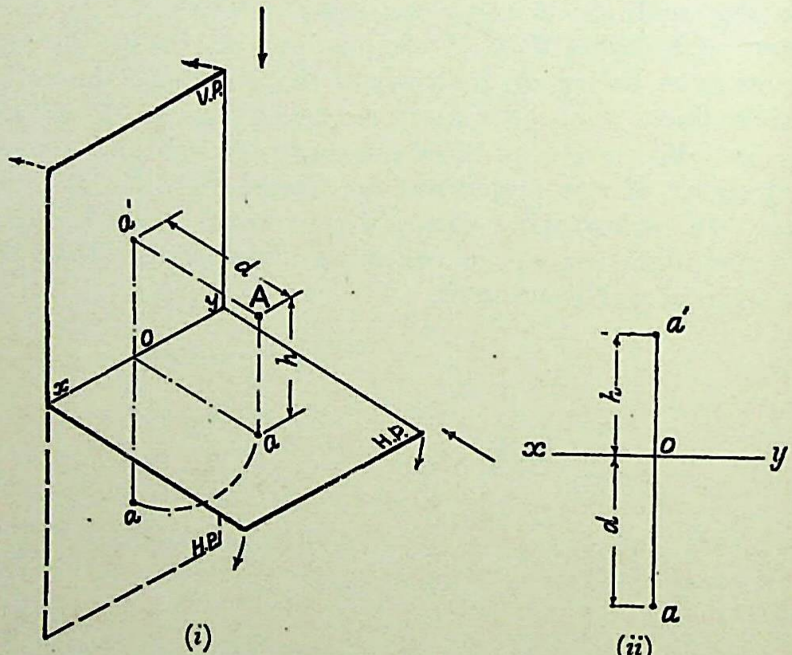


FIG. 8-1

The pictorial view [fig. 8-1(*i*)], shows a point *A*, situated above the H.P. and in front of the V.P., i e. in the first quadrant. a' is its elevation and *a*, the plan. After rotation of the plane, these projections will be seen as shown in

fig. 8-1(*ii*). The elevation a' is above xy and the plan a below it. The line joining a' and a (which also is called a projector), intersects xy at right angles at point o. It is quite evident from the pictorial view that $a'o = Aa$, i.e., the distance of the elevation from xy = the distance of A from the H.P. viz. h. Similarly, $ao = Aa'$, i.e., the distance of the plan from xy = the distance of A from the V.P. viz. d.

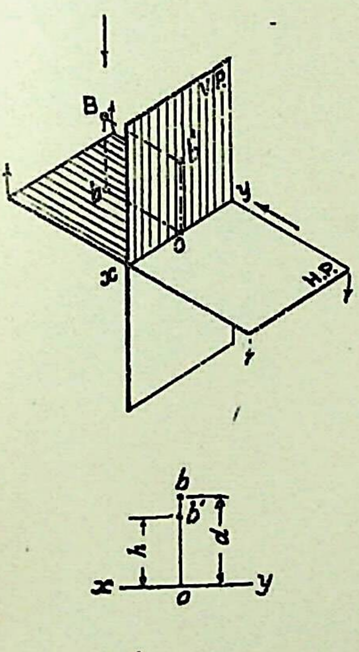


FIG. 8-2

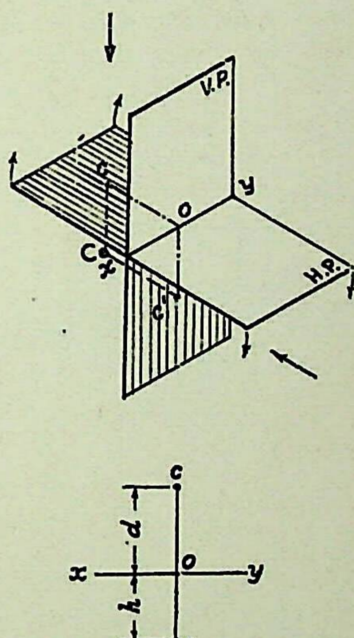


FIG. 8-3

A point B (fig. 8-2) is above the H.P. and behind the V.P., i.e., in the second quadrant. b' is the elevation and b the plan. When the planes are rotated, both the views are seen above xy. Note that $b'o = Bb$ and $bo = Bb'$.

A point C (fig. 8-3) is below the H.P. and behind the V.P., i.e., in the third quadrant. Its elevation c' is below xy and plan c above xy. Also $c'o = Cc$ and $co = Cc'$.

A point E (fig. 8-4) is below the H.P. and in front of the V.P., i.e., in the fourth quadrant. Both its projections are below xy, and $e'o = Ee$ and $eo = Ee'$.

Referring to fig. 8-5, we see that,

(i) A point P is in the H.P. and in front of the V.P. Its elevation p' is in xy and plan p below it.

(ii) A point Q is in the V.P. and above the H.P. Its plan q is in xy, and elevation q' above it.

(iii) A point O is in both the H.P. and the V.P. Its projections o and o' coincide in xy.

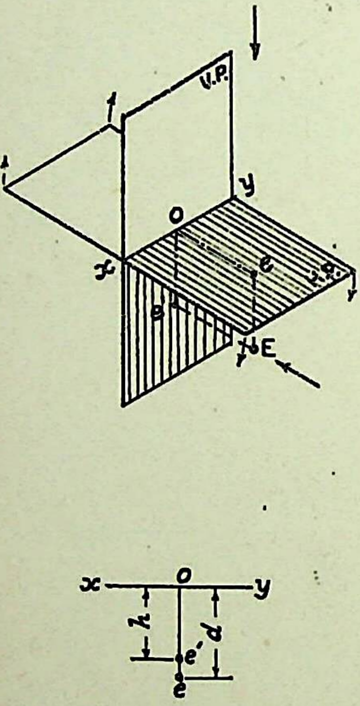


Fig. 8-4

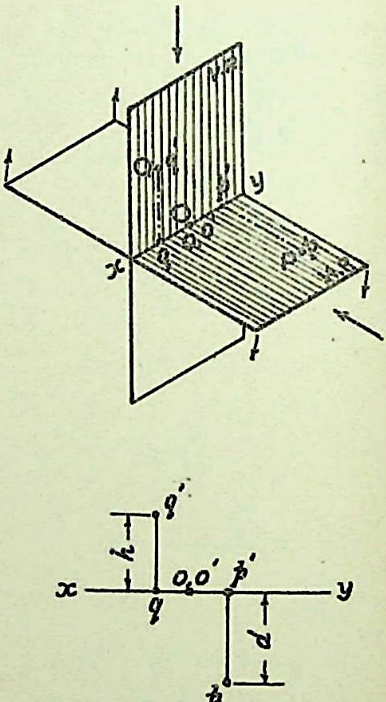


Fig. 8-5

General conclusions:

(i) The line joining the plan and the elevation of a point is always perpendicular to xy. It is called a projector.

(ii) When a point is above the H.P., its elevation is above xy; when it is below the H.P., the elevation is below xy. The distance of a point from the H.P. is shown by the length of the projector from its elevation to xy, e.g. $a'o$, $b'o$ etc.

(iii) When a point is in front of the V.P., its plan is below xy; when it is behind the V.P., the plan is above xy. The distance of a point from the V.P. is shown by the length of the projector, from its plan to xy, e.g. ao, bo etc.

(iv) When a point is in a reference plane, its projection on the other plane is in xy.

Problem 1:

A point A is 2·5 cm (1″) above the H.P. and 3 cm ($1\frac{1}{4}$″) in front of the V.P. Draw its projections.

Draw a ground line xy [fig. 8-1(*ii*)].

Through any point o in it, draw a perpendicular.

As the point is above the H.P. and in front of the V.P. its elevation will be above xy and the plan below xy.

On the perpendicular, mark a point a' above xy, such that $a'a = 2{\cdot}5$ cm (1″). Similarly, mark a point a below xy, so that $ao = 3$ cm ($1\frac{1}{4}$″).

a' and a are the required projections.

EXERCISES VIII

(1) Draw the projections of the following points on the same ground line, keeping the projectors 2·5 cm (1″) apart:

A, in the H.P. and 2 cm ($\frac{3}{4}$″) behind the V.P.

B, 4 cm ($1\frac{1}{2}$″) above the H.P. and 2·5 cm (1″) in front of the V.P.

C, in the V.P. and 4 cm ($1\frac{1}{2}$″) above the H.P.

D, 2·5 cm (1″) below the H.P. and 2·5 cm (1″) behind the V.P.

E, 1·5 cm ($\frac{5}{8}$″) above the H.P. and 5 cm (2″) behind the V.P.

F, 4 cm ($1\frac{1}{2}$″) below the H.P. and 2·5 cm (1″) in front of the V.P.

G, in both the H.P. and the V.P.

(2) A point *P* is 5 cm (2″) from both the reference planes. Draw its projections in all possible positions.

(3) State the quadrants in which the following points are situated:

(a) A point P; its plan is 4 cm ($1\frac{1}{2}''$) below xy; the elevation is 2 cm ($\frac{3}{4}''$) above the plan.

(b) A point Q; its projections coincide with each other 4 cm ($1\frac{1}{2}''$) above xy.

(4) Projections of various points are given in fig. 8-6. State the position of each point with respect to the planes of projection, giving the distances in centimetres.

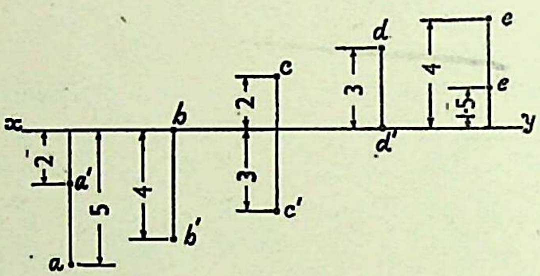


FIG. 8-6

(5) A point P is 1·5 cm ($\frac{5}{8}''$) above the H.P. and 2 cm ($\frac{3}{4}''$) in front of the V.P. Another point Q is 2·5 cm ($1''$) behind the V.P. and 4 cm ($1\frac{1}{2}''$) below the H.P. Draw the projections of P and Q, keeping the distance between their projectors equal to 9 cm ($3\frac{1}{2}''$). Draw straight lines joining (i) their plans and (ii) their elevations.

(6) Two points A and B are in the H.P. The point A is 3 cm ($1\frac{1}{4}''$) in front of the V.P., while B is behind the V.P. The distance between their projectors is 7·5 cm ($3''$) and the line joining their plans makes an angle of 45° with xy. Find the distance of the point B from the V.P.

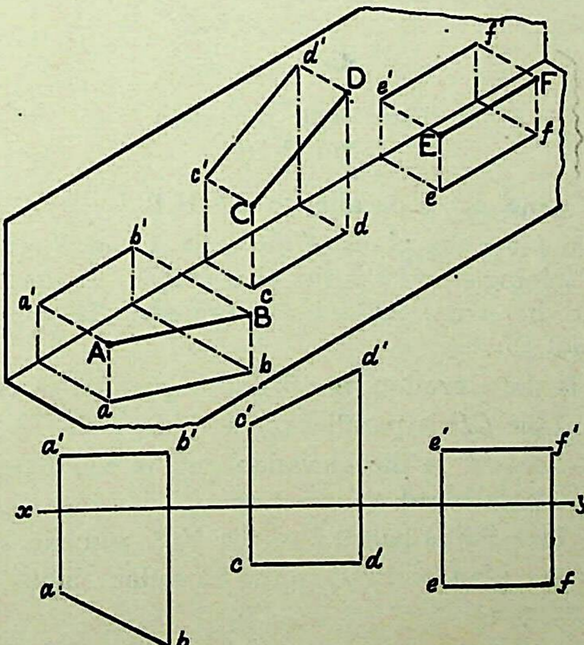
CHAPTER **IX**

PROJECTIONS OF STRAIGHT LINES

A straight line is the shortest distance between two points. Hence, the projections of a straight line may be drawn by joining the respective projections of its ends which are points.

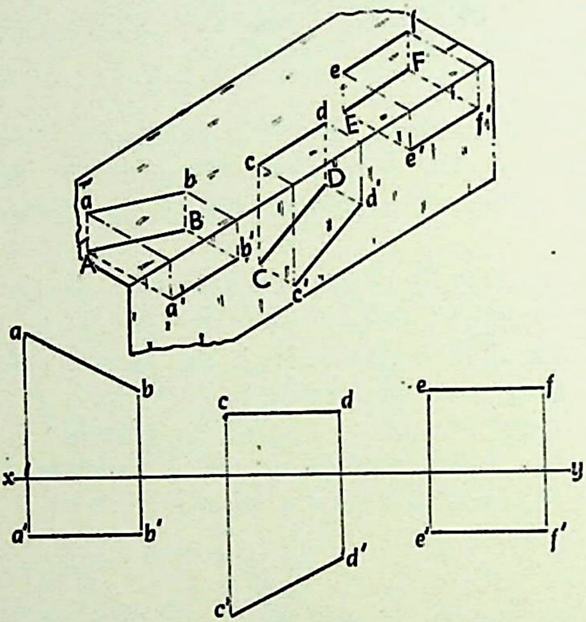
The position of a straight line may also be described with respect to the two reference planes. It may be:

(i) Parallel to one or both the planes.
(ii) Contained by one or both the planes.
(iii) Perpendicular to one of the planes.
(iv) Inclined to one plane and parallel to the other.
(v) Inclined to both the planes.



First-angle projection
FIG. 9-1(*i*)

1. Straight line parallel to one or both the planes: [figs. 9-1(*i*) and 9-1(*ii*)].



Third-angle projection
FIG. 9-1(*i*)

(a) Line *AB* is parallel to the H.P.

a and *b* are the plans of the ends *A* and *B* respectively. The line joining *a* and *b* is the plan of *AB*. It can be clearly seen that the figure *ABba* is a rectangle. Hence, the plan *ab* is equal to *AB*.

a'b' is the elevation of *AB* and is parallel to *xy*.

(b) Line *CD* is parallel to the V.P.

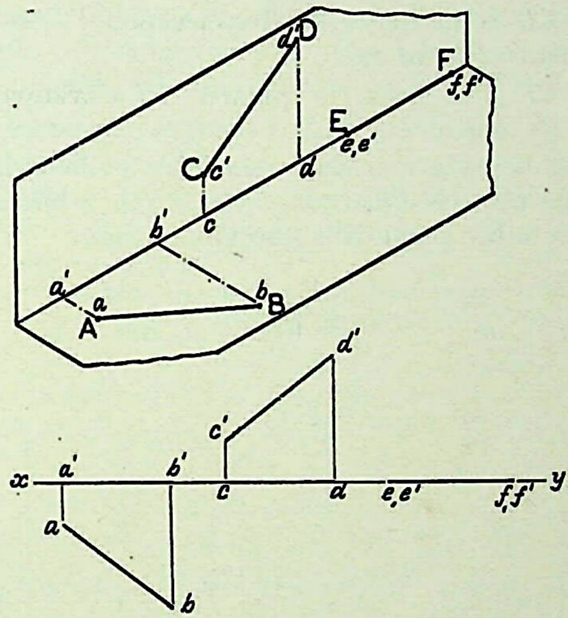
The line *c'd'* is the elevation and is equal to *CD*; the plan *cd* is parallel to *xy*.

(c) Line *EF* is parallel to the H.P. and the V.P.

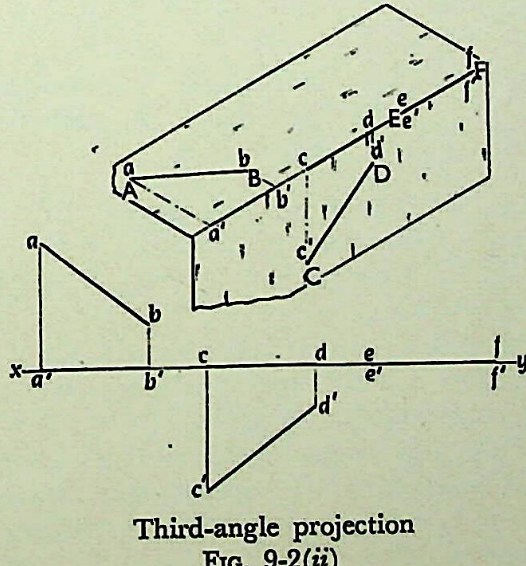
ef is the plan and *e'f'* is the elevation; both are equal to *EF* and parallel to *xy*.

Hence, when a line is parallel to a plane, its projection on that plane is equal to its true length; while its projection on the other plane is parallel to the ground line.

2. Line contained by one or both the planes: [figs. 9-2(*i*) and 9-2(*ii*)].



First-angle projection
FIG. 9-2(*i*)



Third-angle projection
FIG. 9-2(*ii*)

Line *AB* is in the H.P. Its plan *ab* is equal to *AB*; its elevation *a'b'* is in *xy*.

Line *CD* is in the V.P. Its elevation *c'd'* is equal to *CD*; its plan *cd* is in *xy*.

Line *EF* is in both the planes. Its elevation *e'f'* and the plan *ef* coincide in *xy*.

Hence, when a line is contained by a plane, its projection on that plane is equal to its true length; while its projection on the other plane is in the ground line.

3. Line perpendicular to a plane: (fig. 9-3). When a line is perpendicular to one reference plane, it will be parallel to the other.

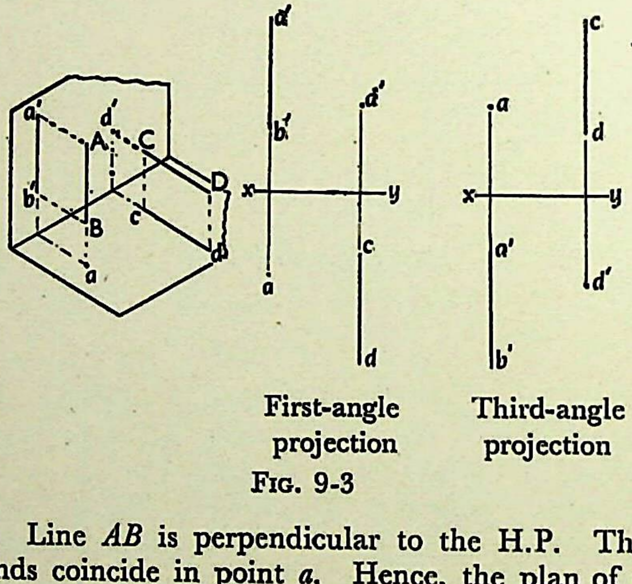


Fig. 9-3

(a) Line *AB* is perpendicular to the H.P. The plans of its ends coincide in point *a*. Hence, the plan of the line *AB* is the point *a*. Its elevation *a'b'* is equal to *AB* and perpendicular to *xy*.

(b) Line *CD* is perpendicular to the V.P. The point *d'* is its elevation and the line *cd* is the plan. *cd* is equal to *CD* and perpendicular to *xy*.

Hence, when a line is perpendicular to a plane its projection on that plane is a point; while its projection on the other plane is equal to its true length and perpendicular to the ground line.

In *first-angle projection method*, when plans of two or more points coincide, the point which is comparatively *farther* away from *xy* in elevation will be visible; and when their elevations coincide, that which is *farther* away from *xy* in plan will be visible.

In *third-angle projection method*, it is just the reverse. When plans of two or more points coincide, the point which is comparatively *nearer xy* in elevation will be visible; and when their elevations coincide, the point which is *nearer xy* in plan will be visible.

4. Line inclined to one plane and parallel to the other:

The inclination of a line to a plane is the angle which the line makes with its projection on that plane.

(a) Line PQ_1 [fig. 9-4(*i*)] is inclined at an angle θ to the H.P. and is parallel to the V.P. The inclination is shown by the angle θ which PQ_1 makes with its own projection on the H.P., viz. the plan pq_1.

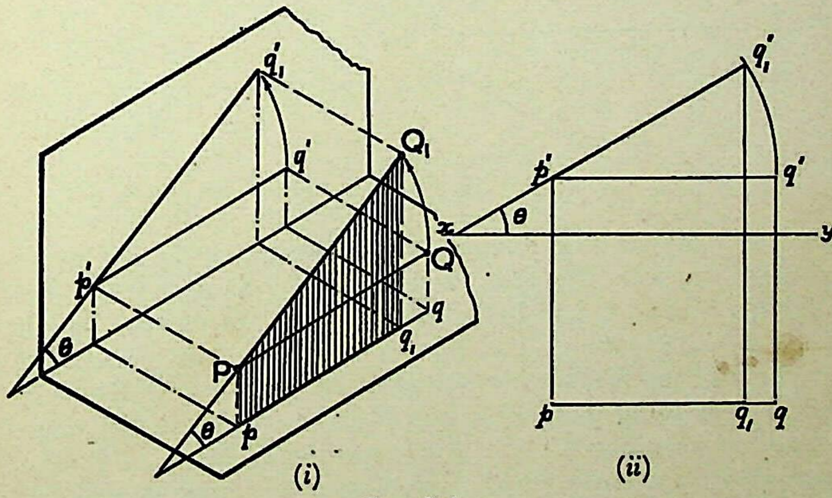


FIG. 9-4

The projections [fig. 9-4((*ii*)] may be drawn by first assuming the line to be parallel to both the H.P. and the V.P. Its elevation $p'q'$ and the plan pq will both be parallel to xy and equal to the true length. When the line is turned

about the end P to the position PQ_1 so that it makes the angle θ with the H.P. while remaining parallel to the V.P., in the elevation, the point q' will move along an arc drawn with p' as centre and $p'q'$ as radius to a point q_1', so that $p'q_1'$ makes the angle θ with xy. In the plan, q will move towards p along pq, to a point q_1 on the projector through q_1'. $p'q_1'$ and pq_1 are the elevation and the plan respectively of PQ_1.

(b) Line RS_1 [fig. 9-5(i)] is inclined at an angle ϕ to the V.P. and is parallel to the H.P The inclination is shown by the angle ϕ which RS_1 makes with it projection

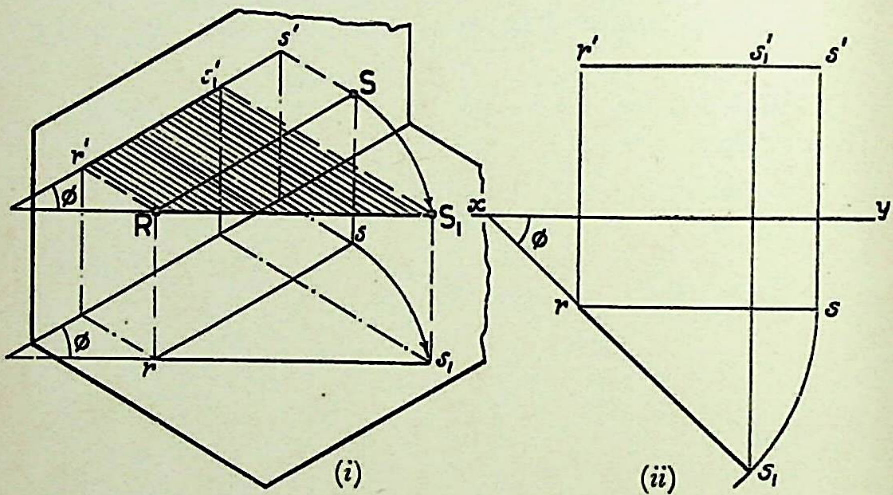


FIG. 9-5

on the V.P., viz the elevation $r's_1'$. Assuming the line to be parallel to both the H.P. and the V.P., its projections $r's'$ and rs are drawn parallel to xy and equal to its true length [fig. 9-5(ii)]. When the line is turned about its end R to the position RS_1 so that it makes the angle ϕ with the V.P. while remaining parallel to the H.P., in the plan, the point s will move along an arc drawn with r as centre and rs as radius, to a point s_1 so that rs_1 makes the angle ϕ with xy. In the elevation, the point s' will move towards r', along the line $r's'$, to a point s_1' on the projector through s_1. rs_1 and $r's_1'$ are the projections of the line RS_1.

Therefore, when the line is inclined to the H.P. and parallel to the V.P., its plan is shorter than its true length,

but parallel to *xy*; its elevation is equal to its true length and is inclined to *xy* at its true inclination with the H.P. And when the line is inclined to the V.P. and parallel to the H.P. its elevation is shorter than its true length, but parallel to *xy*; its plan is equal to its true length and is inclined to *xy* at its true inclination with the V.P.

Hence, when a line is inclined to one plane and parallel to the other, its projection on the plane to which it is inclined is a line, shorter than its true length, but parallel to the ground line; its projection on the plane to which it is parallel, is a line, equal to its true length and inclined to the ground line at its true inclination.

In other words, the inclination of a line with the H.P. is seen in the elevation and that with the V.P. is seen in the plan.

Problem 1:

A line PQ, 9 cm ($3\frac{1}{2}''$) long, is in the H.P. and makes an angle of 30° with the V.P. Draw its projections, if P is 2·5 cm ($1''$) in front of the V.P. [fig. 9-6(*i*)].

As the line is in the H.P., its plan will show the true length and the true inclination with the V.P.; its elevation will be in *xy*.

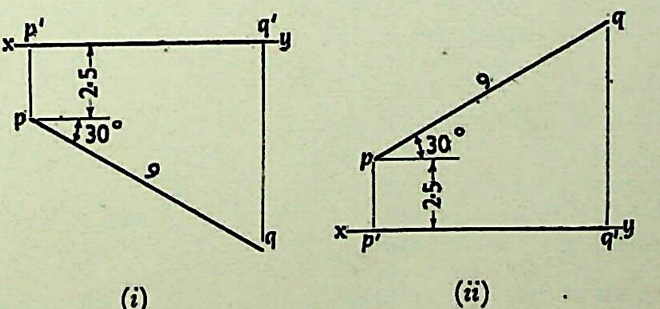


FIG. 9-6

Mark a point *p*, the plan, 2·5 cm ($1''$) below *xy*. Draw a line *pq* equal to 9 cm ($3\frac{1}{2}''$) and inclined at 30° to *xy*. Project *p* to *p'* and *q* to *q'* on *xy*.

pq and *p'q'* are the required plan and elevation respectively.

Fig. 9-6(*ii*) shows the projections when the line is in the third quadrant, the end *P* being 2·5 cm (1″) behind the V.P. Note that the plan *p* is 2·5 cm (1″) above *xy*.

Problem 2:

The length of the plan of a line parallel to the V.P. and inclined at 45° to the H.P. is 5 cm (2″). One end of the line is 1·2 cm (½″) above the H.P. and 2·5 cm (1″) in front of the V.P. Draw the projections and determine the length of the line.

Mark *a*, the plan, 2·5 cm (1″) below *xy* and *a′*, the elevation, 1·2 cm (½″) above *xy* [fig. 9-7(*i*)].

As the line is parallel to the V.P., its plan will be parallel to *xy* and the elevation will show its true length and the true inclination with the H.P.

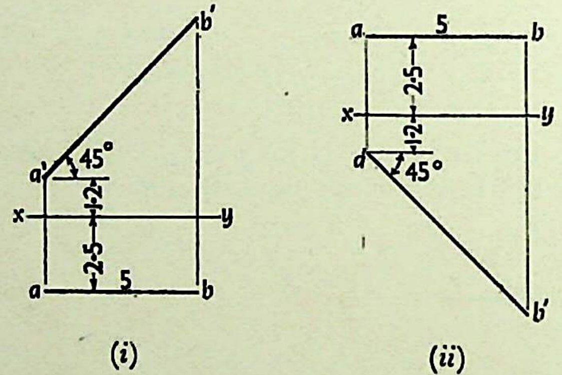


FIG. 9-7

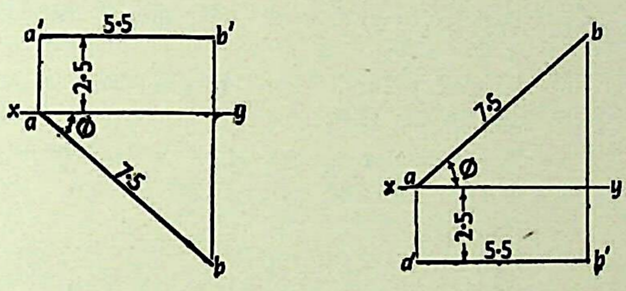
Draw the plan *ab* equal to 10 cm (4″) and parallel to *xy* and draw a projector through *b*.

From *a′* draw a line making 45° angle with *xy* and cutting the projector through *b* at *b′*. Then *a′b′* is the elevation and also the true length of the line.

Fig. 9-7(*ii*) shows the projections when the line is in the third quadrant, one end being 1·2 cm (½″) below the H.P. and 2·5 cm (1″) behind the V.P. The plan *a* is 2·5 cm (1″) above *xy* and the elevation *a′* is 1·2 cm (½″) below *xy*.

Problem 3:

The elevation of a line 7·5 cm (3") long, measures 5·5 cm ($2\frac{1}{4}$"). The line is parallel to the H.P. and one of its ends is in the V.P. and 2·5 cm (1") above the H.P. Draw the projections of the line and determine its inclination with the V.P.



(*i*) (*ii*)

FIG. 9-8

Mark *a*, the plan of one end in *xy*, and *a'*, its elevation, 2·5 cm (1") above *xy* [fig. 9-8(*i*)]. As the line is parallel to the H.P., its elevation will be parallel to *xy*.

Draw the elevation *a'b'* equal to 5·5 cm ($2\frac{1}{4}$") and parallel to *xy*. With *a* as centre and radius equal to 7·5 cm (3"), draw an arc cutting the projector through *b'* at *b*. Join *a* and *b*; *ab* is the plan of the line. Its inclination with *xy*, viz. ϕ is the inclination of the line with the V.P.

Fig. 9-8(*ii*) shows the projections when the line is in the third quadrant, one end being in the V.P. and 2·5 cm (1") below the H.P. The plan *a* is in *xy* and the elevation *a'* is 2·5 cm (1") below *xy*.

EXERCISES IX(a)

(1) Draw the projections of a straight line, 7·5 cm (3″) long in the following positions:

(a) (i) Parallel to both the H.P. and the V.P. and 2·5 cm (1″) from each.

(ii) Parallel to and 3 cm ($1\frac{1}{4}$″) above the H.P. and in the V.P.

(iii) Parallel to and 4 cm ($1\frac{1}{2}$″) in front of the V.P. and in the H.P.

(b) (i) Perpendicular to the H.P., 2 cm ($\frac{3}{4}$″) in front of the V.P. and its one end 1·5 cm ($\frac{5}{8}$″) above the H.P.

(ii) Perpendicular to the V.P., 2·5 cm (1″) above the H.P. and its one end in the V.P.

(iii) Perpendicular to the H.P., in the V.P. and its one end in the H.P.

(c) (i) Inclined at 45° to the V.P., in the H.P. and its one end in the V.P.

(ii) Inclined at 30° to the H.P. and its one end 2 cm ($\frac{3}{4}$″) above it; parallel to and 3 cm ($1\frac{1}{4}$)″ in front of the V.P.

(iii) Inclined at 60° to the V.P. and its one end 1·5 cm ($\frac{5}{8}$″) in front of it; parallel to and 2·5 cm (1″) above the H.P.

(2) A line, 10 cm (4″) long, is parallel to and 4 cm ($1\frac{1}{2}$″) above the H.P. Its two ends are 2·5 cm (1″) and 5 cm (2″) in front of the V.P. respectively. Find its inclination with the V.P.

(3) A line, 9 cm ($3\frac{1}{2}$″) long is parallel to and 2·5 cm (1″) in front of the V.P. Its one end is in the H.P. while the other is 5 cm (2″) above the H.P. Find its inclination with the H.P.

(4) The plan of a line, 7·5 cm (3″) long, measures 5·5 cm ($2\frac{1}{4}$″). The line is in the V.P., its one end being 2·5 cm (1″) above the H.P. Draw its projections.

(5) The elevation of a line, inclined at 30° to the V.P. is 6·5 cm ($2\frac{1}{2}$″) long. Draw the projections of the line, when it is parallel to and 4 cm ($1\frac{1}{2}$″) above the H.P., its one end being 3 cm ($1\frac{1}{4}$″) in front of the V.P.

(6) A vertical line AB 7·5 cm (3") long, has its end A in the H.P. and 2·5 cm (1") in front of the V.P. A line AC, 10 cm (4") long, is in the H.P. and parallel to the V.P. Draw the projections of the line joining B and C, and determine its inclination with the H.P.

(7) Two pegs fixed on a wall are 4·5 metres (15 feet) apart. The distance between the pegs measured parallel to the floor is 3·6 metres (12 feet). If one peg is 1·5 metres (5 feet) above the floor, find the height of the second peg and the inclination of the line joining the two pegs, with the floor, using third-angle projection method.

(8) Draw the projections of the lines in Exs. 1 to 6, assuming them to be in the third quadrant, and taking the given positions to be below the H.P. instead of above the H.P. and behind the V.P. instead of in front of the V.P.

5. Line inclined to both the planes:

(a) A line AB (fig. 9-9), is inclined at θ to the H.P. and is parallel to the V.P. The end A is in the H.P. AB is shown as the hypotenuse of a right-angled triangle, making the angle θ with the base.

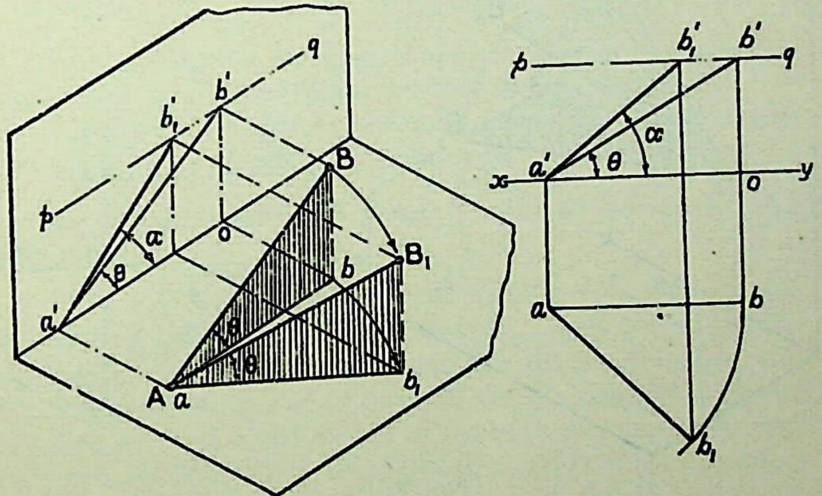


FIG. 9-9

The plan ab is shorter than AB and parallel to xy. The elevation $a'b'$ is equal to AB and makes the angle θ with xy.

Keeping the end A fixed and the angle θ with the H.P. constant, if the end B is moved to any position, say B_1, the line becomes inclined to the V.P. also.

In the plan, b will move along an arc, drawn with a as centre and ab as radius, to a position b_1. The new plan ab_1 is equal to ab, but shorter than AB.

In the elevation, b' will move to a point b_1' keeping its distance from xy constant and equal to $b'o$; i.e., it will move along the line pq, drawn through b' and parallel to xy. This line pq is the locus or path of the end B in the elevation. b_1' will lie on the projector through b_1. The new elevation $a'b_1'$ is shorter than $a'b'$ (i.e. AB) and makes and angle α with xy; α is greater than θ.

Thus, it can be seen that as long as the inclination θ of AB with the H.P. is constant, even when it is inclined to the V.P., (i) its length in the plan, viz. ab, remains constant; and (ii) the distance between the paths of its ends in the elevation, viz. $b'o$, remains fixed.

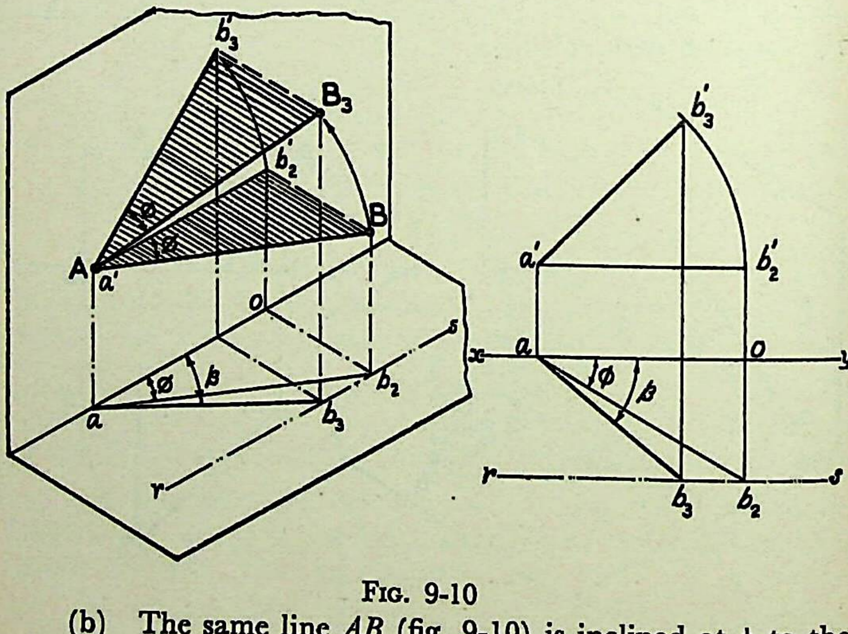


FIG. 9-10

(b) The same line AB (fig. 9-10) is inclined at ϕ to the V.P. and is parallel to the H.P. Its end A is in the V.P. AB is shown as the hypotenuse of a right-angled triangle, making the angle ϕ with the base.

The elevation $a'b_2'$ is shorter than AB and parallel to xy. The plan ab_2 is equal to AB and makes the angle ϕ with xy.

Keeping the end A fixed and the angle ϕ with the V.P. constant, if B is moved to any position, say B_3, the line will become inclined to the H.P. also.

In the elevation, b_2' will move along the arc, drawn with a' as centre and $a'b_2'$ as radius, to a position b_3'. The new elevation $a'b_3'$ is equal to $a'b_2'$ but is shorter than AB.

In the plan, b_2 will move to a point b_3 along the line rs, drawn through b_2 and parallel to xy, thus keeping its distance from the path of a, viz. b_2o constant. rs is the locus or path of the end B in the plan. The point b_3 lies on the projector through b_3'. The new plan ab_3 is shorter than ab_2 (i.e. AB) and makes an angle β with xy; β is greater than ϕ.

Here also we find that, as long as the inclination of AB with the V.P. does not change, even when it becomes inclined to the H.P., (i) its length in the elevation viz. $a'b_2'$, remains constant; and (ii) the distance between the paths of its ends in the plan, viz. b_2o, remains constant.

Hence, when a line is inclined to both the planes, its projections are shorter than the true length and inclined to xy at angles greater than the true inclinations. These angles are called *apparent angles* of inclination.

6. Projections of lines inclined to both the planes:

From art. 5(a) above, we find that as long as the inclination of AB with H.P. is constant, (i) its length in the plan viz. ab remains constant, and (ii) in the elevation, the distance between the loci of its ends, viz. $b'o$ remains constant. In other words if (i) its length in the plan is equal to ab and (ii) the distance between the paths of its ends in the elevation is equal to $b'o$, the inclination of AB with the H.P. will be equal to θ.

Similarly, from art. 5(b) above, we find that as long as the inclination of AB with the V.P. is constant, (i) its length in

the elevation viz. $a'b_2'$, remains constant and (ii) in the plan, the distance between the loci of its ends, viz. b_2o, remains constant. The reverse of this is also true, viz. (i) if its length in the elevation is equal to $a'b_2'$ and (ii) the distance between the paths of its ends in the plan is equal to b_2o, the inclination of AB with the V.P. will be equal to ϕ.

Therefore, in fig. 9-9, if the end B is moved, till in the plan, the distance between a and b perpendicular to xy, is equal to b_2o (see fig. 9-10), the inclination of AB with the V.P. will become equal to θ, and its length in the elevation will automatically become equal to $a'b_2'$.

Similarly, in fig. 9-10, if the end B is moved, so that in the elevation, the distance between a' and b', perpendicular to xy, is equal to $b'o$ (see fig. 9-9), the inclination of AB with the H.P. will become equal to θ, and the length in the plan will become equal to ab.

Combining the above two cases, we conclude that when AB is inclined at θ to the H.P. and at ϕ to the V.P., (i) its lengths in the plan and the elevation will be equal to ab and $a'b_2'$ respectively and (ii) the distances between the paths of its ends in the elevation and the plan, will be equal to $b'o$ and b_2o respectively. The two lengths when arranged with their ends in their respective paths and in projection with each other will be the projections of the line AB, as illustrated in problem 4.

Problem 4:

Given the line AB, its inclinations θ with the H.P. and ϕ with the V.P. and the position of one end A. To draw its projections.

Mark the elevation a' and the plan a, according to the given position of A (fig. 9-12).

Let us first determine the lengths of AB in plan and elevation and the paths of its ends in elevation and plan.

(i) Assume AB to be parallel to the V.P. and inclined at θ to the H.P. As A is above the H.P., AB is shown in the pictorial view as a side of the trapezoid $ABba$ [fig. 9-11(*i*)]. Draw the elevation $a'b'$ equal to AB [fig. 9-12(*i*)]

and inclined at θ to xy. Project the plan ab. Through a' and b', draw lines cd and pq respectively parallel to xy. ab is the length of AB in the plan and, cd and pq are the paths of A and B respectively in the elevation.

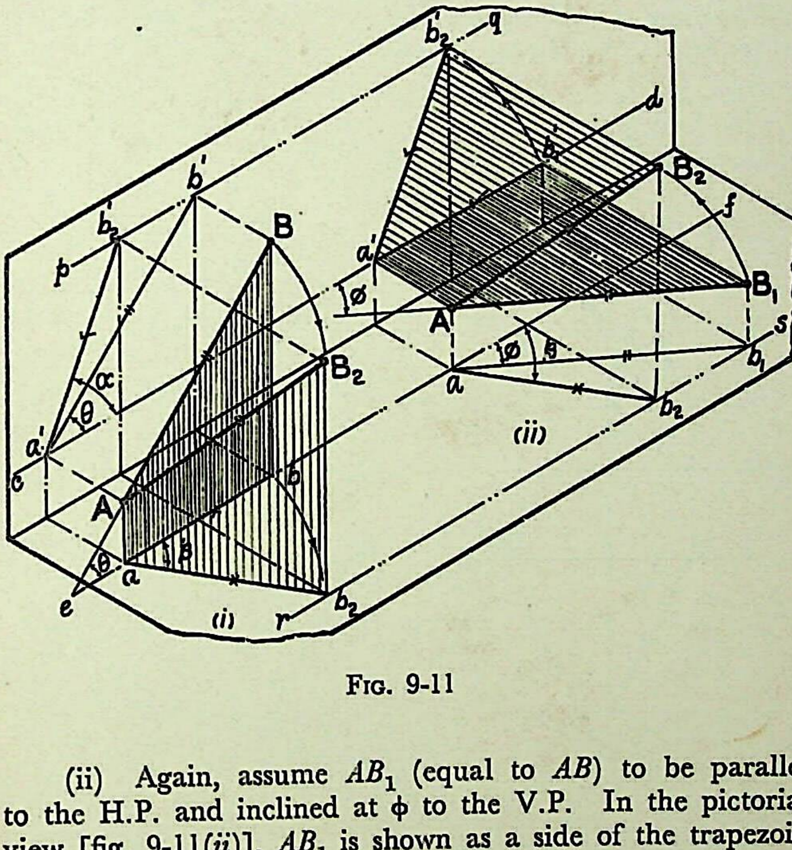


FIG. 9-11

(ii) Again, assume AB_1 (equal to AB) to be parallel to the H.P. and inclined at ϕ to the V.P. In the pictorial view [fig. 9-11(*ii*)], AB_1 is shown as a side of the trapezoid $AB_1b_1'a'$. Draw the plan ab_1 equal to AB [fig. 9-12(*ii*)] and inclined at ϕ to xy. Project the elevation $a'b_1'$. Through a and b_1, draw lines ef and rs respectively parallel to xy. $a'b_1'$ is the length of AB in the elevation and, ef and rs are the paths of A and B respectively in the plan.

We may now arrange (i) ab (the length in plan) between its paths ef and rs, and (ii) $a'b_1'$ (the length in elevation) between the paths cd and pq, keeping them in projection with each other, in one of the following three ways:

(a) In case (i) [fig. 9-11(*i*)], if the side *Bb* is turned about *Aa* so that, *b* comes on the path *rs*, the line *AB* will become inclined at ϕ to the V.P. Therefore, with *a* as centre [fig. 9-12(*i*)] and radius equal to *ab*, draw an arc cutting *rs* at a point b_2. Project b_2 to b_2' on the line *pq*. Draw lines joining *a* with b_2, and a' with b_2'. ab_2 and $a'b_2'$ are the required projections. Check that $a'b_2' = a'b_1'$.

(b) Similarly, in case (ii), if the side B_1b_1' [fig. 9-11(*ii*)], is turned about Aa' till b_1' is on the path *pq*, the line AB_1 will become inclined at θ to the H.P.

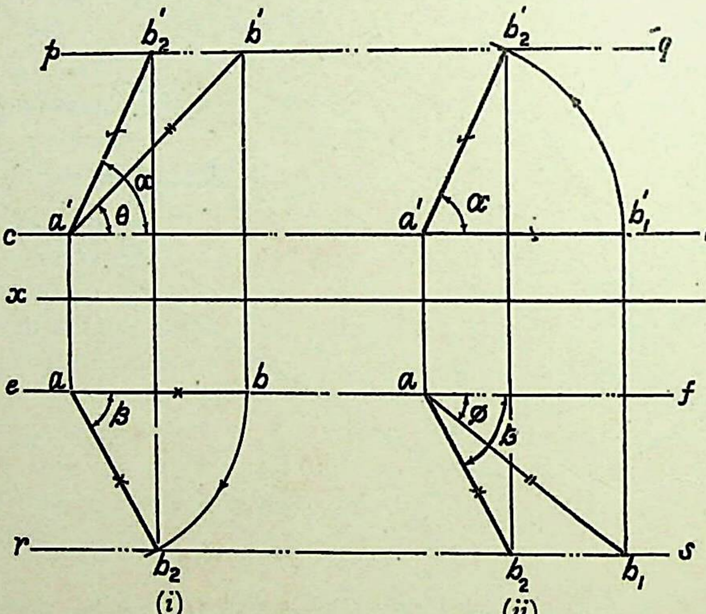


FIG. 9-12 (*i*) and (*ii*)

Hence, with a' as centre [fig. 9-12(*ii*)] and radius equal to $a'b_1'$, draw an arc cutting *pq* at a point b_2'. Project b_2' to b_2 in the plan, on the line *rs*. Draw lines joining *a* with b_2, and a' with b_2'. ab_2 and $a'b_2'$ are the required projections. Check that $ab_2 = ab$.

(c) Combining the above two cases [fig. 9-12(*iii*)]: First, determine in the same figure, the lengths in plan and elevation, viz. *ab* and $a'b_1'$ respectively and also the paths. Then, with *a* as centre and radius equal to *ab*, draw an arc cutting *rs* at a point b_2. With a' as centre and radius equal

to $a'b_1'$, draw an arc cutting pq at b_2'. Draw lines joining a with b_2, and a' with b_2'. ab_2 and $a'b_2'$ are the required projections. Check that b_2 and b_2' lie on the same projector.

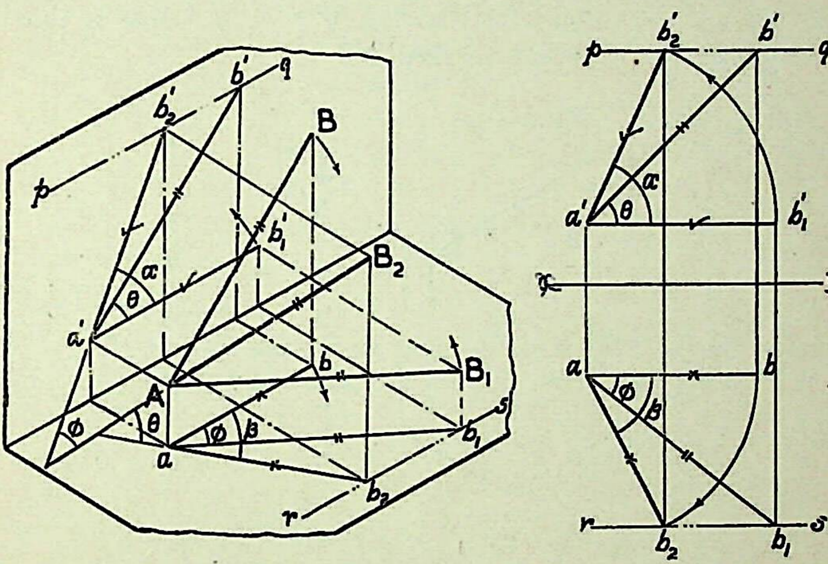


FIG. 9-12(*iii*)

It is quite evident from the figure that the apparent angles of inclination (α and β) are greater than the true inclinations, viz. θ and ϕ respectively.

Fig. 9-12(*iv*) shows the projections of the line AB, when it is in the third quadrant.

7. Line contained by a plane perpendicular to both the reference planes:

As the two reference planes are at right angles to each other, the sum total of the inclinations of a line with the two planes, viz. θ and ϕ, can never be more than 90°. When $\theta + \phi = 90°$, the line will

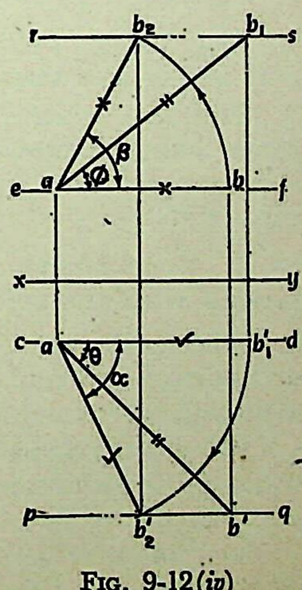
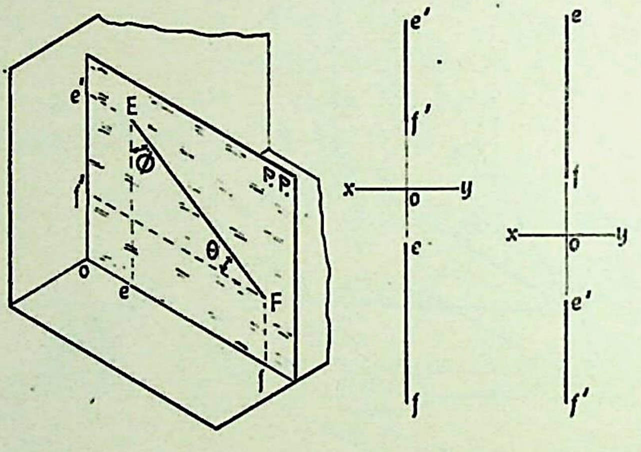


FIG. 9-12(*iv*)

be contained by a third plane perpendicular to both the H.P. and the V.P.

A line *EF* (fig. 9-13), is inclined at θ to the H.P. and at ϕ [equal to $(90° - \theta)$] to the V.P. The line is thus contained by the plane marked P.P.



(*i*)
First-angle projection

(*ii*)
Third-angle projection

FIG. 9-13

The elevation $e'f'$ and the plan ef are both perpendicular to xy and shorter than *EF*.

Therefore, when a line is inclined to both the reference planes and contained by a plane perpendicular to them, its projections are perpendicular to xy and shorter than the true length.

8. True length of a straight line and its inclinations with the reference planes:

When projections of a line are given, its true length and inclinations with the planes are determined by the application of the following rule:

When a line is parallel to plane, its projection on that plane will show its true length and the true inclination with the other plane.

The line may be made parallel to a plane and its true length obtained by the following three methods:

(i) Making each view parallel to the ground line and projecting the other view from it. This is the exact reversal of the processes adopted in Arts. 5(a) and 5(b) for obtaining the projections.

(ii) Rotating the line about its projections till it lies in the H.P. or in the V.P.

(iii) Projecting the views on auxiliary planes parallel to each view. (This method will be dealt with in Chapter XI.)

The following problem shows the application of the first two methods:

Problem 5:

The plan ab and the elevation a'b' of a line AB are given. To determine its true length, and the inclinations with the H.P. and the V.P.

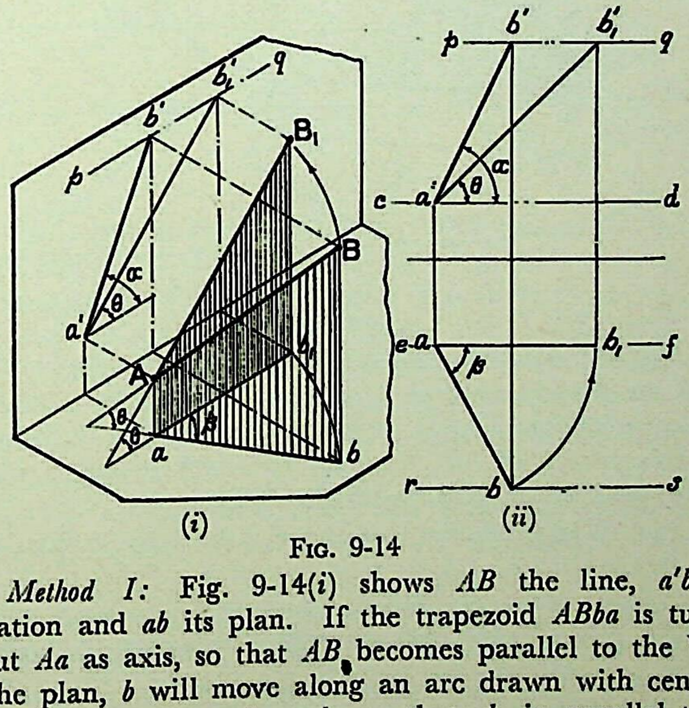


FIG. 9-14

Method I: Fig. 9-14(*i*) shows *AB* the line, *a'b'* its elevation and *ab* its plan. If the trapezoid *ABba* is turned about *Aa* as axis, so that *AB* becomes parallel to the V.P., in the plan, *b* will move along an arc drawn with centre *a* and radius equal to *ab*, to b_1, so that ab_1 is parallel to *xy*. In the elevation, *b'* will move along its locus *pq*, to a point b_1' on the projector through B_1. Therefore, with centre

a and radius equal to *ab* [fig. 9-14(*ii*)], draw an arc to cut *ef* at b_1. Draw a projector through b_1 to cut *pq* (the path of *b'*) at b_1'. Draw the line $a'b_1'$ which is the true length of *AB*; the angle θ, which it makes with *xy* is the inclination of *AB* with the H.P.

Again, in fig. 9-15(*i*), *AB* is shown as a side of a trapezoid *ABb'a'*. If the trapezoid is turned about *Aa'* as axis, so that *AB* is parallel to the H.P., the new plan will show its true length and true inclination with the V.P.

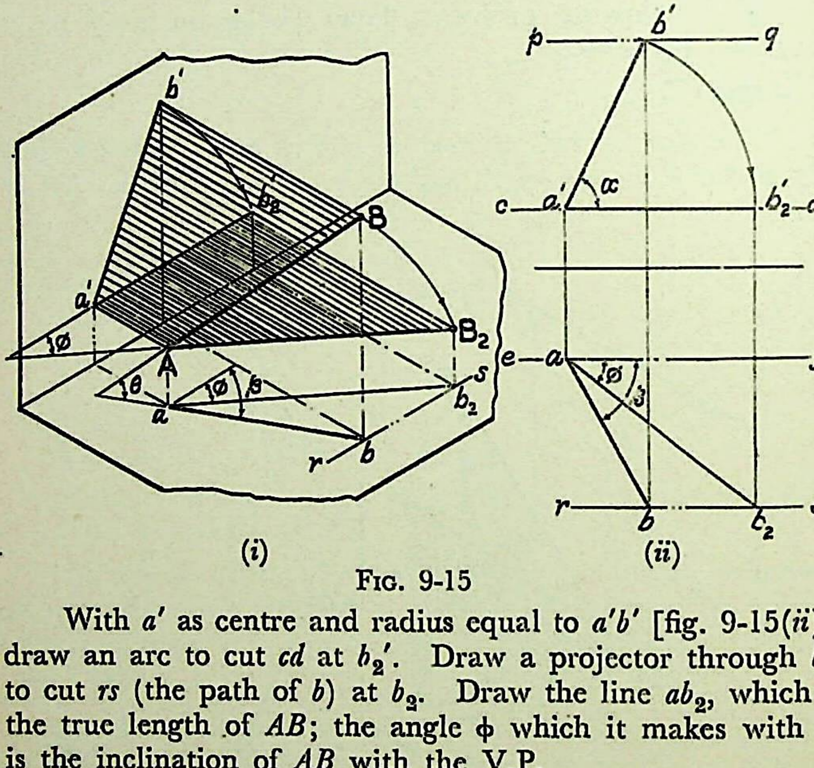


FIG. 9-15

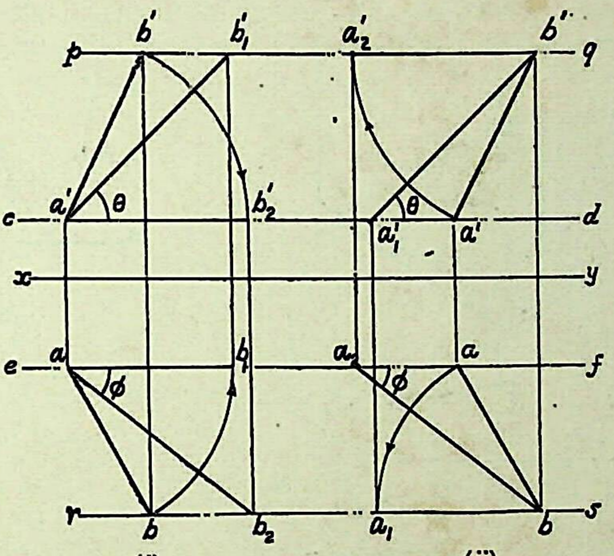
With *a'* as centre and radius equal to *a'b'* [fig. 9-15(*ii*)], draw an arc to cut *cd* at b_2'. Draw a projector through b_2' to cut *rs* (the path of *b*) at b_2. Draw the line ab_2, which is the true length of *AB*; the angle ϕ which it makes with *xy* is the inclination of *AB* with the V.P.

Fig. 9-16(*i*) shows the above two steps combined in one figure.

The same results will be obtained by keeping the end *B* fixed and turning the end *A* [fig. 9-16(*ii*)], as explained below.

With centre *b* and radius equal to *ba*, draw an arc cutting *rs* at a_1 (thus making *ba* parallel to *xy*). Project a_1 to a_1' on *cd* (the path of *a'*). $a_1'b'$ is the true length and θ is the true incli-

nation of AB with the H.P. Similarly, with centre b' and radius equal to $b'a'$, draw an arc cutting pq at a_2'. Project a_2' to a_2 on ef (the path of a). a_2b is the true length and ϕ is the true inclination of AB with the V.P.



(*i*) (*ii*)

Fig. 9-16

Method II: Referring to the pictorial view in fig. 9-17(*i*) we find that AB is the line, ab its plan and $a'b'$ its elevation. In the trapezoid $ABb'a'$, (i) $a'A$ and $b'B$ are both perpendicular to $a'b'$ and are respectively equal to ao_1 and bo_2 (the distances of a and b from xy in the plan), and (ii) the angle between AB and $a'b'$ is the angle of inclination ϕ of AB with the V.P.

Assume that this trapezoid is rotated about $a'b'$, till it lies in the V.P.

In the orthographic view, this trapezoid is obtained by drawing perpendiculars to $a'b'$, viz. $a'A_1$ (equal to ao_1), and $b'B_1$ (equal to bo_2) and then, joining A_1 with B_1. The line A_1B_1 is the true length of AB and its inclination ϕ with $a'b'$ is the inclination of AB with the V.P.

Similarly, in trapezoid $ABba$ in fig. 9-17(*ii*), AB is the line and ab, its plan. Aa and Bb are both perpendicular to ab and are respectively equal to $a'o_1$ and $b'o_2$ (the distances of a' and b' from xy in the elevation). The angle θ between

AB and *ab*, is the inclination of *AB* with the H.P. This figure may now be assumed to be rotated about *ab* as axis, so that it lies in the H.P.

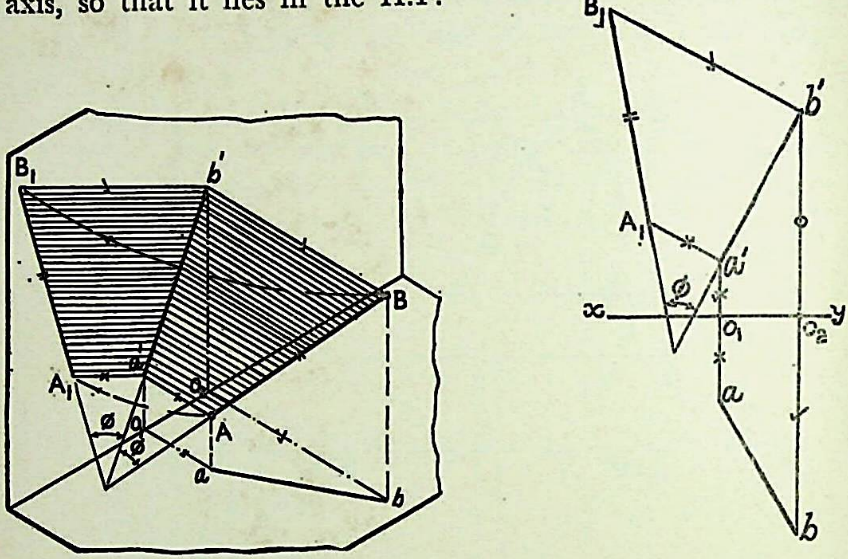


Fig. 9-17(i)

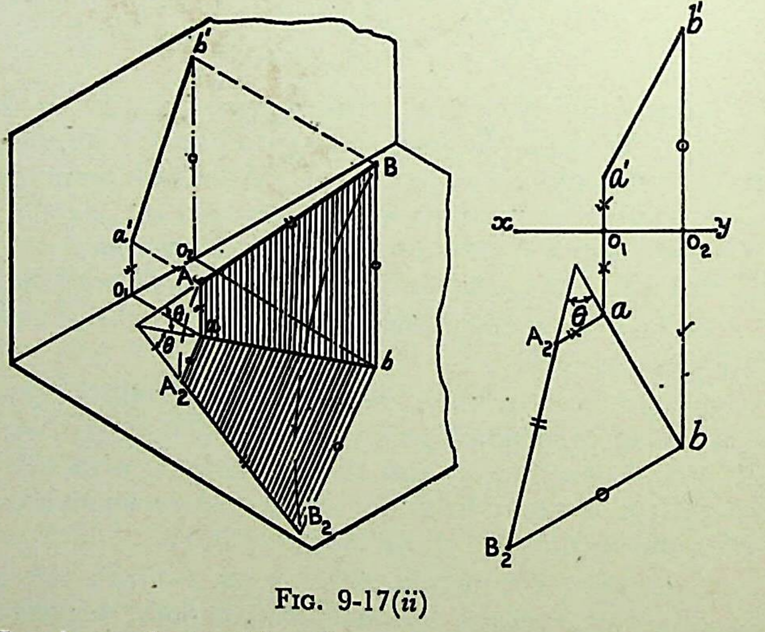


Fig. 9-17(ii)

In the orthographic view, this trapezoid is obtained by erecting perpendiculars to *ab*, viz. aA_2 equal to $a'o_1$

and bB_2 equal to $b'o_2$ and then joining A_2 with B_2. The line A_2B_2 is the true length of AB and its inclination θ with ab is the inclination of AB with the H.P.

When the projections of the line AB are given in third-angle projection, its true length and true inclinations are found by method I as shown in fig. 9-17(*iii*) and by method II as shown in fig. 9-17(*iv*).

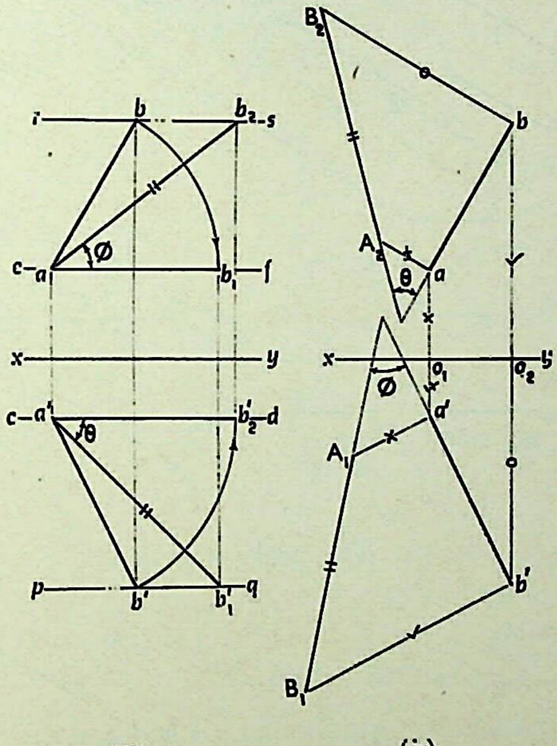


FIG. 9-17(*iii*) and (*iv*)

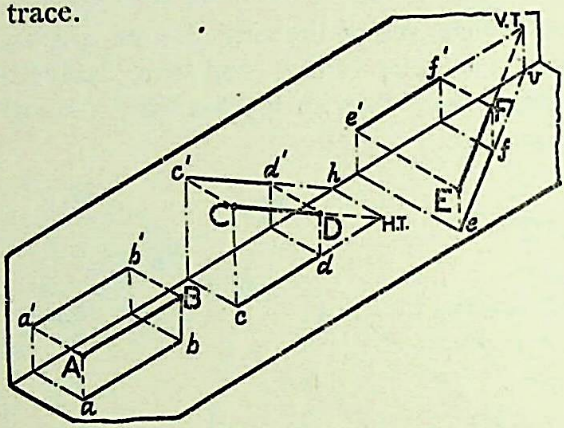
9. Traces of a line:

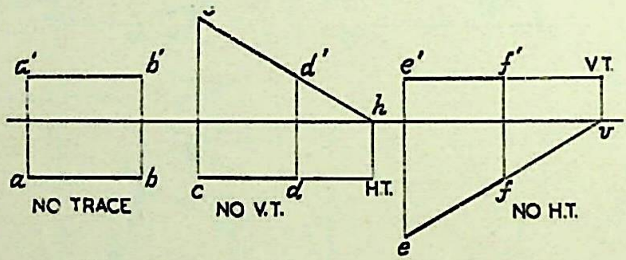
When a line is inclined to a plane, it will meet that plane, produced if necessary. The point in which the line or line-produced meets the plane, is called its *trace*.

The point of intersection of the line with the H.P. is called the *horizontal trace*, usually denoted as H.T. and that with the V.P. is called the *vertical trace* or V.T.

Refer to fig. 9-18.

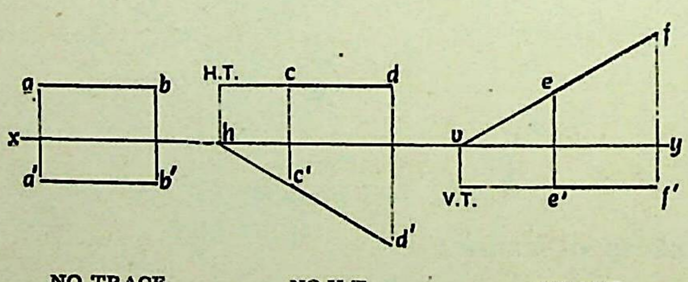
(i) A line AB is parallel to the H.P. and the V.P. It has no trace.





(i) (ii) (iii)

(a) First-angle projection



(i) (ii) (iii)

(b) Third-angle projection

Fig. 9-18

(ii) A line CD is inclined to the H.P. and parallel to the V.P. It has only the H.T. but no V.T.

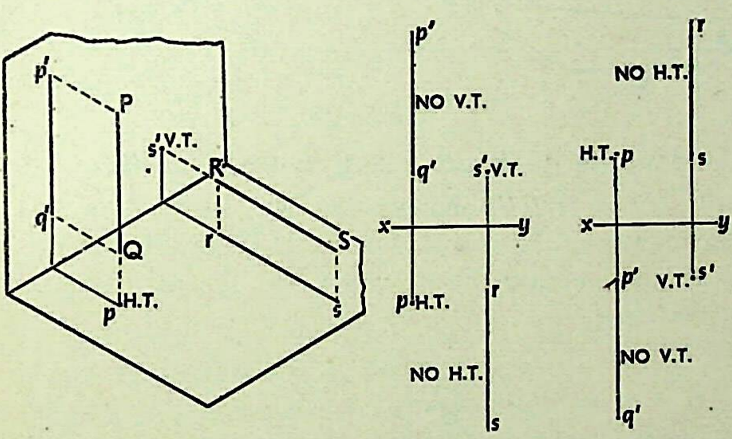
(iii) A line *EF* is inclined to the V.P. and parallel to the H.P. It has only the V.T. but no H.T.

Thus, when a line is parallel to a plane it has no trace upon that plane.

Refer to fig. 9-19.

(i) A line *PQ* is perpendicular to the H.P. Its H.T. coincides with its plan which is a point. It has no V.T.

(ii) A line *RS* is perpendicular to the V.P. Its V.T. coincides with its elevation which is a point. It has no H.T.



(*i*)
First-angle
projection

(*ii*)
Third-angle
projection

Fig. 9-19

Hence, when a line is perpendicular to a plane its trace on that plane, coincides with its projection on that plane. It has no trace on the other plane.

Refer to fig. 9-20.

(i) A line *AB* has its end *A* in the H.P. and the end *B* in the V.P. Its H.T. coincides with *a* the plan of *A* and the V.T. coincides with *b'* the elevation of *B*.

(ii) A line *CD* has its end *C* in both the H.P. and the V.P. Its H.T. and V.T. coincide with *c* and *c'* (the projections of *C*).

Hence, when a line has an end in a plane, its trace upon that plane coincides with the projection of that end on that plane.

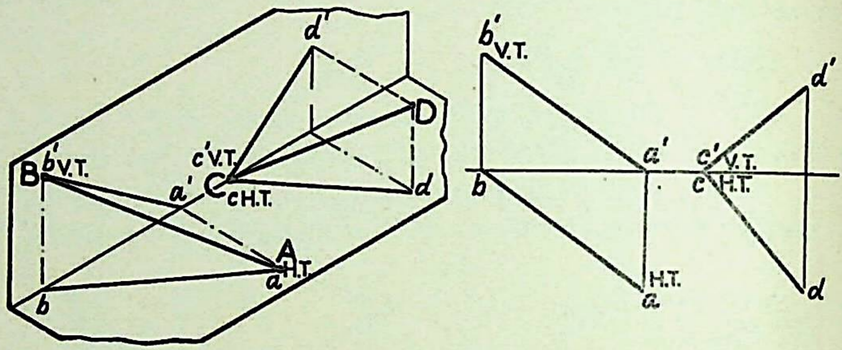
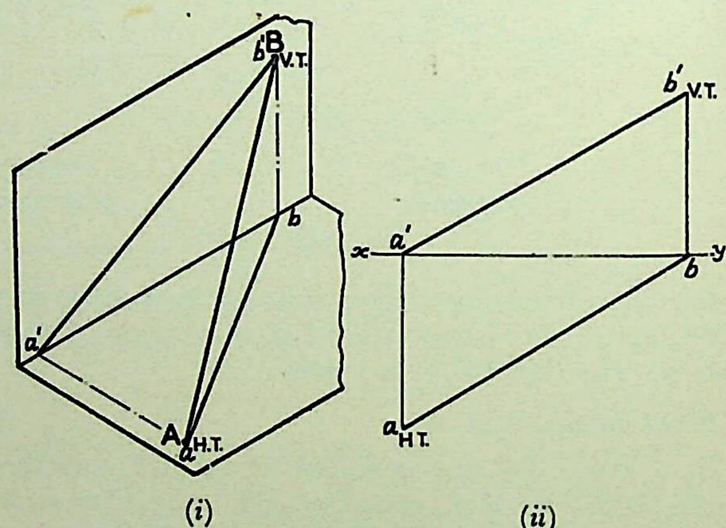


Fig. 9-20

10. Methods of determining traces of a line:

I. Fig. 9-21(*i*) shows a line *AB*, inclined to both the reference planes. Its end *A* is in the H.P. and *B* is in the V.P.

a'b' and *ab* are the elevation and the plan respectively [fig. 9-21(*ii*)].



(*i*) (*ii*)

Fig. 9-21

The H.T. of the line is on the projector through *a'* and coincides with *a*. The V.T. is on the projector through *b* and coincides with *b'*.

Let us now assume that *AB* is shortened from both its ends, its inclination with the planes remaining constant. The H.T. and V.T. of the new line *CD* are still the same as can be seen clearly in fig. 9-22(*i*).

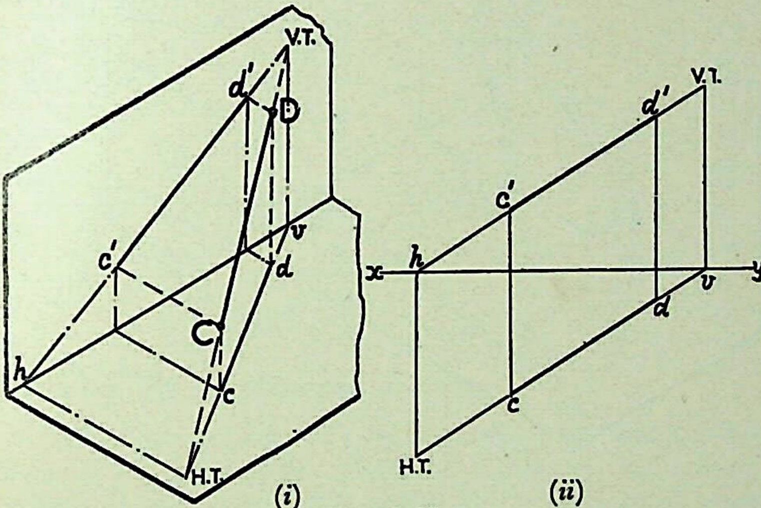


FIG. 9-22

c'd' and *cd* are the projections of *CD* [fig. 9-22(*ii*)]. Its traces may be determined as shown below:

Produce the elevation *d'c'* to meet *xy* at a point *h*. Through *h*, draw a projector to meet the plan *dc*-produced, at the H.T. of the line.

Similarly, produce the plan *cd* to meet *xy* at a point *v*. Through *v*, draw a projector to meet the elevation *c'd'*-produced, at the V.T. of the line.

II. *c'd'* and *cd* are the projections of the line *CD* [fig. 9-23(*ii*)]. Determine the true length C_1D_1 from the elevation *c'd'* by trapezoid method. The point of intersection between *c'd'*-produced and C_1D_1-produced, is the V.T. of the line.

Similarly, determine the true length C_2D_2 from the plan *cd*. Produce them to intersect at the H.T. of the line.

The above is quite evident from the pictorial view shown in fig. 9-23(*i*).

When the projections of the line *CD* are given in third-angle projection, its traces are determined by method I as

shown in fig. 9-23(*iii*) and by method II as shown in fig. 9-23(*iv*).

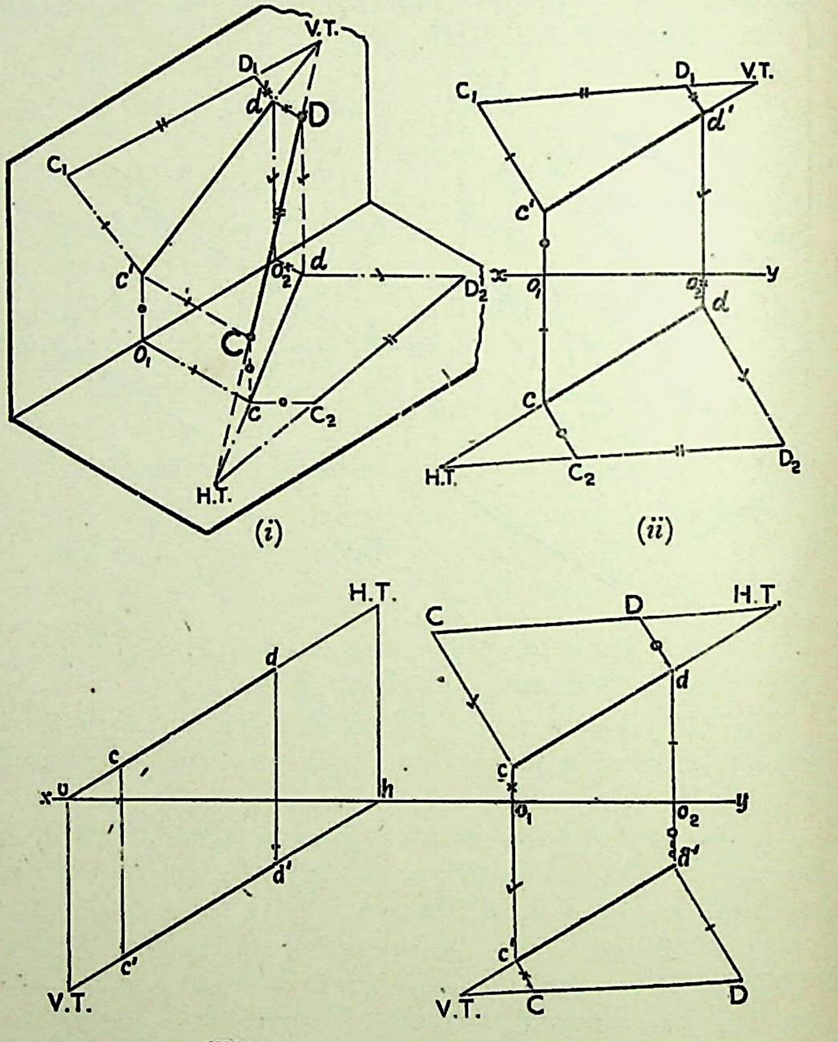


FIG. 9-23

TRACES OF A LINE, THE PROJECTIONS OF WHICH ARE PERPENDICULAR TO XY:

When the projections of a line are perpendicular to *xy*, i.e. when the sum of its inclinations with the two principal planes of projection is 90°, it is not possible to find the traces by the first method. Method II must, therefore, be adopted as shown in fig. 9-24.

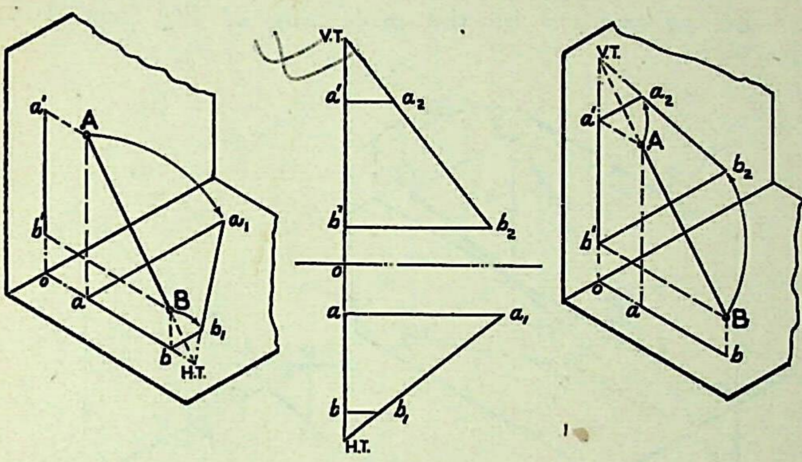


FIG. 9-24

POSITIONS OF TRACES OF A LINE:

Although the line may be situated in the first quadrant, its both traces may be above or below *xy*, as shown in problem 6 and in figs. 9-25 and 9-26. When a line intersects a plane, its trace on that plane will be contained by its projection on that plane as shown in problem 7.

Problem 6:

Projections of a line PQ are given. Determine the positions of its traces.

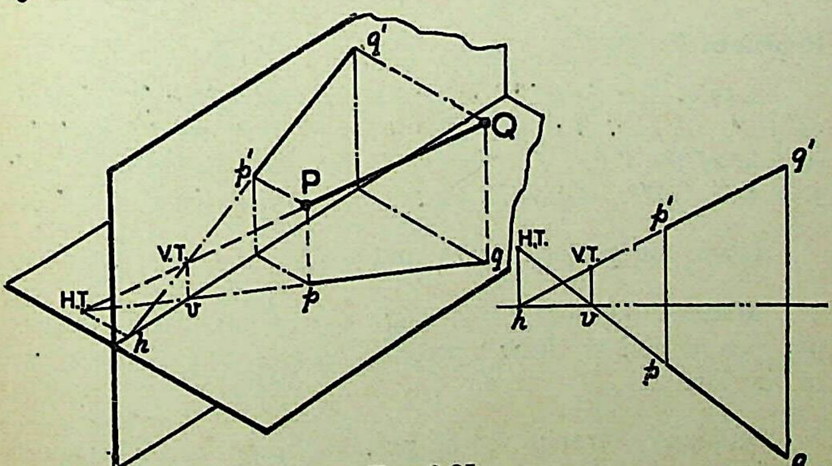


FIG. 9-25

Let pq and $p'q'$ be the projections of PQ (figs. 9-25 and 9-26).

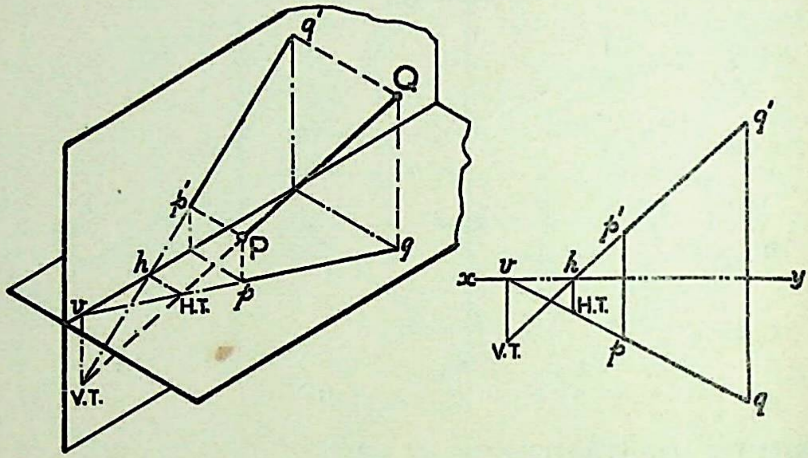


FIG. 9-26

Produce the plan qp to meet xy at v. Draw a projector through v, to meet the elevation $q'p'$-produced at the V.T. Through h, the point of intersection between $q'p'$-produced and xy, draw a projector to meet the plan qp-produced at the H.T.

Note that in fig. 9-25, the traces are above xy; in fig. 9-26 they are below it.

Problem 7:

A point A is 5 cm (2") below the H.P. and 1·2 cm ($\frac{1}{2}$") behind the V.P. A point B is 1 cm ($\frac{3}{8}$") above the H.P. and 2·5 cm (1") in front of the V.P. The distance between the projectors of A and B is 4 cm ($1\frac{1}{2}$"). Determine the traces of the line joining A and B.

Draw the projections ab and $a'b'$ of the line AB.

Method I (fig. 9-27): Through v, the point of intersection between ab and xy, draw a projector to meet $a'b'$ at the V.T. of the line.

Similarly, through h, the point of intersection between $a'b'$ and xy, draw a projector to cut ab at the H.T. of the line.

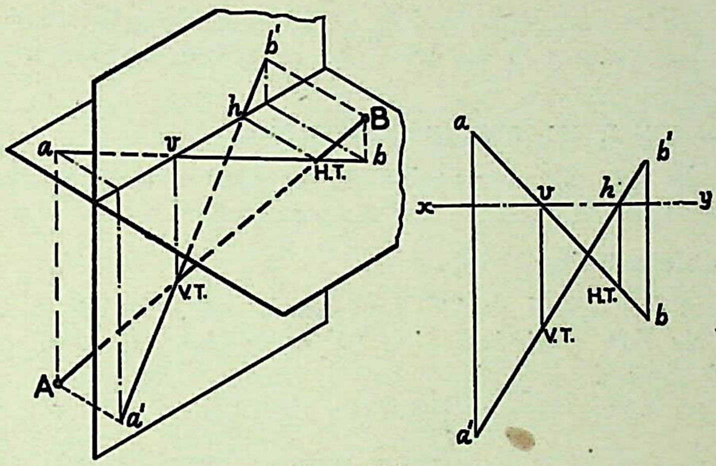


FIG. 9-27

Method II (fig. 9-28): At the ends a and b, draw perpendiculars to ab, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$, on its opposite sides (as a' and b' are on opposite sides of xy). Join A_2 with B_2 cutting ab at the H.T. of the line.

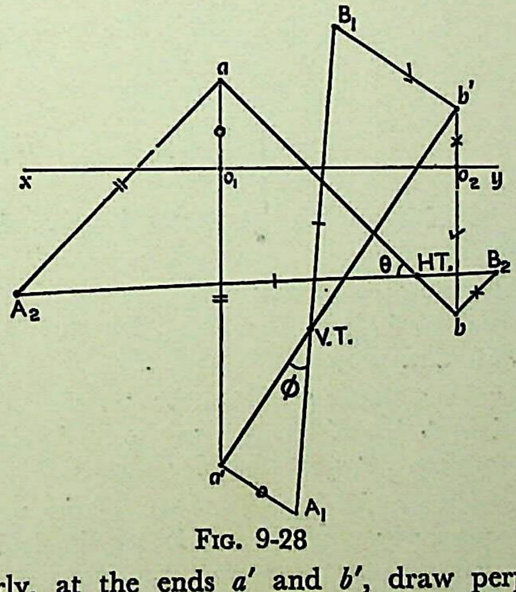


FIG. 9-28

Similarly, at the ends a' and b', draw perpendiculars to $a'b'$ on its opposite sides, viz. $a'A_1$ equal to ao_1 and $b'B_1$ equal to bo_2. Draw the line A_1B_1 intersecting $a'b'$ at the V.T. of the line.

Note that $A_1B_1 = A_2B_2 = AB$ and that θ and ϕ are the inclinations of AB with the H.P. and the V.P. respectively.

ADDITIONAL ILLUSTRATIVE PROBLEMS:

In the following problems, the ends of the lines should be assumed to be in the first quadrant, unless otherwise stated.

Problem 8 (fig. 9-29):

A line AB, 5 cm (2") long, has its end A in both the H.P. and the V.P. It is inclined at 30° to the H.P. and at 45° to the V.P. Draw its projections.

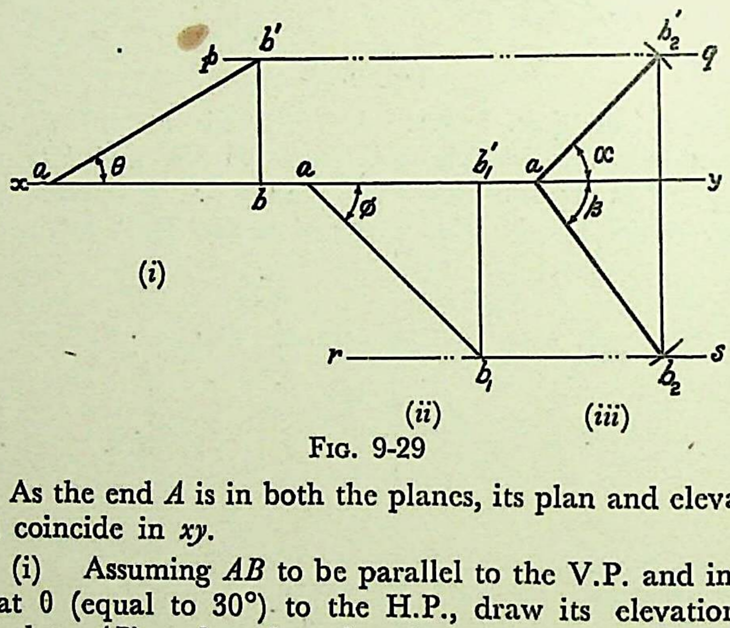


FIG. 9-29

As the end A is in both the planes, its plan and elevation will coincide in xy.

(i) Assuming AB to be parallel to the V.P. and inclined at θ (equal to 30°) to the H.P., draw its elevation ab' (equal to AB) and project the plan ab.

(ii) Again assuming AB to be parallel to the H.P. and inclined at ϕ (equal to 45°) to the V.P., draw its plan ab_1 (equal to AB). Project the elevation ab_1'.

ab and ab_1' are the lengths of AB in the plan and the elevation respectively, and pq and rs are the loci of the end B in the elevation and the plan respectively.

(iii) With a as centre and radius equal to ab_1', draw an arc cutting pq in b_2'. With the same centre and radius equal to ab, draw an arc cutting rs in b_2.

Draw lines joining a with b_2' and b_2. ab_2' and ab_2 are the required projections.

Fig. 9-30 shows (in pictorial and orthographic views), the solution obtained with all the above steps combined in one figure only.

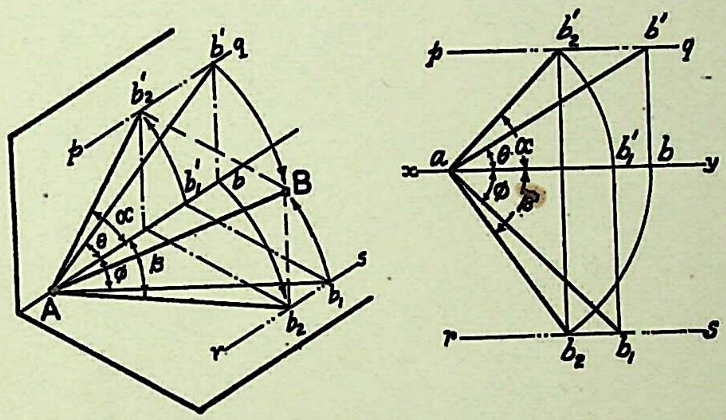


Fig. 9-30

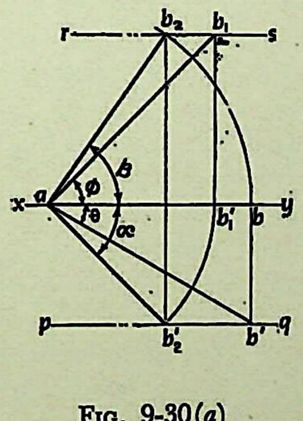
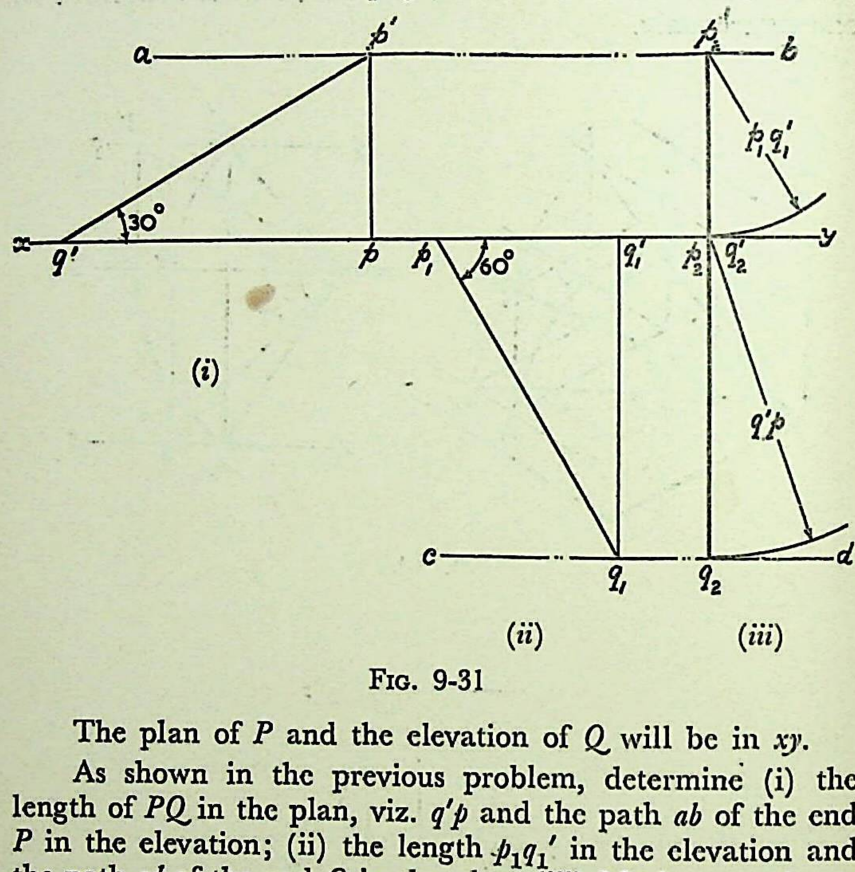


Fig. 9-30(a)

Fig. 9-30(a) shows the projections assuming the line to be in the third quadrant, i.e. by third-angle projection method.

Problem 9 (fig. 9-31):

A line PQ 7·5 cm (3") long has its end P in the V.P. and the end Q in the H.P. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.



(ii) (iii)

FIG. 9-31

The plan of *P* and the elevation of *Q* will be in *xy*.

As shown in the previous problem, determine (i) the length of *PQ* in the plan, viz. *q'p* and the path *ab* of the end *P* in the elevation; (ii) the length p_1q_1' in the elevation and the path *cd* of the end *Q* in the plan. (iii) Mark any point p_2 (the plan of *P*) in *xy* and project its elevation p_2' in *ab*. With p_2' as centre and radius equal to p_1q_1', draw an arc cutting *xy* in q_2'. It coincides with p_2. With p_2 as centre and radius equal to *q'p*, draw an arc cutting *cd* in q_2. p_2q_2 and $p_2'q_2'$ are the required projections. They lie in a line perpendicular to *xy* because the sum of the two inclinations is equal to 90°.

Problem 10 (fig. 9-32):

A line PQ 10 cm (4") long, is inclined at 30° to the H.P. and at 45° to the V.P. Its mid-point is in the V.P. and 2 cm (¾")

above the H.P. Draw its projections, if its end P is in the third quadrant and Q in the first quadrant.

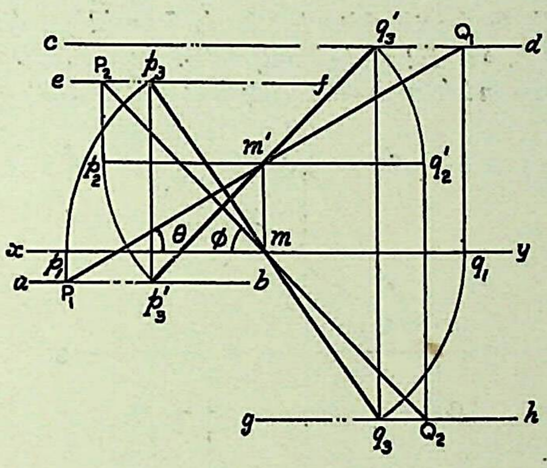


FIG. 9-32

The elevation and the plan of *P* will be below and above *xy* respectively, while those of *Q* will be above and below *xy* respectively.

Mark *m*, the plan of the mid-point in *xy* and project its elevation *m'*, 2 cm ($\frac{3}{4}''$) above *xy*.

Through *m'*, draw a line making an angle θ (equal to 30°) with *xy* and with the same point as centre and radius equal to $\frac{1}{2}PQ$, cut it at P_1 below *xy* and at Q_1 above *xy*. Project P_1Q_1 to p_1q_1 on *xy*; p_1q_1 is the length of *PQ* in plan. *ab* and *cd* are the paths of *P* and *Q* respectively in the elevation

Similarly, through *m*, draw a line making an angle ϕ (equal to 45°) with *xy* and cut it with the same radius at P_2 above *xy* and at Q_2 below it. Project P_2Q_2 to $p_2'q_2'$ on the horizontal line through *m'*. $p_2'q_2'$ is the length of *PQ* in the elevation and *ef* and *gh* are the paths of *P* and *Q* respectively in the plan.

With *m* as centre and radius equal to mp_1 or mq_1, draw arcs cutting *ef* at p_3 and *gh* at q_3.

With *m'* as centre and radius equal to $m'p_2'$ or $m'q_2'$, draw arcs cutting *ab* at p_3' and *cd* at q_3'.

p_3q_3 and $p_3'q_3'$ are the required projections.

Problem 11 (fig. 9-33):

The plan of a line AB 7·5 cm (3″) long, measures 6·5 cm (2½″), while the length of its elevation is 5 cm (2″). Its one end A is in the H.P. and 1·2 cm (½″) in front of the V.P. Draw the projections of AB and determine its inclinations with the H.P. and the V.P.

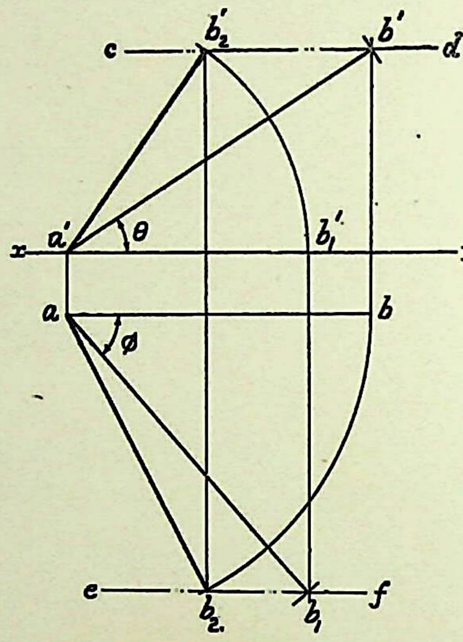


FIG. 9-33

Mark the elevation *a′* and the plan *a* of the given end *A*.

Assuming *AB* to be parallel to the V.P., draw a line *ab* equal to 6·5 cm (2½″) and parallel to *xy*. With *a′* as centre and radius equal to 7·5 cm (3″), draw an arc cutting the projector through *b* at *b′*. The line *cd* through *b′* and parallel to *xy*, is the locus of *B* in the elevation and θ is the inclination of *AB* with the H.P.

Similarly, draw a line $a'b_1'$ in *xy* and equal to 5 cm (2″). With *a* as centre and radius equal to *AB*, draw an arc cutting the projector through b_1' at b_1. *ef* is the locus of *B* in the plan and φ is the inclination of *AB* with the V.P.

With *a′* as centre and radius equal to $a'b_1'$, draw an arc cutting *cd* in b_2'. With *a* as centre and radius equal to

ab, draw an arc cutting *ef* in b_2. $a'b_2'$ and ab_2 are the required projections.

Problem 12 (fig. 9-34):

A line AB, 6·5 cm (2½") long, has its end A 2 cm (¾") above the H.P. and 2·5 cm (1") in front of the V.P. The end B is 4 cm (1½") above the H.P. and 6·5 cm (2½") in front of the V.P. Draw the projections of AB and show its inclinations with the H.P. and the V.P.

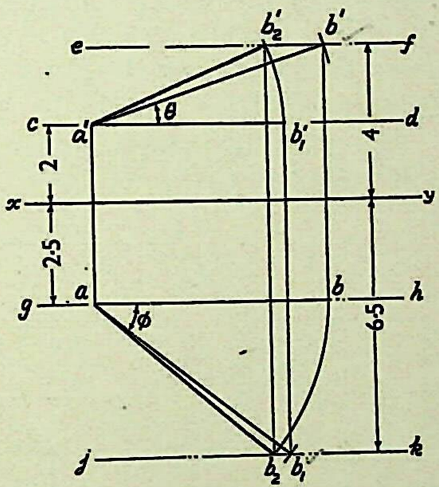


FIG. 9-34

As per given positions, draw the loci *cd* and *gh* of the end *A*, and *ef* and *jk* of the end *B* in the elevation and the plan respectively.

Mark any point *a* (the plan of *A*) in *gh* and project it to *a'* on *cd*.

With *a'* as centre and radius equal to 6·5 cm (2½"), draw an arc cutting *ef* in *b'*. Join *a'* with *b'*. θ, the inclination of *a'b'* with *xy*, is the inclination of *AB* with the H.P. Project *b'* to *b* on *gh*. *ab* is the length of *AB* in the plan.

With *a* as centre and radius equal to 6·5 cm (2½"), draw an arc cutting *jk* in b_1. Join *a* with b_1. φ, the inclination of ab_1 with *xy*, is the inclination of *AB* with the V.P Project b_1 to b_1' on *cd*. $a'b_1'$ is the length of *AB* in the elevation.

Arrange ab and $a'b_1'$ between their respective paths as shown. $a'b_2'$ and ab_2 are the required projections of AB.

Problem 13 (figs. 9-35 and 9-36):

XX *The projectors of the ends of a line AB are 4·5 cm (1¾") apart. The end A is 2 cm (¾") above the H.P. and 2·5 cm (1") in front of the V.P. The end B is 1·2 cm (½") below the H.P. and 4 cm (1½") behind the V.P. Determine the true length and traces of AB, and its inclinations with the two planes.*

Draw two projectors 4·5 cm (1¾") apart. On one projector, mark the plan a and the elevation a' of the end A; on the other, mark the plan b and the elevation b' of B, as per given distances. ab and $a'b'$ are the projections of AB.

Determine the true length, traces and inclinations by any one of the following two methods:

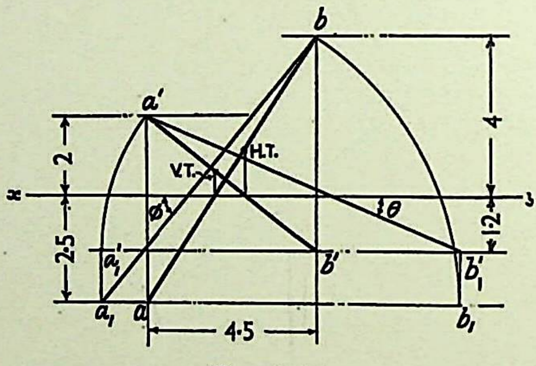


Fig. 9-35

Method I (fig. 9-35): By making the line parallel to a plane.

Keeping a fixed, turn ab to a position ab_1, thus making it parallel to xy. Project b_1 to b_1' on the locus of b'. $a'b_1'$ is the true length of AB and θ is its true inclination with the H.P.

Similarly, turn $a'b'$ to a position $a_1'b'$ and project a_1' to a_1 on the path of a (because the end a has been moved). a_1b is the true length of AB and ϕ is its inclination with the V.P.

Traces: Through the point of intersection of the plan ab with xy, draw a projector to cut $a'b'$ at the V.T. Through the point of intersection of $a'b'$ with xy, draw a projector to cut ab at the H.T. of the line.

Method II (fig. 9-36): At the ends a and b of the plan ab, draw perpendiculars to ab, viz. aA_1 equal to $a'o_1$ and bB_1 equal to $b'o_2$, on opposite sides of it (because a' and b' are on opposite sides of xy). A_1B_1 is the true length of AB; θ (its inclination with ab) is the inclination of AB with the H.P. and the point at which A_1B_1 intersects ab is the H.T. of AB.

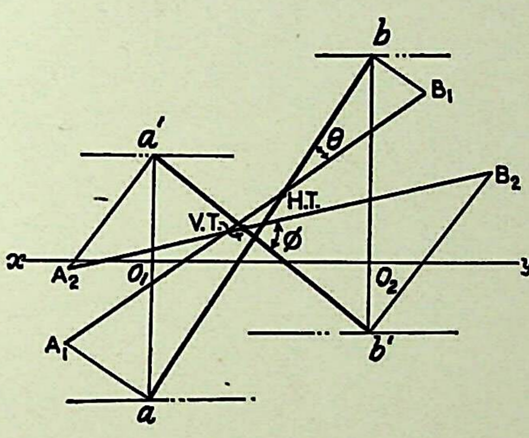


FIG. 9-36

Similarly, at the ends a' and b', draw perpendiculars to $a'b'$, viz. $a'A_2$ equal to ao_1 and $b'B_2$ equal to bo_2, on opposite sides of it. A_2B_2 is the true length of AB; ϕ (its inclination with $a'b'$) is the inclination of AB with the V.P. and the point at which A_2B_2 intersects $a'b'$ is the V.T. of AB.

Problem 14 (fig. 9-37):

A line AB, 9 cm ($3\frac{1}{2}''$) long, is inclined at 45° to the H.P. and its plan makes an angle of 60° with the V.P The end A is in the H P. and 1·2 cm ($\frac{1}{2}''$) in front of the V.P. Draw its elevation and find its true inclination with the V.P.

Mark a and a', the projections of the end A.

Draw a line $a'b'$ equal to AB and making an angle θ (equal to 45°) with xy. Project b' to b so that ab the plan is parallel to xy. Keeping the end a fixed, turn the plan ab to a position ab_1 so that it makes an angle β (equal to 60°) with xy. Project b_1 to b_1' on the locus of b'. $a'b_1'$ is the elevation of AB.

To find the true inclination with the V.P., draw an arc with a as centre and radius equal to AB, cutting the locus of b_1 in b_2. Join a with b_2. ϕ is the true inclination of AB with the V.P.

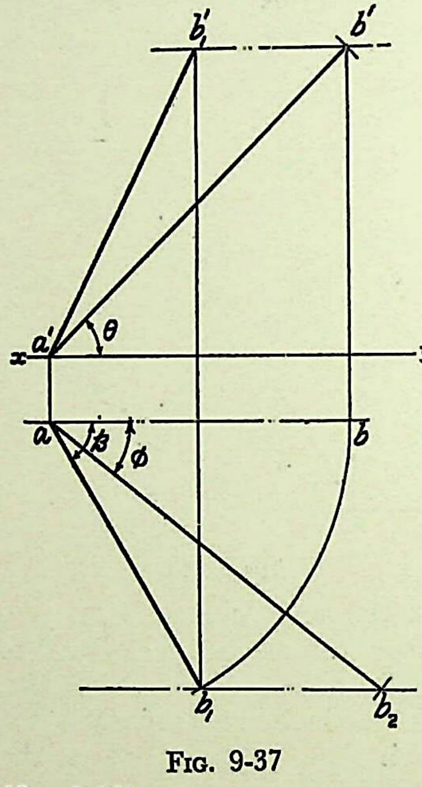


FIG. 9-37

Problem 15 (fig. 9-38):

Incomplete projections of a line PQ, inclined at 30° to the H.P. are given in fig. 9-38(i). Complete the projections and determine the true length of PQ and its inclination with the V.P.

Turn the plan pq to a position pq_1, so that it is parallel to xy. Through p', draw a line making an angle θ (equal to 30°) with xy and cutting the projector through q_1 at q_1'. $p'q_1'$ is the true length of PQ.

Through q_1', draw a line parallel to xy and cutting the projector through q at q'. $p'q'$ is the elevation of PQ.

With p as centre and radius equal to $p'q_1'$, draw an arc cutting the locus of q at q_2. Join p with q_2. ϕ is the inclination of PQ with the V.P.

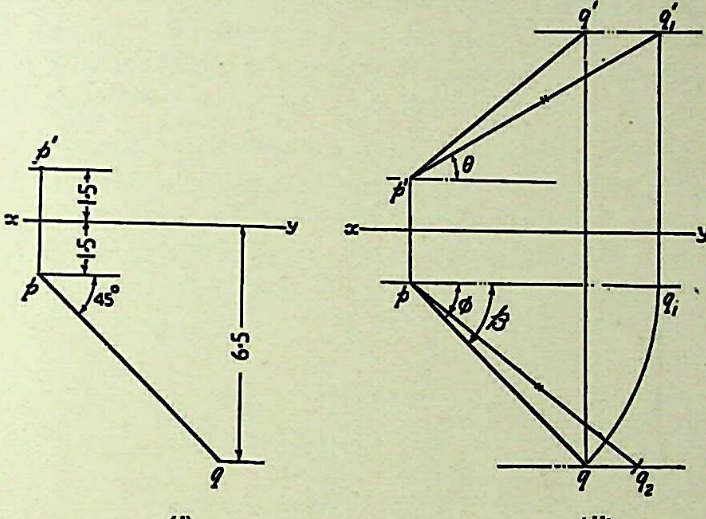


FIG. 9-38

Problem 16 (fig. 9-39):

The end A of a line AB is 2·5 cm (1") behind the V.P. and is below the H.P. The end B is 1·3 cm (½") in front of the V.P. and is above the H.P. The distance between the end-projectors is 6·5 cm (2½"). The line is inclined at 40° to the H.P. and its H.T. is 2 cm (¾") behind the V.P. Draw the projections of the line and determine its true length and the V.T.

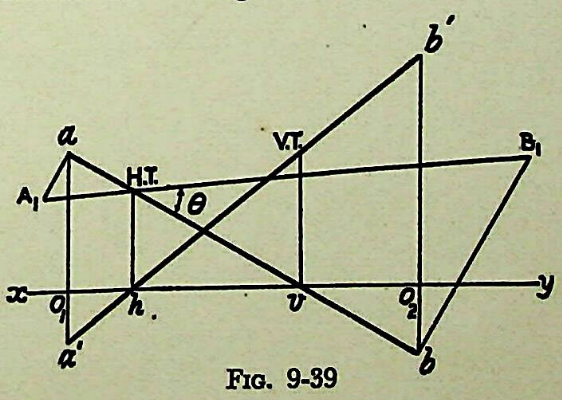


FIG. 9-39

Draw the plan *ab* and mark the H.T. on it, 2 cm (¾") above *xy*.

We have seen that the line representing the true length obtained by the trapezoid method, intersects the plan or the plan-produced at the H.T. at an angle equal to the true inclination of the line with the V.P.

Hence, at the ends *a* and *b*, draw perpendiculars to *ab* on its opposite sides (as one end is below the H.P. and the other end above it). Through the H.T., draw a line making angle θ (equal to 40°) with *ab* and cutting the perpendiculars at A_1 and B_1, as shown. A_1B_1 is the true length of *AB*. aA_1 and bB_1 are the distances of the ends *A* and *B* respectively, from the H.P.

Project *a* and *b* to *a'* and *b'*, making $a'o_1$ equal to aA_1 and $b'o_2$ equal to bB_1. *a'b'* is the elevation of *AB*. Through *v*, the point of intersection between *ab* and *xy*, draw a projector cutting *a'b'* at the V.T. of the line.

Problem 17 (fig. 9-40):

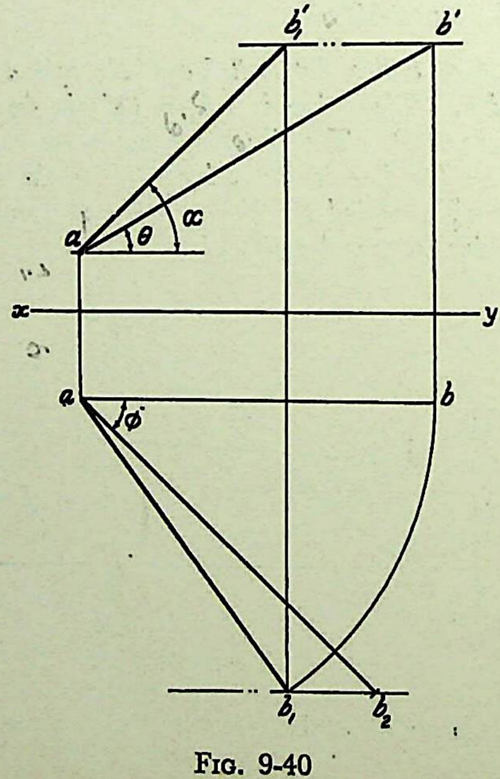


Fig. 9-40

A line AB, 9 cm ($3\frac{1}{2}''$) long, is inclined at 30° to the H.P. Its end A is 1·3 cm ($\frac{1}{2}''$) above the H.P. and 2 cm ($\frac{3}{4}''$) in front of the V.P. Its elevation measures 6·5 cm ($2\frac{1}{2}''$). Draw the plan of AB and determine its inclination with the V.P.

Mark a and a' the projections of the end A. Through a', draw a line $a'b'$ equal to 9 cm ($3\frac{1}{2}''$) and making an angle θ (equal to 30°) with xy.

With a' as centre and radius equal to 6·5 cm ($2\frac{1}{2}''$), draw an arc cutting the path of b' at b_1'. $a'b_1'$ is the elevation of AB.

Project b' to b, so that ab is parallel to xy. ab is the length of AB in the plan.

With a as centre and radius equal to ab, draw an arc cutting the projector through b_1' at b_1. ab_1 is the required plan.

Determine ϕ as described in the previous problem.

Problem 18 (fig. 9-41):

The ends of a line PQ are on the same projector. The end P is 3 cm ($1\frac{1}{4}''$) below the H.P. and 1·3 cm ($\frac{1}{2}''$) behind the V.P. The end Q is 5·5 cm ($2\frac{1}{4}''$) above the H.P. and 4·5 cm ($1\frac{3}{4}''$) in front of the V.P. Determine the true length and traces of PQ and its inclinations with the two planes.

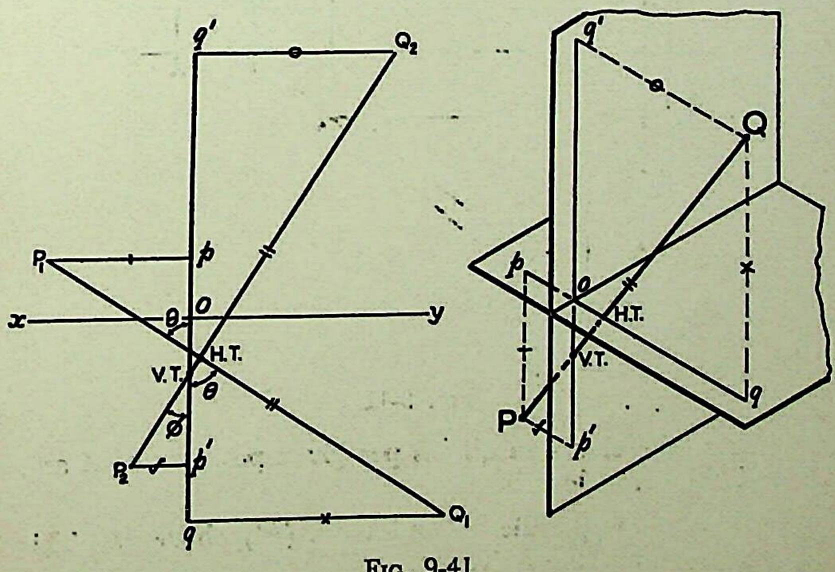


FIG. 9-41

At the ends p and q of the plan pq, erect perpendiculars, viz. pP_1 equal to $p'o$, and qQ_1 equal to $q'o$ and on opposite sides of pq. P_1Q_1 is the true length of PQ; θ is the inclination of PQ with the H.P., and the point of intersection between P_1Q_1 and pq is the H.T. of PQ.

Similarly, draw perpendiculars to $p'q'$, viz. $p'P_2$ equal to po and $q'Q_2$ equal to qo and on opposite sides of $p'q'$. P_2Q_2 is the true length; ϕ is the true inclination of PQ with the V.P., and the point where P_2Q_2 cuts $p'q'$, is the V.T. of PQ.

Problem 19 (fig. 9-42):

A line AB, inclined at 40° to the V.P., has its ends 5 cm (2″) and 2 cm (¾″) above the H.P. The length of its elevation is 6·5 cm (2½″) and its V.T. is 1 cm (⅜″) above the H.P. Determine the true length of AB, its inclination with the H.P. and its H.T.

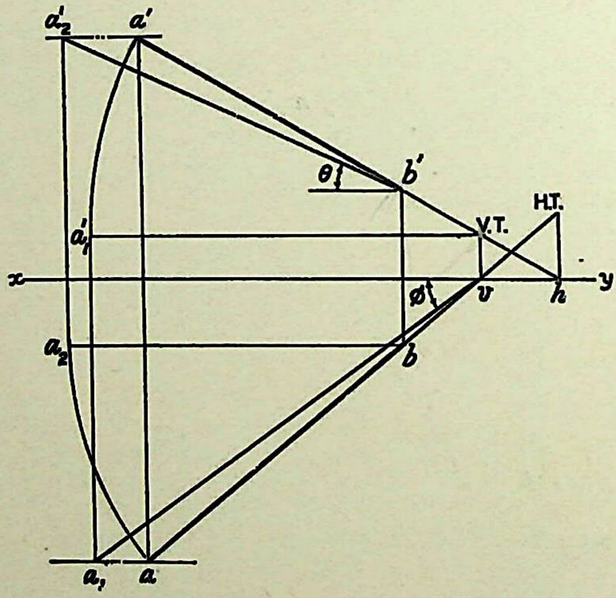


FIG. 9-42

Draw the elevation $a'b'$ as per given positions of A and B and the given length.

Draw a line parallel to and 1 cm (⅜″) above xy. This line will contain the V.T.

Produce $a'b'$ to cut this line at the V.T. Draw a projector through V.T. to v on xy.

Assuming a'V.T. to be the elevation of a line, which makes 40° angle with the V.P. and whose one end v is in the V.P., let us determine its true length.

Keeping V.T. fixed, turn the end a' to a_1' so that the line becomes parallel to xy. Through v, draw a line making an angle ϕ (equal to 40°) with xy and cutting the projector through a_1' at a_1. The line through a_1, drawn parallel to xy, is the locus of A in the plan. Project a' to a on this line. av is the plan of the line, whose elevation is a' V.T. and whose true length is equal to a_1v.

But $a'b'$ is the given elevation of AB. Therefore, project b' to b on av. ab is the plan of AB. Obtain the inclination θ with the H.P. by making the plan ab parallel to xy, as shown.

Produce $a'b'$ to meet xy at h. Draw a projector through h to cut ab-produced, at the H.T. of the line.

Problem 20 (fig. 9-43):

The elevation $a'b'$ and the H.T. of a line AB, inclined at 20° to the H.P. are given in fig. 9-43(i). Determine the true length of AB, its inclination with the V.P. and its V.T.

Consider that hb' is the elevation of a line inclined at 20° to the H.P. and the plan of whose one end is in H.T.

Through h, draw a line making an angle θ (equal to 20°) with xy and cutting the locus of B in elevation, in b_1'. hb_1' is the true length of the line whose length in the plan is b_1H.T.

With H.T. as centre and radius equal to H.T.b_1, draw an arc cutting the projector through b' at b. H.T.b is the plan and hb' is the elevation of a line which contains AB.

Therefore, through a', draw a projector cutting H.T.b at a.

ab is the plan of AB.

Obtain the true length ab_2 (of AB) and its inclination ϕ with the V.P., by making $a'b'$ parallel to xy.

Produce *ba* to meet *xy* in *v*. Draw a projector through *v* to cut *b'a'*-produced, at the V.T. of the line.

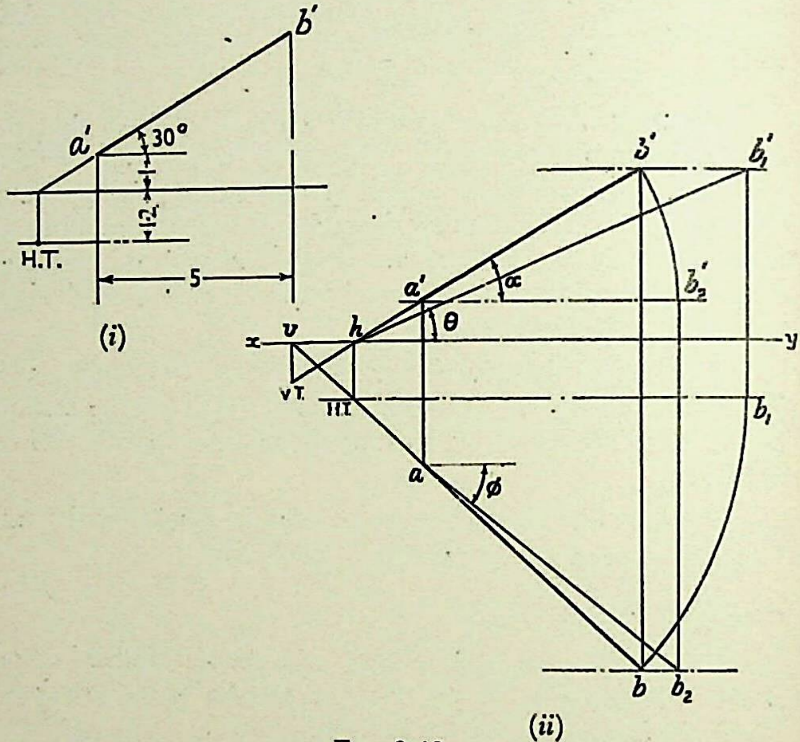


FIG. 9-43

Problem 21 (fig. 9-44):

A tripod stand rests in the H.P. One of its legs is 15 cm (6") long and makes an angle of 70° with the floor. The other two legs are 16·3 cm ($6\frac{1}{2}$") and 17·5 cm (7") long respectively. The upper ends of the legs are attached to the corners of a horizontal equilateral triangular frame of 5 cm (2") side, one side of which is parallel to the V.P. In the plan, the legs appear as lines 120° apart, which if produced, would meet in a point. Draw the plan and elevation of the tripod, and determine the angle, which each of the other two legs makes with the H.P. Assume the thickness of the frame and of the legs to be equal to that of the line.

At any point *P* on *xy*, draw a line *PA*, 15 cm (6") long and making 70° angle with *xy*. *h* is the height of the tripod and PA_1 is the length of the leg in the plan.

Draw an equilateral triangle *abc* of 5 cm (2″) side with one side parallel to *xy*. Project the elevation *a′b′c′*, at the height *h* above *xy*.

Determine the lengths of the other two legs in the plan. With *b′* as centre and radius equal to 16·3 cm ($6\frac{1}{2}$″), draw an arc cutting *xy* in q_1'. Similarly, with *c′* as centre and radius equal to 17·5 cm (7″), draw an arc cutting *xy* in r_1'. bq_1 and cr_1 are the lengths of the two legs in the plan, and α and β respectively are their inclinations with the H.P.

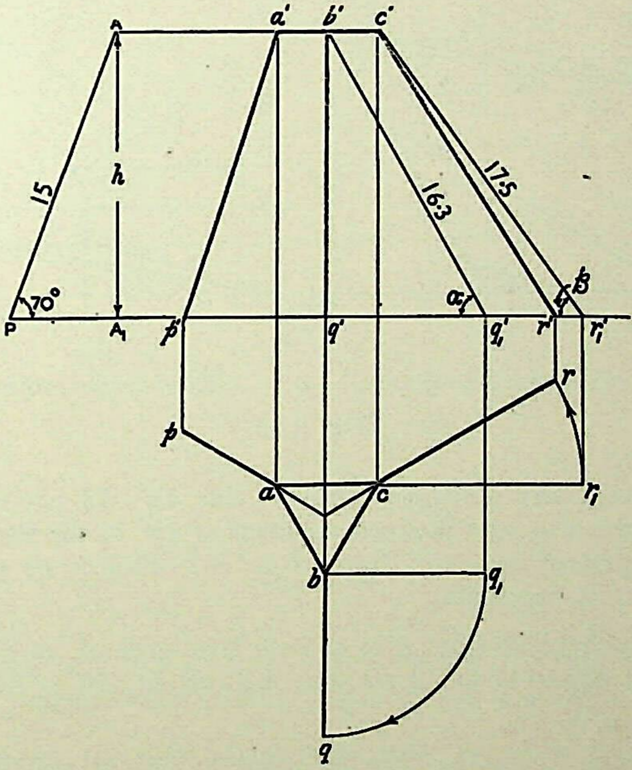


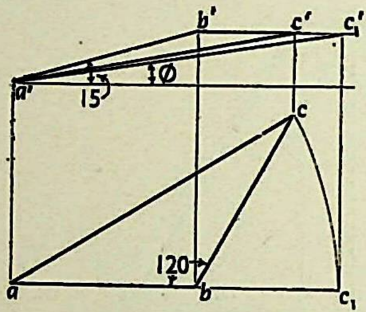
FIG. 9-44

The legs in the plan are to be inclined at 120° to each other and to meet at a point, if produced. Therefore, draw lines bisecting the angles of the triangle, making *ap* equal to PA_1, *bq* equal to bq_1 and *cr* equal to cr_1, thus completing the plan.

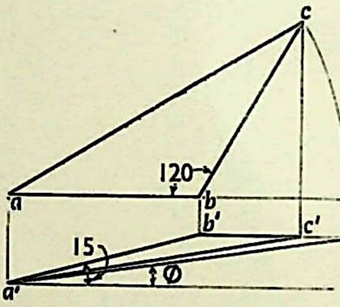
Project p, q and r to p, q' and r' respectively on xy. Complete the elevation by drawing lines $a'p'$, $b'q'$ and $c'r'$.

Problem 22 (fig. 9-45):

A straight road going uphill from a point A, due East to another point B, is 4 Km long and has a slope of 15°. Another straight road from B, due 30° East of North, to a point C is also 4 Km long but is on level ground. Determine the length and slope of the straight road joining the points A and C. Scale 3 cm = 1 Km.



(*i*)
First-angle projection



(*ii*)
Third-angle projection

FIG. 9-45

Mark any point a'. Draw a line $a'b'$, 12 cm long, to the right of a' and inclined upwards at 15° to the horizontal (to represent the road from A to B). Project its plan ab keeping it horizontal.

As the road from B to C is on level ground, in the plan bc will be equal to 12 cm and inclined at (90° + 30°), i.e. 120° to ab.

From b, draw a line bc equal to 12 cm and making 120° angle with ab. Project c to c' making $b'c'$ horizontal. $a'c'$ and ac are the elevation and the plan respectively of the road from A to C.

Determine the true length $a'c_1'$ and the angle ϕ as shown, which are the length and slope respectively, of the road from A to C.

Problem 23 (fig. 9-46):

Two lines AB and AC make an angle of 120° between them in their elevations and plans. AB is parallel to both the H.P. and the V.P. Determine the real angle between AB and AC.

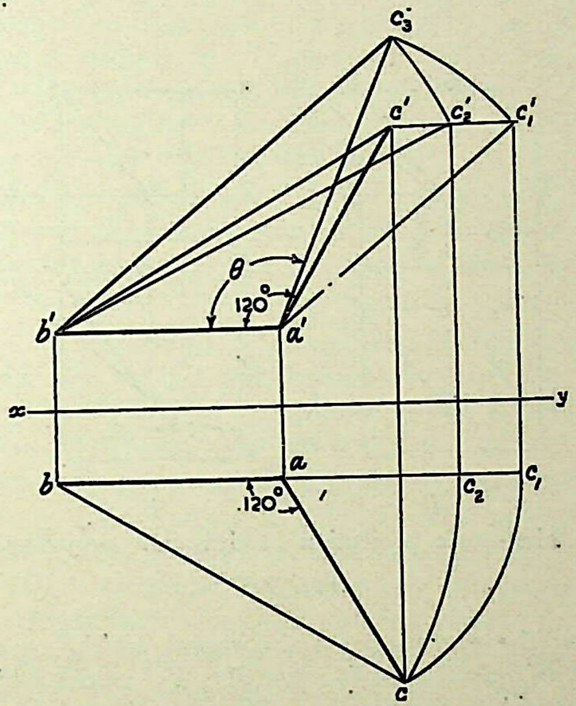


FIG. 9-46

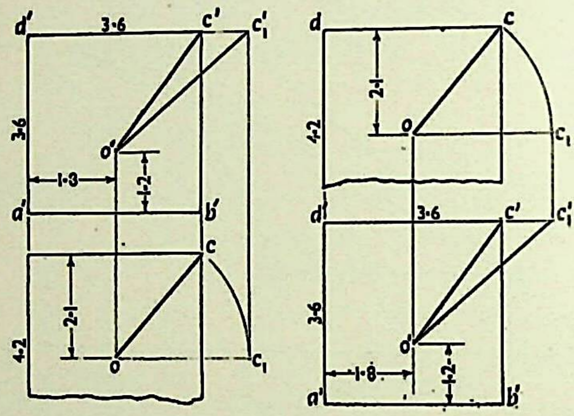
Draw any line $a'b'$ parallel to xy and another line $a'c'$ of any length, making 120° angle with $a'b'$. Join b' with c'.

Project the plan ab of $a'b'$ parallel to xy and the plan ac of $a'c'$, making 120° angle with ab. Join b with c. $a'b'$ or ab is the true length of AB. Determine the true lengths of AC and BC, viz. $a'c_1'$ and $b'c_2'$, as shown.

Draw a triangle $a'b'c_3'$ making $a'c_3'$ equal to $a'c_1'$ and $b'c_3'$ equal to $b'c_2'$. $\angle b'a'c_3'$ is the real angle between AB and AC.

Problem 24 (fig. 9-47):

An object O is placed 1·2 m (4′) above the ground and in the centre of a room 4·2 m × 3·6 m × 3·6 m high (14′ × 12′ × 12′ high). Determine graphically its distance from one of the corners between the roof and two adjacent walls. Scale 3 cm = 1 m.



(*i*) First-angle projection (*ii*) Third-angle projection

Fig. 9-47

Draw the elevation (of the room) *a′b′c′d′* as seen from the front of, say 3·6 m (12′) wall. *a′b′* is the width of the room and *a′d′* is the height. The elevation *o′* of the object will be seen 1·2 m above the mid-point of *a′b′*. *c′* and *d′* are the top corners of the room. *o′c′* is the elevation of the line joining the object with a top corner.

Draw the plan of the room. It will be a rectangle of sides equal to 3·6 m and 4·2 m. The plan *o* of the object will be in the centre of the rectangle. *oc* is the plan of *o′c′*.

Determine the true length $o'c_1'$, which will show the distance of the object from one of the top corners of the room.

EXERCISES IX(b)

(1) A line *AB*, 7·5 cm (3″) long, is inclined at 45° to the H.P. and 30° to the V.P. Its end *B* is in the H.P. and 4 cm ($1\frac{1}{2}''$) in front of the V.P. Draw its projections and determine its traces.

(2) Draw the projections of a line *AB*, 9 cm ($3\frac{1}{2}''$) long, its mid-point *M* being 5 cm (2″) above the H.P. and 4 cm ($1\frac{1}{2}''$) in front of the V.P.; the end *A* is 2 cm ($\frac{3}{4}''$) above the H.P. and 1·3 cm ($\frac{1}{2}''$) in front of the V.P. Show the traces and the inclinations of the line with the H.P. and the V.P.

(3) The elevation of a line *PQ*, 12·5 cm (5″) long, measures 7·5 cm (3″) and its plan measures 10 cm (4″). Its end *Q* and the mid-point *M* are in the first quadrant, *M* being 2·5 cm (1″) from both the planes. Draw the projections of the line *PQ*.

(4) A line *AB*, 7·5 cm (3″) long, is in the second quadrant, with the end *A* in the H.P. and the end *B* in the V.P. The line is inclined at 30° to the H.P. and at 45° to the V.P. Draw the projections of *AB* and determine its traces.

(5) The end *A* of a line *AB* is in the H.P. and 2·5 cm (1″) in front of the V.P. The end *B* is in the V.P. and 5 cm (2″) below the H.P. The distance between the end projectors is 7·5 cm (3″). Draw the projections of *AB* and determine its true length, traces and inclinations with the two planes.

(6) The plan of a line *CD*, 7·5 cm (3″) long, measures 5 cm (2″). *C* is 5 cm (2″) in front of the V.P. and 1·5 cm ($\frac{5}{8}''$) below the H.P. *D* is 1·5 cm ($\frac{5}{8}''$) in front of the V.P. and is above the H.P. Draw the elevation of *CD* and find its inclinations with the H.P. and the V.P. Show also its traces.

(7) A line *PQ*, 10 cm (4″) long, is inclined at 45° to the H.P. and at 30° to the V.P. Its end *P* is in the second quadrant and *Q* is in the fourth quadrant. A point *R* on *PQ*, 4 cm ($1\frac{1}{2}''$) from *P* is in both the planes. Draw the projections of *PQ*.

(8) A line *AB*, 6·5 cm ($2\frac{1}{2}''$) long, has its end *A* in the H.P. and 1·5 cm ($\frac{5}{8}''$) in front of the V.P.; the end *B* is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

(9) The elevation of a line *AB* measures 6·5 cm ($2\frac{1}{2}''$) and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T.

of the line is 1·5 cm ($\frac{5}{8}''$) below the H.P. The line is inclined at 30° to the V.P. Draw the projections of AB and find its true length and inclination with the H.P. Also locate its H.T.

× (10) A line AB is in the first quadrant. Its ends A and B are 2 cm ($\frac{3}{4}''$) and 6 cm ($2\frac{3}{8}''$) in front of the V.P. respectively. The distance between the end projectors is 7·5 cm (3″). The line is inclined at 30° to the H.P. and its H.T. is 1 cm ($\frac{3}{8}''$) below the H.P. Draw the projections of AB and determine its true length and the V.T.

(11) A room is 4·8 m×4·2 m×3·6 m high (16′×14′×12′ high). Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it. Use third-angle projection method.

(12) Two oranges on a tree are respectively 1·8 m (6 feet) and 3 m (10 feet) above the ground, and 1·2 m (4 feet) and 2·1 m (7 feet) from a 0·3 m (1 foot) thick wall, but on the opposite sides of it. The distance between the oranges, measured along the ground and parallel to the wall is 2·7 m (9 feet). Determine the real distance between the oranges.

(13) Draw an isosceles triangle abc, of base ab equal to 4 cm ($1\frac{1}{2}''$) and altitude 7·5 cm (3″) with a in xy and ab inclined at 45° to xy. The figure is the plan of a triangle whose corners A, B and C are respectively 7·5 cm (3″), 2·5 cm (1″) and 5 cm (2″) above the H.P. Determine the true shape of the triangle and the inclination of AB with the two planes.

(14) Three points A, B and C are 7·5 m (25 ft) above the ground level, on the ground level and 9 m (30 ft) below the ground level respectively. They are connected by roads with each other and are seen at angles of depression of 10°, 15° and 30° respectively from a point O on a hill 30 m (100 ft) above the ground level. A is due north-east, B is due north and C is due south-east of O. Find the lengths of the connecting roads. Use third-angle projection method.

(15) A pipe-line from a point A, running due north-east has a downward gradient of 1 in 5. Another point B is 12 m (40 ft) away from and due east of A and on same level. Find the length and slope of a pipe-line from B which runs due 15° east of north and meets the pipe-line from A.

(16) A plate chimney, 18 m (60 feet) high and 0·9 m (3 feet) diameter is supported by two sets of three guy wires each, as shown in fig. 9-48. One set is attached at 3 m (10 feet) from the top and anchored 6 m (20 feet) above the ground level. The other set is fixed to the chimney at its mid-height and anchored on the ground. Determine the length and slope with the ground, of one of the wires from each set.

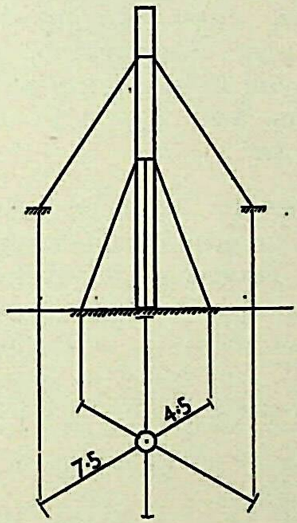


FIG. 9-48

(17) The guy-ropes of two poles 12 m (40 feet) apart, are attached to a point 15 m (50 feet)above the ground on the corner of a building. The points of attachment on the poles are 7·5 m (25 feet) and 4·5 m (15 feet) above the ground and the ropes make 45° and 30° respectively with the ground. Draw the projections and find the distances of the poles from the building and the lengths of the guy ropes. Use third-angle projection method.

CHAPTER X

PROJECTIONS OF PLANES

Plane figures: Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

Traces of planes: A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them. These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

Types of planes: Planes may be divided into two main types:

(i) Perpendicular planes

(ii) Oblique planes.

Perpendicular planes can be sub-divided into the following sub-types:

(a) Perpendicular to both the reference planes.

(b) Perpendicular to one plane and parallel to the other.

(c) Perpendicular to one plane and inclined to the other.

Planes which are inclined to both the reference planes are called *oblique planes.*

PERPENDICULAR PLANES:

(i) Plane, perpendicular to both the H.P. and the V.P. (fig. 10-1).

A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

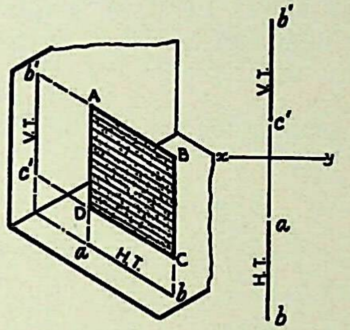


FIG. 10-1

The elevation $b'c'$ and the plan *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 10-2(*i*)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

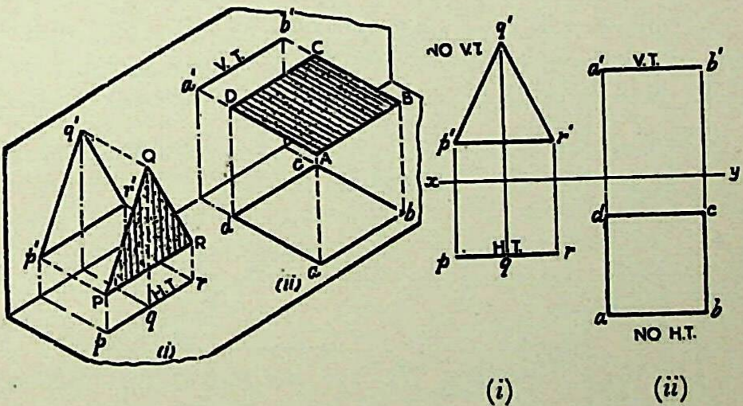


FIG. 10-2

The elevation $p'q'r'$ shows the exact shape and size of the triangle. The plan *pqr* is a line, parallel to *xy*; it coincides with the H.T.

(iii) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 10-2(*ii*)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The plan *abcd* shows the true shape and true size of the square. The elevation *a'b'* is a line, parallel to *xy*; it coincides with the V.T.

(iv) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 10-3).

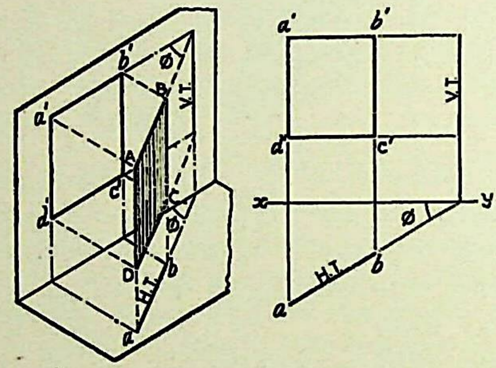


Fig. 10-3

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ϕ to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ϕ to *xy*.

Its plan *ab* is a line, inclined at ϕ to *xy*. The elevation *a'b'c'd'* is smaller than *ABCD*.

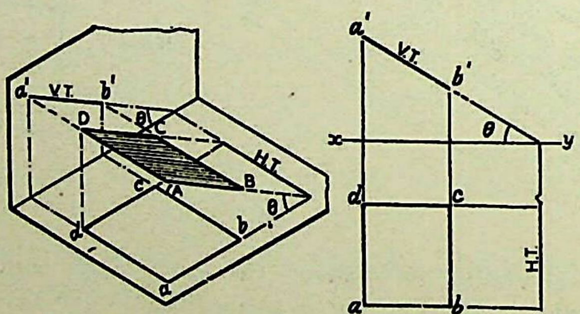


Fig. 10-4

(v) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 10-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its elevation *a'b'* is a line inclined at θ to *xy*. The plan *abcd* is a rectangle, which is smaller than the square *ABCD*.

Fig. 10-4(*a*) shows the projections and the traces of all these perpendicular planes in third-angle projection.

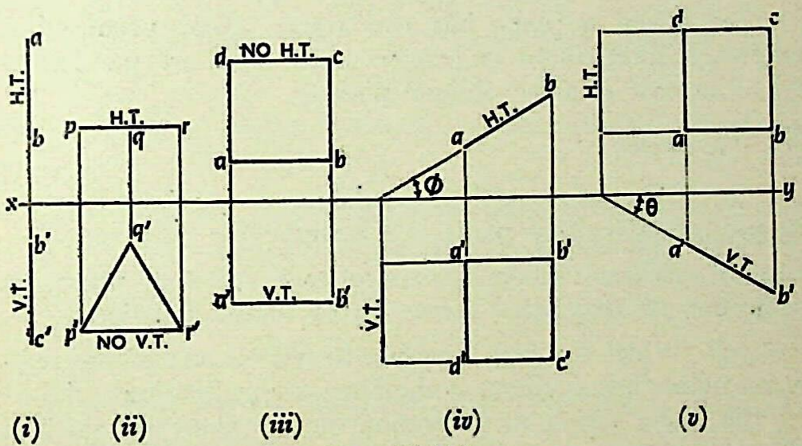


FIG. 10-4(*a*)

OBLIQUE PLANES:

Representation of oblique planes by their traces, is too advanced to be included in this book. A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

GENERAL CONCLUSIONS:

I. Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*; when it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy*, as in case of some oblique planes).

II. Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 1:

Show by means of traces, each of the following planes:

(a) Perpendicular to the H.P. and the V.P.

(b) Perpendicular to the H.P. and inclined at 30° to the V.P.

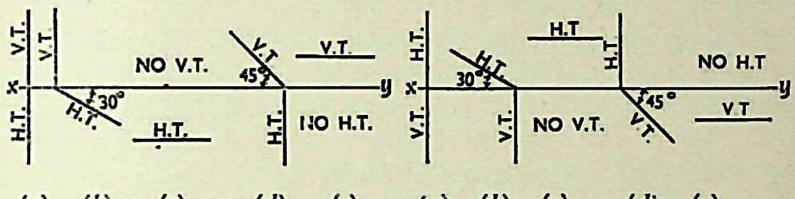
(c) Parallel to and 4 cm ($1\frac{1}{2}''$) away from the V.P.

(d) Inclined at 45° to the H.P. and perpendicular to the V.P.

(e) Parallel to the H.P. and 2·5 cm (1″) away from it.

Fig. 10-5 shows the various traces. (*a*) The H.T. and the V.T. are in a line perpendicular to *xy*. (*b*) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*. (*c*) The H.T. is parallel to and 4 cm ($1\frac{1}{2}''$) away from *xy*. It has no V.T. (*d*) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(*e*) The V.T. is parallel to and 2·5 cm (1″) away from *xy*. It has no H.T.



(*a*) (*b*) (*c*) (*d*) (*e*)
First-angle projection

(*a*) (*b*) (*c*) (*d*) (*e*)
Third-angle projection

FIG. 10-5

Projections of planes parallel to one of the reference planes:

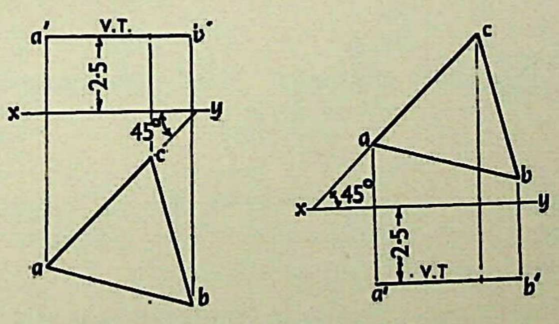
The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(*a*) *When the plane is parallel to the H.P.*

The plan should be drawn first and the elevation projected from it.

Problem 2:

An equilateral triangle of 5 cm (2″) side has its V.T. parallel to and 2·5 cm (1″) away from xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.



(*i*)
First angle projection

(*ii*)
Third-angle projection

FIG. 10-6

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the plan (fig. 10-6).

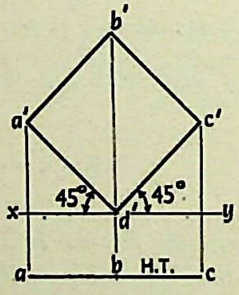
Draw an equilateral triangle *abc* of 5 cm (2″) side, keeping one side, say. *ac*, inclined at 45° to *xy*. Project the elevation, 2·5 cm (1″) away from *xy*, as shown.

(*b*) *When the plane is parallel to the V.P.*

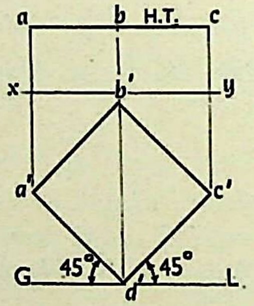
Beginning should be made with the elevation and the plan projected from it.

Problem 3:

A square ABCD of 4 cm (1½″) side has a corner on the ground and 2 cm (¾″) away from the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.



(*i*)
First-angle projection



(*ii*)
Third-angle projection

FIG. 10-7

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The elevation will show the true shape and position of the square.

Draw a square *a′b′c′d′* in elevation with one corner in the ground and all its sides inclined at 45° to *xy* (fig. 10-7). Project the plan keeping the line *ac* parallel to *xy* and 2 cm (¾″) away from it. The plan is its H.T. It has no V.T.

Projections of a plane inclined to one reference plane and perpendicular to the other:

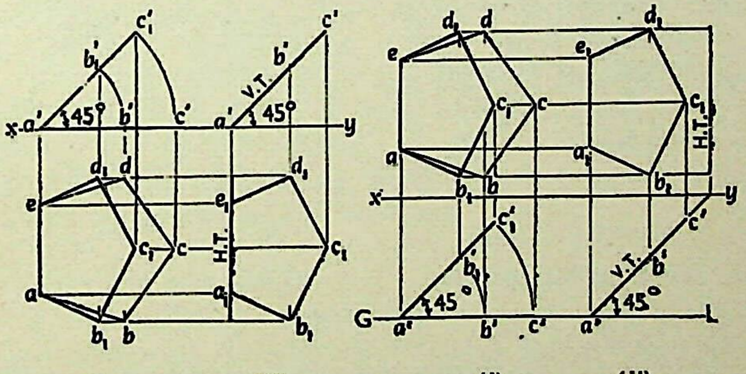
When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(*a*) *Plane, inclined to the H.P. and perpendicular to the V.P.*

When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its plan will show the true shape; the elevation will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new elevation will be inclined to *xy* at the true inclination. In plan, the corners will move along their respective paths (parallel to *xy*).

Problem 4:

A regular pentagon of 2·5 cm (1″) side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and traces.



(*i*) (*ii*)
First-angle projection

(*i*) (*ii*)
Third-angle projection

FIG. 10-8

Assuming it to be parallel to the H.P., draw the pentagon in plan with one side perpendicular to *xy* [fig. 10-8(*i*)]. Project the elevation. Tilt the elevation about the point a', so that it makes 45° angle with *xy*. Project the new plan $ab_1c_1d_1e$, downwards (or upwards) from this elevation and hori-

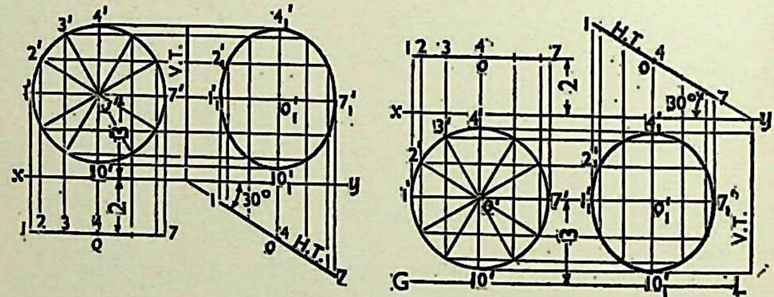
zontally from the first plan. It will be more convenient if the elevation is reproduced in the new position separately and the plan projected from it, as shown in fig. 10-8(*ii*). The V.T. coincides with the elevation and the H.T. is perpendicular to *xy*, through the point *a'*.

(*b*) *Plane, inclined to the V.P. and perpendicular to the H.P.*

In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 5:

Draw the projections of a circle of 5 cm (2") diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 3 cm ($1\frac{3}{16}$") above the ground and 2 cm ($\frac{3}{4}$") away from the V.P. Show also its traces.



(*i*) (*ii*)
First-angle projection

(*i*) (*ii*)
Third-angle projection

FIG. 10-9

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

Assuming the circle to be parallel to the V.P., draw its projections. The elevation will be a circle [fig. 10-9(*i*)], having its centre 3 cm ($1\frac{3}{16}$") above *xy* (or *GL*). The plan will be a line, parallel to and 2 cm ($\frac{3}{4}$") from *xy*. Divide the circumference into twelve equal parts (with a 30°-60° set-

square) and mark the points as shown. Project these points in the plan. The centre O will coincide with the point 4. When the circle is tilted, so as to make 30° angle with the V.P., its plan will become inclined at 30° to xy; in the elevation, all the points will move along their respective paths (parallel to xy). Reproduce the plan, keeping the centre O at the same distance, viz. 2 cm ($\frac{3}{4}''$) from xy [fig. 10-9(*ii*)]. For the final view, project all the points upwards (or downwards) from this plan and horizontally from the first elevation. Draw a freehand curve through the twelve points $1_1'$, $2_1'$ etc. This curve will be an ellipse.

The plan is the H.T. of the circle. Produce it to meet xy in v. Through v, draw a vertical line, which is the required V.T.

Projections of oblique planes:

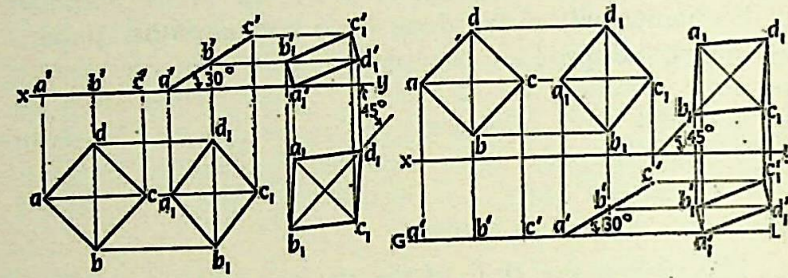
When a plane is inclined to both the H.P. and the V.P., its projections are drawn in three stages. In the initial stage, the plane is assumed to be parallel to the H.P. It is then tilted so as to make the required angle with the H.P. As already explained, its elevation in this position will be a line, while its plan will be smaller in size. In the final position, when the plane is turned to the required inclination with the V.P., only the position of the plan will change. Its shape and size will not be affected. In the final elevation, the corresponding distances of all the corners from xy will remain the same as in the second elevation.

Problem 6:

A square ABCD of 5 cm (2") side has its corner A on the ground, its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections (fig. 10-10).

In the initial stage, keep the square parallel to the H.P. with AC parallel to the V.P. (i) Draw the plan and elevation. When the square is tilted about the corner A so that AC makes 30° angle with the H.P., BD remains perpendicular to the V.P. and parallel to the H.P. (ii) Draw the second elevation with $a'c'$ inclined at 30° to the ground line. Project

the second plan. The square may now be turned so that *BD* makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the plan will change. Its shape



(*i*) (*ii*) (*iii*)
First-angle projection

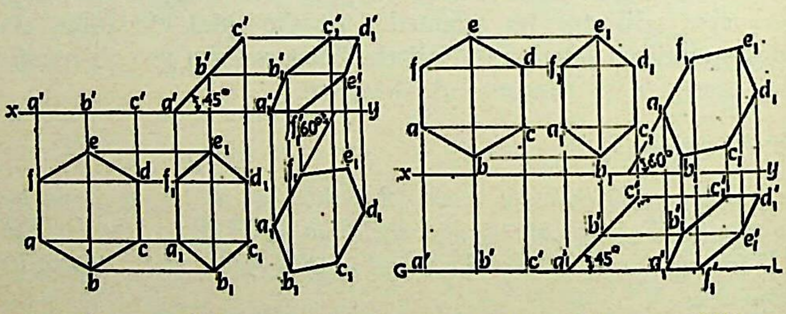
(*i*) (*ii*) (*iii*)
Third-angle projection

Fig. 10-10

will remain the same. (iii) Reproduce the plan so that b_1d_1 is inclined at 45° to *xy*. Project the final elevation upwards (or downwards) from this plan and horizontally from the second elevation.

Problem 7:

Draw the projections of a regular hexagon of 2·5 cm (1″) side, having one of its sides on the ground and inclined at 60° to the V.P. and its surface making an angle of 45° with the ground (fig. 10-11).



(*i*) (*ii*) (*iii*)
First-angle projection

(*i*) (*ii*) (*iii*)
Third-angle projection

Fig. 10-11

(i) Draw the hexagon in the plan with one side perpendicular to *xy*. Project *a′c′*, the elevation. (ii) Draw *a′c′* inclined at 45° to the ground line and project the second plan.

(iii) Reproduce this plan, making $a_1 f_1$ inclined at 60° to *xy* and project the final elevation.

Problem 8:

Draw the plan and the elevation of a circle of 5 cm (2") diameter resting in the H.P. on a point A on the circumference, its plane inclined at 45° to the H.P. and (a) the plan of the diameter AB making 30° angle with the V.P.; (b) the diameter AB making 30° angle with the V.P. (fig. 10-12).

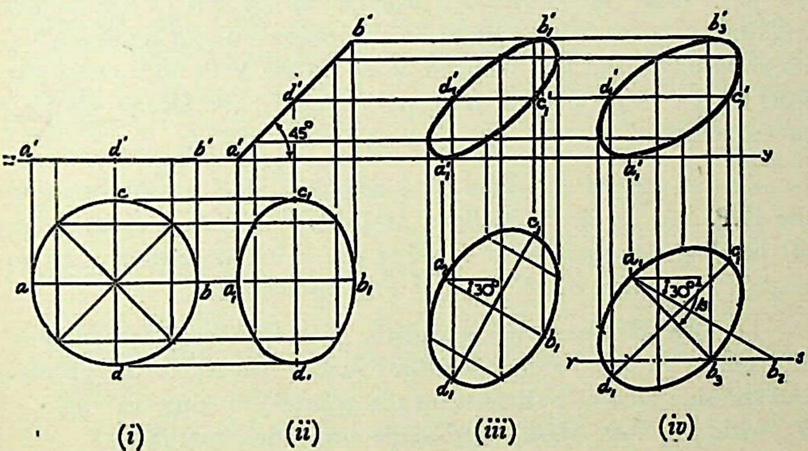


Fig. 10-12

Draw the projections of the circle with *A* in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [figs. 10-12(*i*) and (*ii*)].

(a) In the second plan, the line a_1b_1 is the plan of the diameter *AB*. Reproduce this plan, so that a_1b_1 (the plan of diameter *AB*), makes 30° angle with *xy* [fig. 10-12(*iii*)]. Project the required elevation.

(b) If the diameter *AB*, which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its plan a_1b_1 will make an angle greater than 30° with *xy*. This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to *AB* and inclined at 30° to *xy* [fig. 10-12(*iv*)]. With a_1 as centre and radius equal to the

plan of AB, viz. a_1b_1, draw an arc cutting rs (the path of B in the plan) at b_3. Draw the line joining a_1 and b_3, and around it, reproduce the second plan. Project the final elevation. It is evident that a_1b_3 is inclined to xy at an angle β which is greater than 30°.

EXERCISES X

(1) Draw an equilateral triangle of 7·5 cm (3") side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P. Use third-angle projection method.

(2) A regular hexagon of 4 cm ($1\frac{1}{2}$") side has a corner in the H.P. Its surface is inclined at 45° to the H.P. and the plan of the diagonal through the corner on which it rests, makes an angle of 60° with the V.P. Draw its projections.

(3) Draw the elevation and the plan of a regular pentagon of 4 cm ($1\frac{1}{2}$") side, having its surface inclined at 30° to the ground and the side on which it rests on the ground, making an angle of 60° with the V.P. Use third-angle projection method.

(4) Draw the projections of a rhombus having diagonals 12·5 cm (5") and 5 cm (2") long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to the H.P.

(5) Draw a regular hexagon of 4 cm ($1\frac{1}{2}$") side, with its two sides vertical. Draw a circle of 4 cm ($1\frac{1}{2}$") diameter in its centre. The figure represents a hexagonal plate with a hole in it and having its surface parallel to the V.P. Draw its plan and elevation when the surface is vertical and inclined at 30° to the V.P. Assume the thickness of the plate to be equal to that of a line. Use third-angle projection method.

(6) Draw the projections of a circle of 7·5 cm (3") diameter having the end A of the diameter AB in the H.P., the end B in the V.P., and the surface inclined at 30° to the H.P. and at 60° to the V.P.

CHAPTER **XI**

PROJECTIONS ON AUXILIARY PLANES

Two views of an object, viz. the elevation and the plan (projected on the principal planes of projection), are sometimes not sufficient to convey all the information regarding the object. Additional views, called *auxiliary views*, are therefore, projected on other planes, known as *auxiliary planes*. These views are often found necessary in technical drawings. Auxiliary views may also be used for determining (i) the true length of a line, (ii) the point-view of a line, (iii) the edge-view of a plane, (iv) the true size and form of a plane etc. They are thus very useful in finding solutions of problems in practical solid geometry.

AUXILIARY PLANES AND VIEWS:

Auxiliary planes are of two types: (i) *auxiliary vertical plane* or A.V.P. and (ii) *auxiliary inclined plane* or A.I.P.

Auxiliary vertical plane is perpendicular to the H.P. and inclined to the V.P. Projection on an A.V.P. is called *auxiliary elevation.*

Auxiliary inclined plane is perpendicular to the V.P. and inclined to the H.P. Projection on an A.I.P. is called *auxiliary plan.*

The orthographic views of the auxiliary projections are drawn by rotating the auxiliary plane about that principal plane to which it is perpendicular.

PROJECTION OF A POINT ON AN AUXILIARY VERTICAL PLANE:

A point A [fig. 11-1(i)] is situated in front of the V.P. and above the H.P. A.V.P. is the auxiliary vertical plane, inclined at an angle α to the V.P. The H.P. and the A.V.P. meet at right angles in the line x_1y_1.

a' and a are the elevation and the plan respectively, of the point A. a_1' is the auxiliary elevation obtained by drawing a projector Aa_1' perpendicular to the A.V.P. It can be

clearly seen that $a_1'o_1$ (the distance of the auxiliary elevation a_1' from x_1y_1) $= a'o$ (the distance of the elevation a' from xy) $=Aa$ (the distance of the point A from the H.P.).

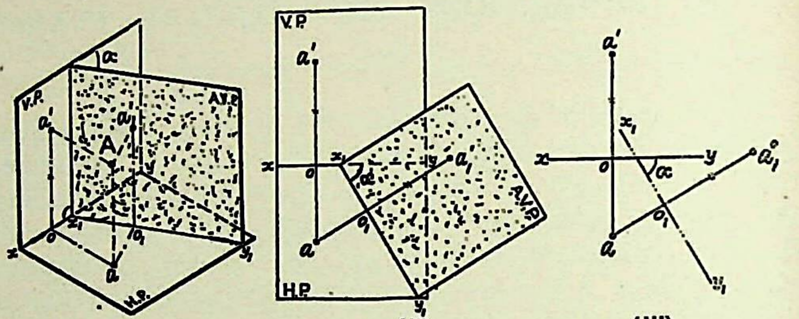
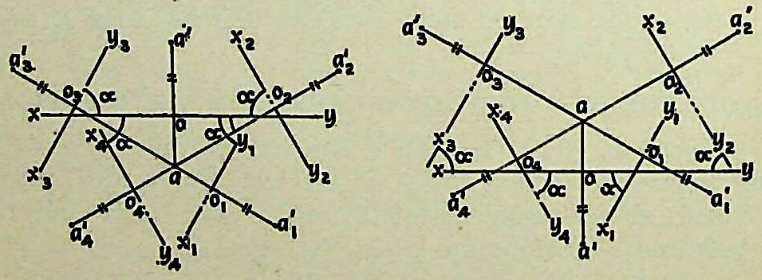


FIG. 11-1

Fig. 11-1(*ii*) shows the V.P. and the A.V.P. rotated about the H.P., to which they are perpendicular. The line of intersection x_1y_1, between the A.V.P. and the H.P., is inclined at the angle α to xy. The line joining the plan a with the auxiliary elevation a_1', is at right angles to x_1y_1 and intersects it at o_1. Note that, $a_1'o_1 = a'o$.

To draw the orthographic views [fig. 11-1(*iii*)], start with the ground line xy and mark the elevation a' and the plan a. Draw a new ground line x_1y_1, making the angle α with xy. Through the plan a, draw a projector aa_1' perpendicular to and intersecting x_1y_1 at o_1 and such that $a_1'o_1 = a'o$. a_1' is the required auxiliary elevation.



First-angle projection

Third-angle projection

FIG. 11-2

The new ground line making the angle α with xy, can be drawn in four different positions, as shown by lines x_1y_1, x_2y_2 etc. in fig. 11-2. All the elevations are projected from the plan a and their distances from their respective ground lines are equal, i.e. $a_1'o_1 = a_2'o_2 \ldots\ldots\ldots\ldots = a'o$.

PROJECTION OF A POINT ON AN AUXILIARY INCLINED PLANE:

A point P [fig. 11-3(i)] is situated above the H.P. and in front of the V.P. A.I.P. is an auxiliary inclined plane inclined at an angle β to the H.P. It meets the V.P. at right angles and in a line x_1y_1.

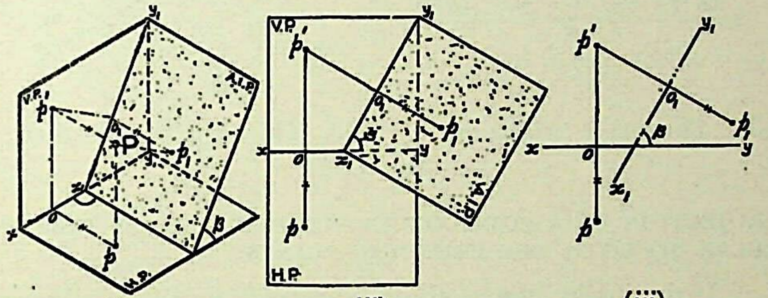


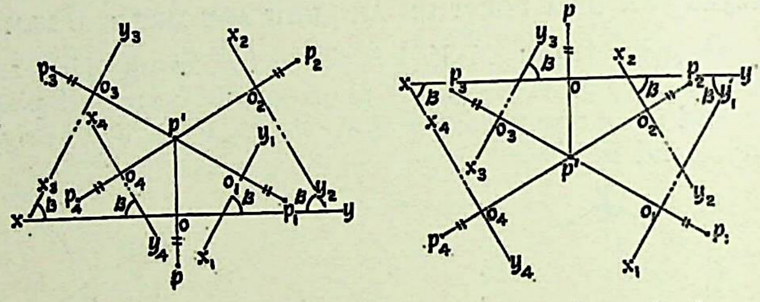
FIG. 11-3

p' and p are respectively the elevation and the plan of the point P. p_1 is the auxiliary plan obtained by drawing the projector Pp_1, perpendicular to the A.I.P. It can be seen that p_1o_1 (the distance of the auxiliary plan p_1 from x_1y_1) $= po$ (the distance of the plan p from xy) $= Pp'$ (the distance of the point P from the V.P.).

The H.P. and the A.I.P. are then rotated about the V.P. to which they are perpendicular [fig. 11-3(ii)]. x_1y_1, the line of intersection between the V.P. and the A.I.P. makes the angle β with xy. The line joining the elevation p' and the auxiliary plan p_1 is at right angles to x_1y_1 and intersects it at o_1. Note that $p_1o_1 = po$.

To draw the orthographic view [fig. 11-3(iii)], draw xy and mark p' and p. Draw x_1y_1 making the angle β with xy. Through the elevation p', draw a projector $p'p_1$, perpendicular to and intersecting x_1y_1 at o_1 and such that $p_1o_1 = po$. p_1 is the required auxiliary plan.

In this case also, the new ground line can be drawn in four different positions as shown in fig. 11-4 by lines x_1y_1, x_2y_2 etc., each inclined at β to xy. All the plans are projected from the elevation p' and their distances from their respective ground lines are equal, i.e. $p_1o_1 = p_2o_2 \cdots\cdots\cdots\cdots = po$.



First-angle projection Third-angle projection

FIG. 11-4

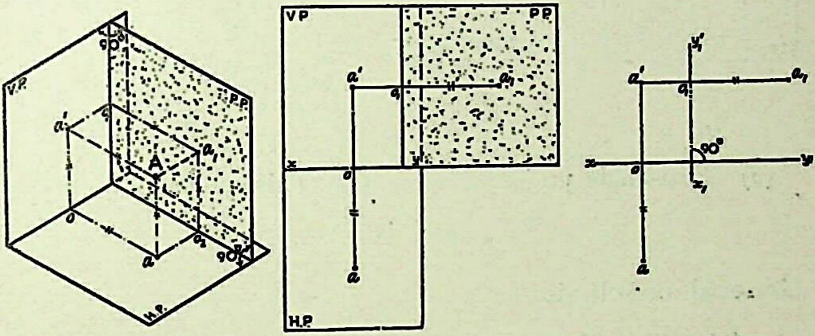
PROJECTION OF A POINT ON AN AUXILIARY PLANE PERPENDICULAR TO BOTH THE PRINCIPAL PLANES:

If the inclination of the A.V.P. with the V.P. is increased, so that $\alpha = 90°$, the A.V.P. will be perpendicular to both the planes. Similarly, if the inclination of the A.I.P. with the H.P. is increased, so that $\beta = 90°$, it will also be perpendicular to both the H.P. and the V.P. This plane may be rotated about any one of the principal planes. The view on this plane can, therefore, be projected from either the plan or the elevation and named accordingly.

In fig. 11-5(*i*), A is a point. P.P. is an auxiliary plane perpendicular to the H.P. and the V.P. a_1 is the auxiliary view projected on the P.P., It can be seen that $a_1o_2 = a'o = Aa$ (the distance of A from the H.P.). Also, $a_1o_1 = ao = Aa'$ (the distance of A from the V.P.).

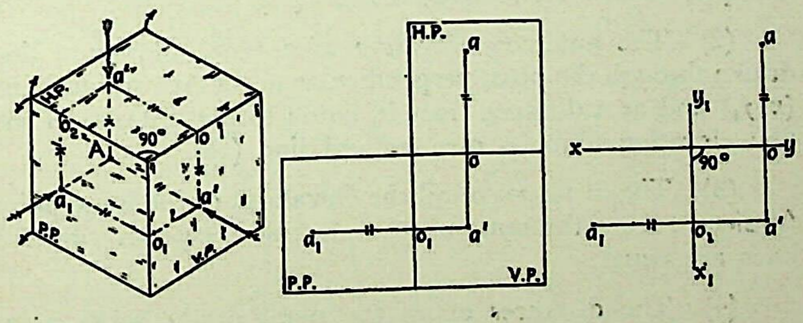
Fig. 11-5(*ii*) shows the P.P. rotated about the V.P. a_1 lies on the projector drawn through the elevation a' and perpendicular to the line of intersection between the V.P. and the P.P. It is thus projected from the elevation and hence, called the auxiliary plan. In technical drawings, this view is generally termed as the *end elevation* or *end view*. Note that $a_1o_1 = ao$.

Also note that in first-angle projection, when seen from the left the new ground line and the end view come to the right of the elevation; while in third-angle projection, when seen from the left they come to the left of the elevation. Thus, in the former method, the view looking from any side of the elevation is placed on its other side; while in the latter method, it is placed on the same side of the elevation.



(*i*) (*ii*) (*iii*)

(*a*) First-angle projection



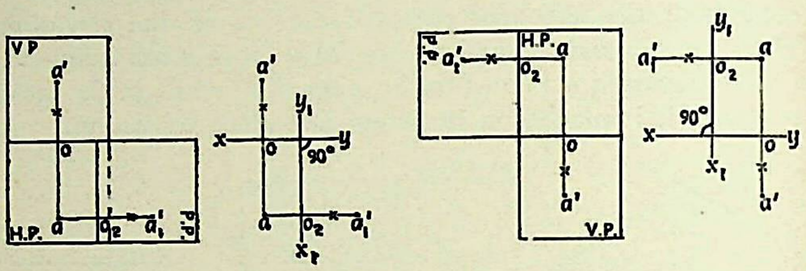
(*i*) (*ii*) (*iii*)

(*b*) Third-angle projection

FIG. 11-5

Fig. 11-6 shows the P.P. rotated about the H.P. The view on the P.P. now lies on the projector through the plan *a*. Hence, it is called the auxiliary elevation. In this case, $a_1'o_2 = a'o$.

The orthographic views [figs. 11-5(*iii*) and 11-6(*ii*)] in both cases are self-explanatory.



(*i*) (*ii*) (*i*) (*ii*)

(*a*) First-angle projection (*b*) Third-angle projection

FIG. 11-6

General conclusions:

(1) The auxiliary plan of a point lies on a line drawn through the elevation, perpendicular to the new ground line (x_1y_1) and at a distance from it, equal to the distance of the first plan from its own ground line (xy).

(2) The auxiliary elevation of a point lies on a line drawn through the plan, perpendicular to the new ground line (x_1y_1) and at a distance from it, equal to the distance of the first elevation from its own ground line (xy).

(3) The distances of all the elevations of the same point (projected from the same plan) from their respective ground lines are equal.

(4) The distances of all the plans of the same point (projected from the same elevation) from their respective ground lines are equal.

Problem 1:

The projections of a line AB are given. Draw (i) an auxiliary elevation of the line on an A.V.P. inclined at 60° to the V.P. and (ii) an auxiliary plan on an A.I.P. making an angle of 75° with the H.P.

Let *ab* and *a′b′* be the given projections (fig. 11-7). Draw a new ground line x_1y_1, inclined at 60° to *xy* to represent the A.V.P. Project the auxiliary elevation $a_1'b_1'$ from the

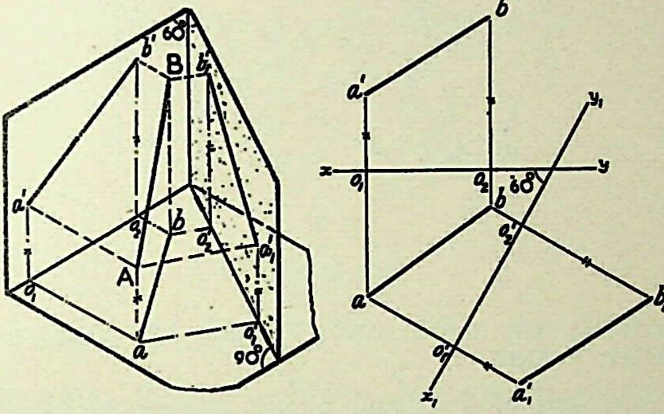


FIG. 11-7

plan *ab*, by making $a_1'o_1'$ equal to $a'o_1$, and $b_1'o_2'$ equal to $b'o_2$. Similarly, draw x_2y_2 for the A.I.P. (fig. 11-8), inclined at 75° to *xy*. Project the auxiliary plan a_2b_2 from the elevation *a′b′*, by making a_2o_1'' equal to ao_1 and b_2o_2'' equal to bo_2.

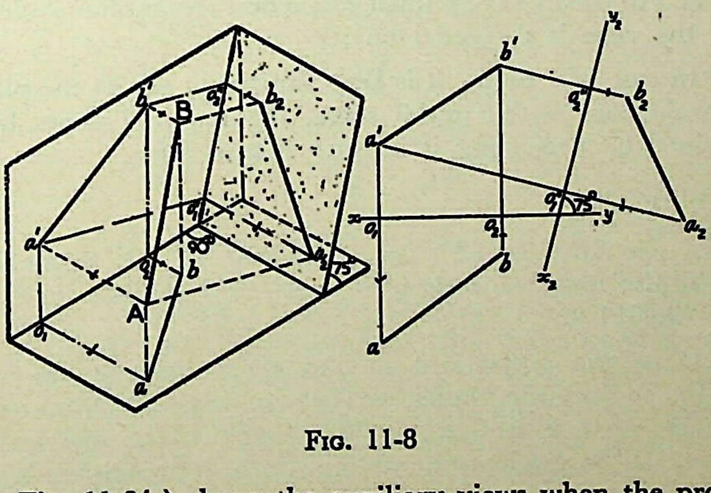


FIG. 11-8

Fig. 11-8(*a*) shows the auxiliary views when the projections of *AB* .are given by third-angle projection method.

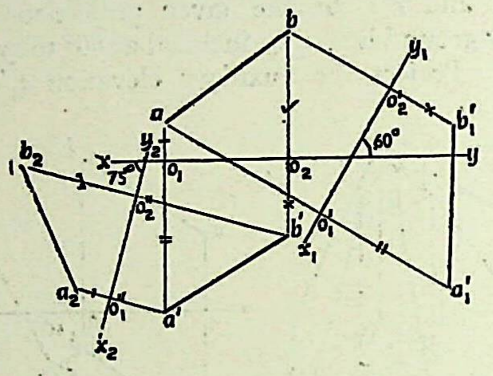


FIG. 11-8(a)

PROJECTIONS OF LINES AND PLANES BY THE USE OF AUXILIARY PLANES:

Projections of lines and planes at given inclinations to one or both the planes may also be obtained by the use of auxiliary planes. The method adopted is called the *alteration or change of ground line method.*

The line, in its initial position, is assumed to be parallel to both the planes of projection. Then, instead of making the line inclined to one of the planes, an auxiliary plane inclined to the line is assumed, i.e., a new ground line is drawn and the view is projected on it.

In case of a plane, it is kept parallel to one of the planes of projection in the initial stage, the required views being obtained by projecting it on new ground lines.

Problem 2:

A line AB 5 cm (2") long, is inclined at 30° to the H.P. and its plan makes an angle of 60° with the V.P. Draw its projections (fig. 11-9).

Draw the projections ab and $a'b'$, assuming AB to be parallel to both the planes. Project a new plan a_1b_1 on a new ground line x_1y_1 inclined at 30° to $a'b'$. a_1b_1 is still parallel to x_1y_1. Draw another ground line x_2y_2 to represent an A.V.P. inclined at 60° to the plan a_1b_1. Project the required elevation $a_1'b_1'$ on x_2y_2. Note that, $a_1o_1 = ao$, $a_1'o_2 = a'o_1$ etc.

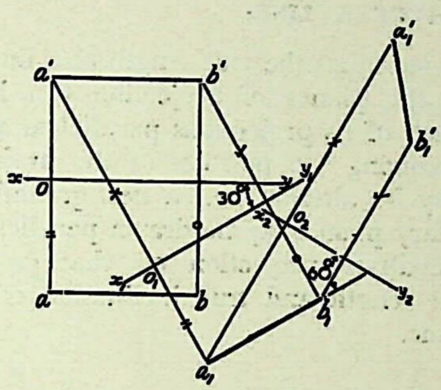


FIG. 11-9

Problem 3:

An equilateral triangle of 4 cm ($1\frac{1}{2}''$) side has an edge in the H.P. and inclined at 60° to the V.P. Its plane makes an angle of 45° with the H.P. Draw its projections (fig. 11-10).

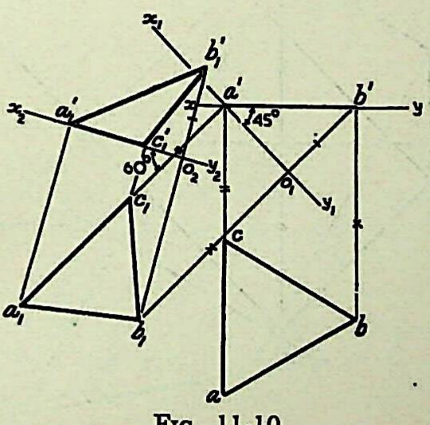


FIG. 11-10

Draw the plan *abc* and the elevation $a'b'$, assuming the triangle to be parallel to the H.P. and one side perpendicular to the V.P. Draw x_1y_1 inclined at 45° to $a'b'$ (the elevation) and passing through a' (because the triangle has its edge in the H.P.). Project the new plan $a_1b_1c_1$. Again, draw a ground line x_2y_2 inclined at 60° to the edge a_1c_1 (which is to make 60° angle with the V.P.). Project the final elevation $a_1'b_1'c_1'$ on x_2y_2. Note that $b_1o_1 = bb'$, $b_1'o_2 = b'o_1$ etc.

TRUE LENGTH OF A LINE:

We have seen that the true length of a line and its inclinations with the planes of projection can be determined by making each of its projections parallel to xy (Chap. IX). Instead of changing the position of the projection, that of the plane may be altered, i.e., a new ground line representing an auxiliary plane may be drawn parallel to the projection. The auxiliary projection on that ground line will show the true length and true inclination of the line with the other plane.

Problem 4:

The projections of a line AB are given. To determine its true length and true inclinations with the reference planes.

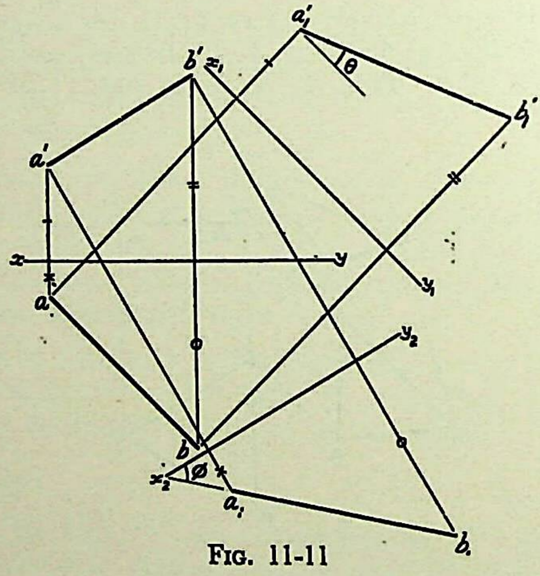


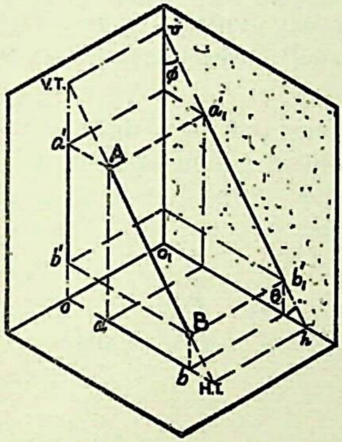
FIG. 11-11

Let ab and $a'b'$ be the given projections of AB (fig. 11-11). Draw a ground line x_1y_1 to represent an A.V.P. parallel to ab, the plan. Project the auxiliary elevation $a_1'b_1'$, which is the true length of AB. θ is its true inclination with the H.P.

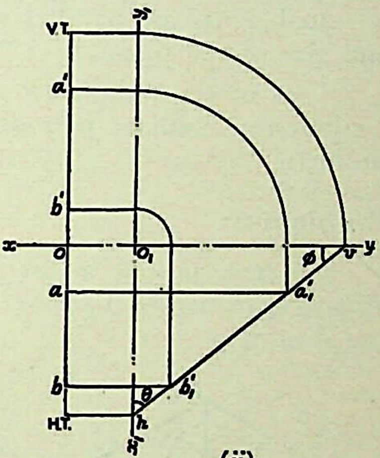
Similarly, draw x_2y_2 parallel to $a'b'$, the elevation and project the auxiliary plan a_1b_1. It is the true length of AB and ϕ is its true inclination with the V.P.

Problem 5:

The projections of a line AB viz. ab and a'b' are on the same projector as shown in fig. 11-12(ii). Find the true length, inclinations with the H.P. and the V.P. and the traces of AB.



(*i*)



(*ii*)

FIG. 11-12

Draw a ground line x_1y_1 parallel to the projections. It will be perpendicular to *xy* and will represent an auxiliary plane at right angles to both the H.P. and the V.P. as shown in fig. 11-12(*i*). Project the auxiliary elevation $a_1'b_1'$, which will be the true length of *AB*. θ and ϕ are its true inclinations with the H.P. and the V.P. respectively. Produce $a_1'b_1'$ to meet *xy* at *h* and x_1y_1 at *v*. The H.T. will be on the plan *ab*-produced, so that *o*H.T. = o_1h. The V.T. is on the elevation *a'b'*-produced, so that *o*V.T. = o_1v

The distances of the ends of the line from x_1y_1 are kept equal to their respective distances above *xy* by drawing quarter circles with o_1 as centre, as shown.

POINT-VIEW OF A LINE AND EDGE-VIEW OF A PLANE:

We have seen in Chapter IX that when a line is perpendicular to a reference plane, its projection on that plane is a point; while its projection on the other reference plane

shows its true length. In other words, the projection of the view of a line showing its true length, on an auxiliary plane perpendicular to that view will be a point. Similarly, a plane will be seen as a line, when it is projected on a plane, perpendicular to the true length of any one of its elements. The projection of the line-view or edge-view of a plane on an auxiliary plane parallel to it, will show the true shape and size of the plane.

Uses of the point-view of a line and the edge-view of a plane on auxiliary planes are illustrated in the following problems:

Problem 6:

The projections of a line PQ are given. Determine, (i) the distance of its mid-point from xy and (ii) the shortest distance of the line from xy.

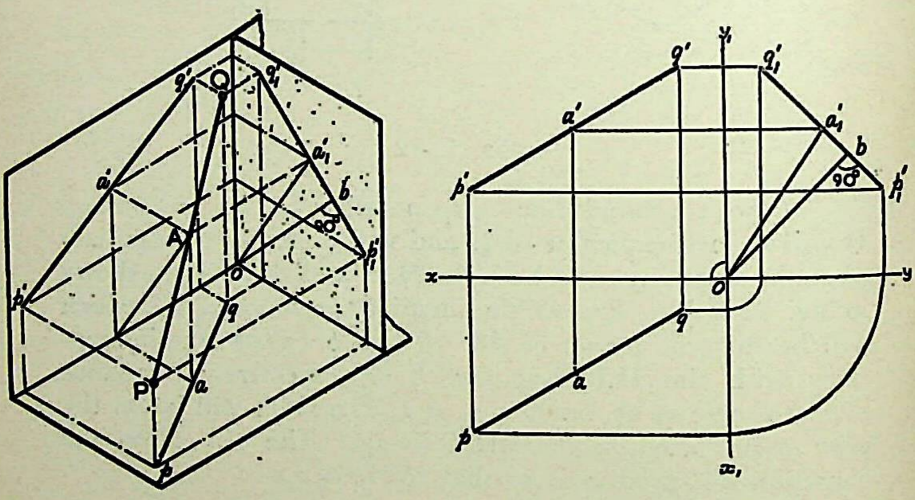


FIG. 11-13

Let pq and $p'q'$ be the given projections of PQ (fig. 11-13). Draw a new ground line x_1y_1, perpendicular to xy and project the end elevation $p_1'q_1'$ on it.

When considering the end elevation, x_1y_1 is the edge-view of the V.P. and xy is that of the H.P. The point o is the end elevation or the point-view of the line of intersection of the

V.P. and the H.P., i.e. xy. The line joining any point on $p_1'q_1'$ with o, will show the shortest distance of that point from xy.

(i) Find the mid-point a_1' of $p_1'q_1'$ and join it with o. oa_1' is the distance of the mid-point of PQ from xy.

(ii) From o, draw a line ob, perpendicular to $p_1'q_1'$. ob is the shortest distance of PQ from xy. It will be perpendicular to both PQ and xy.

Problem 7:

An isosceles triangle ABC, base 6·5 cm (2½") and altitude 4 cm (1½") has its base AC in the H.P. and inclined at 30° to the V.P. The corners A and B are in the V.P. Draw its projections (fig. 11-14).

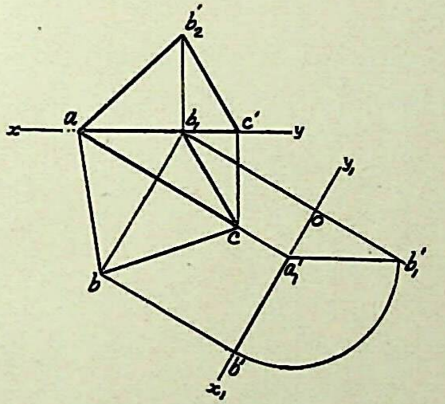


FIG. 11-14

Assume the triangle to be lying in the H.P. with the base AC inclined at 30° to the V.P. and A in the V.P. Its true shape will be seen in the plan.

Therefore, in plan, draw ac 6·5 cm (2½") long and inclined at 30° to xy and complete the triangle abc. Project c to c' on xy.

When the triangle is tilted about the edge AC, so that the corner B is in the V.P., in the plan, the point b will move along a line perpendicular to ac, to a point b_1 in xy. The distance of the elevation of B above xy may now be determined by means of an auxiliary plane. Draw a ground

line x_1y_1 perpendicular to ac. Project an auxiliary elevation of the triangle abc. It will be a line $b'a_1'$, showing the edge-view of the triangle; a_1' is the point-view of the line ac. When b moves to b_1, b' will move along the arc, drawn with a_1' as centre and radius equal to $a_1'b'$, to b_1' on the line through b_1 drawn perpendicular to x_1y_1. ob_1' is the required distance.

Therefore, through b_1, draw a line perpendicular to xy and on it, mark a point b_2' such that, $b_1b_2' = ob_1'$.

Join a and c with b_2'. $ab_2'c'$ and ab_1c are the required projections.

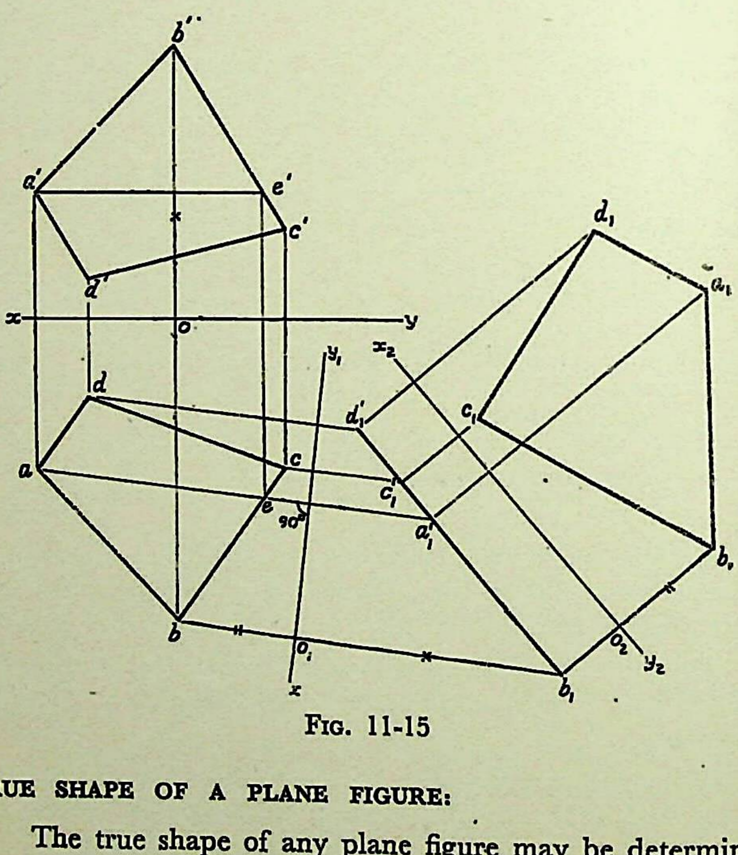


Fig. 11-15

TRUE SHAPE OF A PLANE FIGURE:

The true shape of any plane figure may be determined by means of its projections on auxiliary planes, as illustrated in problems 8 and 9.

Problem 8:

Fig. 11-15 shows the plan abcd and elevation $a'b'c'd'$ of a quadrilateral. Determine its true-shape.

Through any corner, say a', draw a line parallel to xy and meeting $b'c'$ at e'. Project e' to e on the line bc in plan. ae is the true length of the element $a'e'$.

Draw a ground line x_1y_1 perpendicular to ae. Project a new elevation from the plan. It is a line $d_1'b_1'$.

Again, draw another ground line x_2y_2 parallel to the line-view $d_1'b_1'$ and project on it, a new plan $a_1b_1c_1d_1$, which will show the true shape of the quadrilateral.

Note that $b_1'o_1 = b'o$, $b_1o_2 = bo_1$ etc.

Problem 9:

Projections of a pentagon resting in the H.P. on one of its sides are given. Determine the true shape of the pentagon (fig. 11-16).

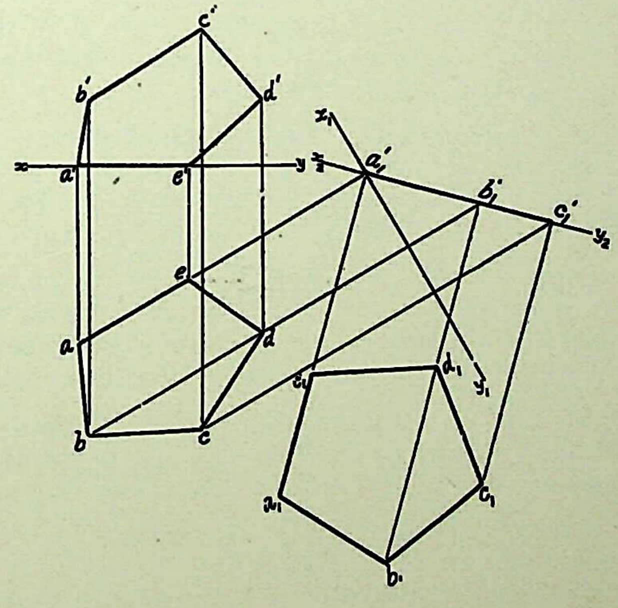


FIG. 11-16

ae is the true length of the side because it is parallel to the H.P. Draw a ground line x_1y_1 perpendicular to ae.

Project an auxiliary elevation on it. It will give a point-view a_1' of ae and an edge-view $a_1'c_1'$ of the pentagon. Draw another ground line x_2y_2 containing $a_1'c_1'$ and project an auxiliary plan $a_1b_1c_1d_1e_1$, which will be the true shape of the pentagon.

EXERCISES XI

Solve the following exercises by applying the method of projections on auxiliary planes:

(1) Determine the true length, inclinations with the H.P. and the V.P., and the traces of the line PQ in Prob. 17, Chapter IX.

(2) Find the true length and the distance of the mid-point from xy of the line whose projections are given in Prob. 13, Chapter IX.

(3) Draw the projections of the various plane figures as required in Exs. 1, 2 and 3 of Chapter X.

(4) abc is an equilateral triangle of altitude 5 cm (2″) with ab in xy and c below it. abc' is an isosceles triangle of altitude 7·5 cm (3″). c' is above xy. Determine the true shape of a triangle ABC, of which abc is the plan and abc' is the elevation.

(5) Determine the true shape of the figure, the plan of which is a regular pentagon of 3·5 cm ($1\frac{3}{8}$″) side, having one side inclined at 30° to xy and whose elevation is a straight line making an angle of 45° with xy. Use third-angle projection method.

(6) An equilateral triangle ABC of sides 7·5 cm (3″) long has its side AB in the V.P. and inclined at 60° to the H.P. Its plane makes an angle of 45° with the V.P. Draw its projections.

(7) An isosceles triangle PQR having the base PQ 5 cm (2″) long and altitude 7·5 cm (3″) has its corners P, Q and R 2·5 cm (1″), 5 cm (2″) and 7·5 cm (3″) respectively above the ground. Draw its projections. Use third-angle projection method.

CHAPTER XII

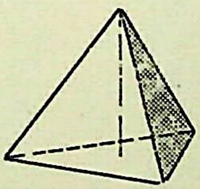
PROJECTIONS OF SOLIDS

A solid has three dimensions viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, atleast two orthographic views are necessary. Sometimes, additional views, projected on auxiliary planes, become necessary to make the description of a solid complete.

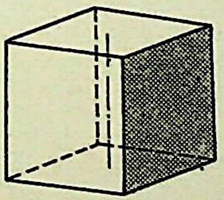
Solids may be divided into two main groups: (i) *polyhedra* and (ii) *solids of revolution.*

Polyhedra: A polyhedron is defined as a solid bounded by planes called *faces*. When all the faces are equal and regular, the polyhedron is said to be regular.

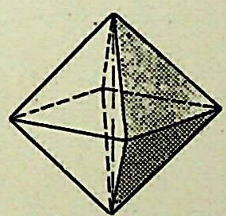
There are five regular polyhedra which may be defined as stated below:



Tetrahedron
FIG. 12-1



Cube
FIG. 12-2



Octahedron
FIG. 12-3

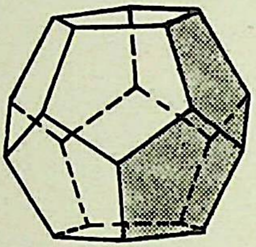
(i) *Tetrahedron:* It has four equal faces, each an equilateral triangle (fig. 12-1).

(ii) *Cube* or *hexahedron:* It has six faces, all equal squares (fig. 12-2).

(iii) *Octahedron:* It has eight equal equilateral triangles as faces (fig. 12-3).

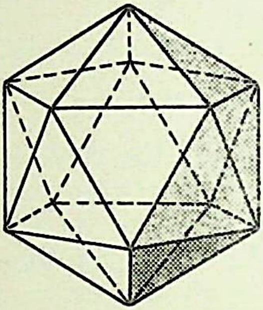
(iv) *Dodecahedron:* It has twelve equal and regular pentagons as faces (fig. 12-4).

(v) *Icosahedron:* It has twenty equal faces, all equilateral triangles (fig. 12-5).



Dodecahedron

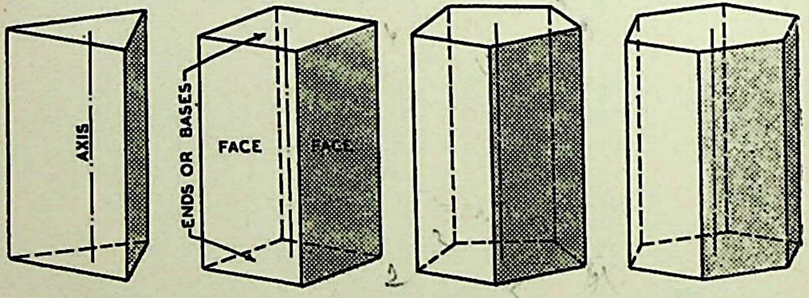
FIG. 12-4



Icosahedron

FIG. 12-5

Prism is a polyhedron having two equal and similar faces, called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.



Triangular Square Pentagonal Hexagonal

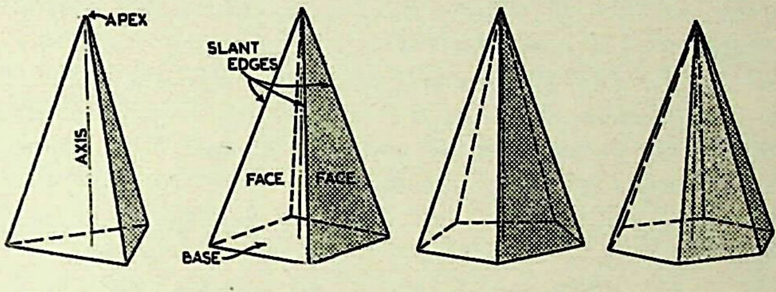
Prisms

FIG. 12-6

A right regular prism (fig. 12-6) has its axis perpendicular to the bases. All its faces are equal rectangles.

A *pyramid* is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

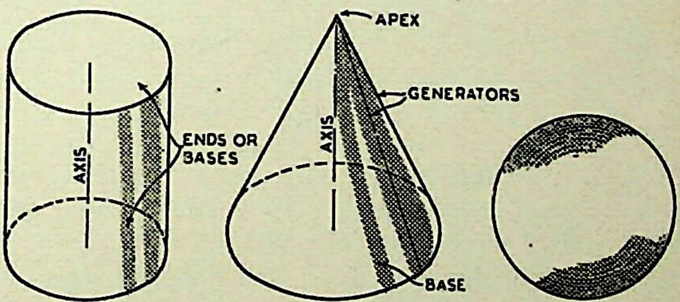
A right regular pyramid (fig. 12-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.



Pyramids
FIG. 12-7

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



Cylinder
FIG. 12-8

Cone
FIG. 12-9

Sphere
FIG. 12-10

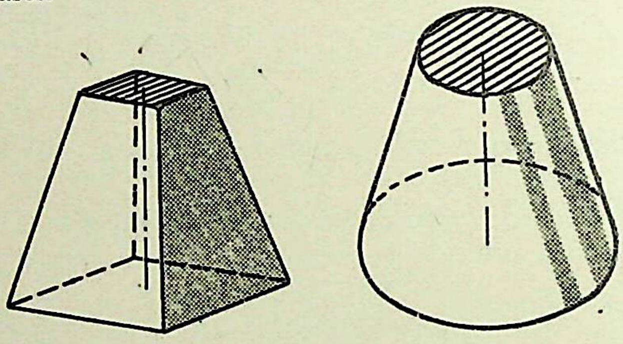
Solids of revolution: A *right circular cylinder* (fig. 12-8) is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.

A *right circular cone* (fig. 12-9) is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed. It has one circular base.

Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.

A *sphere* (fig. 12-10) is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.



Frustums
FIG. 12-11

When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 12-11).

When a solid is cut by a plane inclined to the base it is said to be *truncated*.

In this book, mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.

PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS:

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen

that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size. Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the plan should be drawn first and the elevation projected from it.

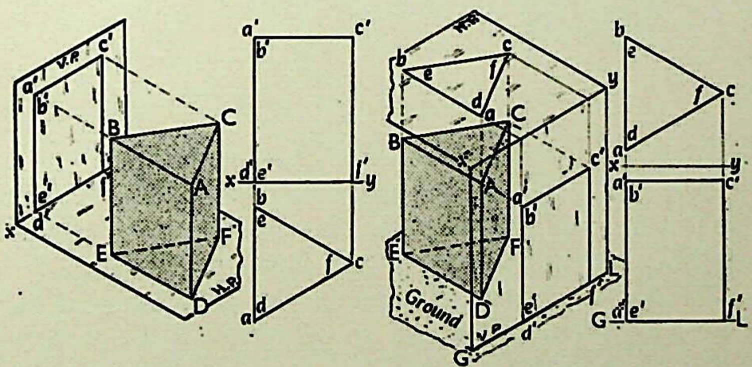
When the axis is perpendicular to the V.P., beginning should be made with the elevation. The plan should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the plan nor the elevation will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the end elevation must be drawn first. The elevation and the plan are then projected from the end elevation. The projections in such cases may also be drawn in two stages.

AXIS PERPENDICULAR TO THE H.P.:

Problem 1 (fig. 12-12):

Draw the plan and the elevation of a triangular prism, base 4 cm ($1\frac{1}{2}$") side, axis 5 cm (2") long, resting on one of its bases on the ground with a vertical face perpendicular to the V.P.



(i) First-angle projection (ii) Third-angle projection

FIG. 12-12

As the axis is perpendicular to the ground, begin with the plan (fig. 12-12). It will be an equilateral triangle of

sides 4 cm ($1\frac{1}{2}''$) long, with one of its sides perpendicular to *xy*. Name the corners as shown, thus completing the plan. The corners *d*, *e* and *f* are hidden and coincide with the top corners *a*, *b* and *c* respectively.

Project the elevation, which will be a rectangle. Name the corners. The line *b'e'* coincides with *a'd'*.

Problem 2:

Draw the projections of a pentagonal pyramid, base 3 cm ($1\frac{1}{4}''$) edge, axis 5 cm (2") long, having its base on the ground and an edge of the base parallel to V.P. Also draw its end elevation.

(a) First-angle projection [fig. 12-13(*i*)]:

Assume the side *DE* which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.

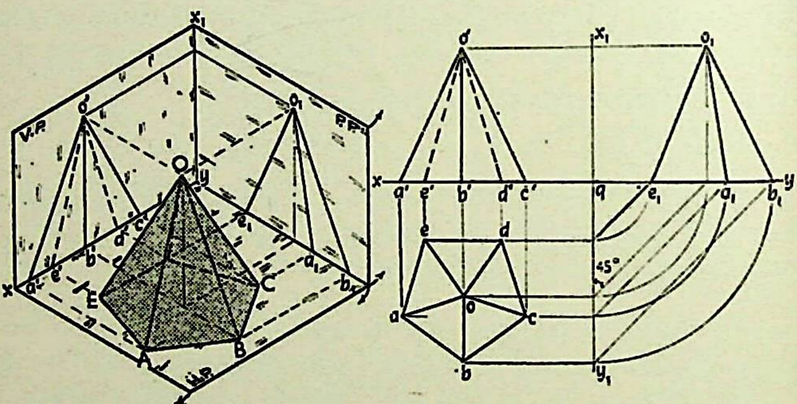


FIG. 12-13(*i*)

In the plan, draw a regular pentagon *abcde* with *de* parallel to and nearer *xy* [fig. 12-13(*i*)]. Locate its centre *o* and join it with the corners to indicate the slant edges.

Through *o*, project the axis in the elevation and mark the apex *o'*, 5 cm (2") above *xy*. Project all the corners of the base on *xy*. Draw lines *o'a'*, *o'b'* and *o'c'* to show the visible edges. Show the hidden edges *o'd'* and *o'e'* by dashed lines.

For the end elevation looking from the left, draw a new ground line x_1y_1 perpendicular to *xy* and to the right of the elevation. Project the end elevation on it, horizontally from

the elevation as shown. The respective distances of all the points in the end elevation from x_1y_1, should be equal to their distances in the plan from xy, e.g. $qa_1 = aa'$, $qb_1 = bb'$ etc. This is done systematically as explained below:

From each point in the plan draw horizontal lines upto x_1y_1. With q, the point of intersection between xy and x_1y_1 as centre, draw quarter circles as shown. Project up all the points to intersect the corresponding horizonal lines from the elevation. Instead of quarter circles, lines drawn at 45° to xy and x_1y_1 will also give the same result.

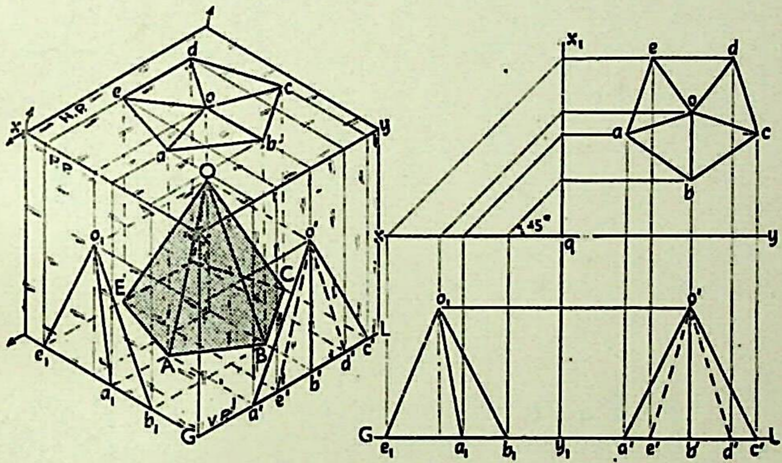


Fig. 12-13(*ii*)

(b) Third-angle projection [fig. 12-13(*ii*)]:

The pyramid is kept in the same position and hence, *DE* is away from the V.P.

Draw the plan, with *de* away from *xy*.

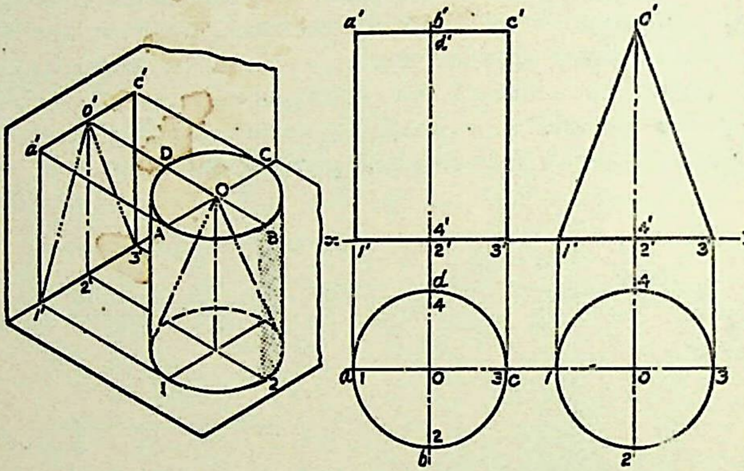
Project all the corners of the base on *GL* (the line for the ground) and complete the elevation.

End elevation from the left will come to the left of the elevation. Hence, draw x_1y_1 to the left of the elevation and project the end elevation as shown.

Note that the views obtained by the two methods are identical except for their positions.

Problem 3 (fig. 12-14):

Draw the projections of (i) a cylinder, base 4 cm (1½") diameter, axis 5 cm (2") long, and (ii) a cone, base 4 cm (1½") diameter, axis 5 cm (2") long, resting on the ground on their respective bases.



(*i*) (*ii*) (*iii*)

FIG. 12-14

(i) Draw a circle of 4 cm (1½") dia. in the plan and project the elevation, which will be a rectangle [fig. 12-14(*ii*)].

(ii) Draw the plan [fig. 12-14(*iii*)]. Through the centre *o*, project the apex *o'*, 5 cm (2") above *xy*. Complete the triangle in the elevation, as shown.

In the pictorial view [fig. 12-14(*i*)], the cone is shown as contained by the cylinder.

Problem 4 (fig. 12-15):

A cube of 5 cm (2") side is resting on the ground, with its vertical faces equally inclined to the V.P. Draw its projections.

Begin with the plan.

Draw a square *abcd* with a side making 45° angle with *xy*. Project up the elevation. The edge *d'h'* will coincide with *b'f'*.

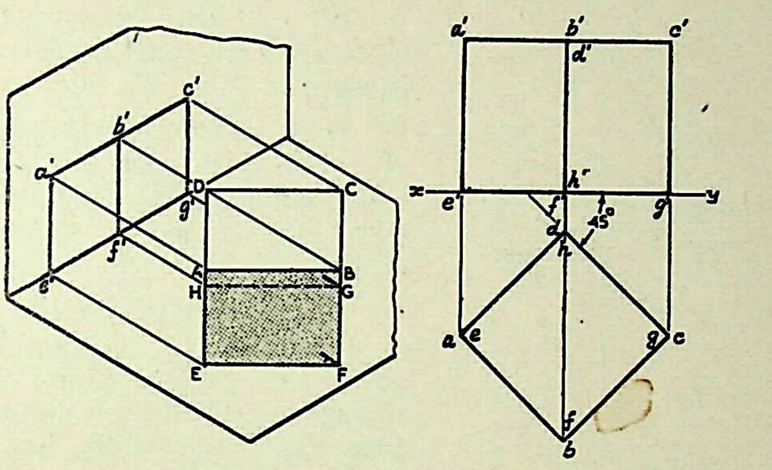


FIG. 12-15

Problem 5 (fig. 12-16):

Draw the projections of a hexagonal pyramid, base 4 cm ($1\frac{1}{2}''$) side, axis 6·5 cm ($2\frac{1}{2}''$) long, having its base on the ground and one of the edges of the base inclined at 45° to the V.P.

In the plan, draw a line *af* equal to 4 cm ($1\frac{1}{2}''$) and inclined at 45° to *xy*. Construct a regular hexagon on *af*. Mark its centre *o* and complete the plan by drawing lines joining it with the corners.

Project up the elevation as described in problem 2, showing the hidden edges *o'e'* and *o'f'* as dashed lines.

Problem 6 (fig. 12-17):

A tetrahedron of 5 cm (2") edge is resting on the ground on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.

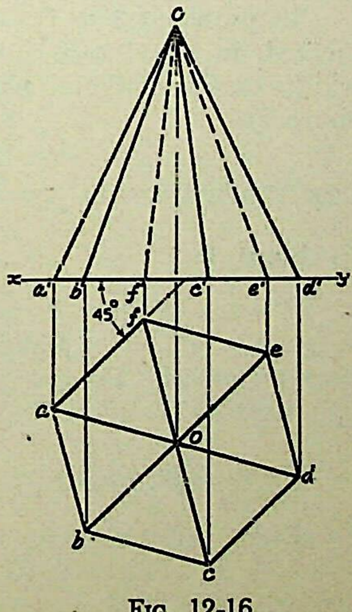


FIG. 12-16

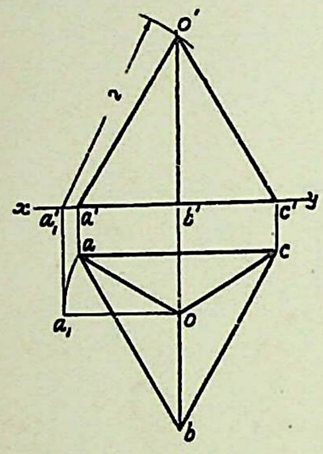


FIG. 12-17

All the four faces of the tetrahedron are equal equilateral triangles of 5 cm (2") side.

Draw an equilateral triangle *abc* in the plan with one side, say *ac*, parallel to *xy*. Locate its centre *o* and join it with the corners.

In the elevation, the corners *a'*, *b'* and *c'* will be in *xy*. The apex *o'* will lie on the projector through *o*, so that its true distance from the corners of the base is equal to 5 cm (2").

To locate *o'*, make *oa* (or *ob* or *oc*) parallel to *xy*. Project a_1 to a_1' on *xy*. With a_1' as centre and radius equal to 5 cm (2"), cut the projector through *o* in *o'*. Join *o'a'*, *o'b'* and *o'c'* to complete the elevation. *o'b'* will be the distance of the apex from the ground.

In problems 3 to 6, according to third-angle projection method, in the elevation, the base of the solid in each case will be on *GL*, while the plan will be above the elevation and above *xy*.

AXIS PERPENDICULAR TO THE V.P.:

Problem 7:

A hexagonal prism has one of its rectangular faces parallel to the ground. Its axis is perpendicular to the V.P. and 3·5 cm ($1\frac{3}{8}$") above the ground. Draw its projections when the nearer end is 2 cm ($\frac{3}{4}$") from the V.P. Side of base 2·5 cm (1") long; axis 5 cm (2") long.

(a) First-angle projection [fig. 12-18(*i*)]:

Begin with the elevation. Construct a regular hexagon of 2·5 cm (1") side with its centre 3·5 cm ($1\frac{3}{8}$") above *xy* and one side parallel to it.

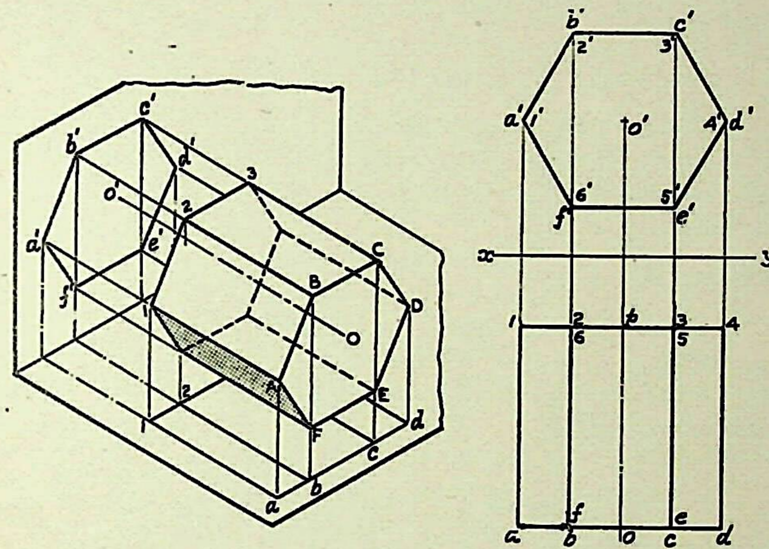


FIG. 12-18(*i*)

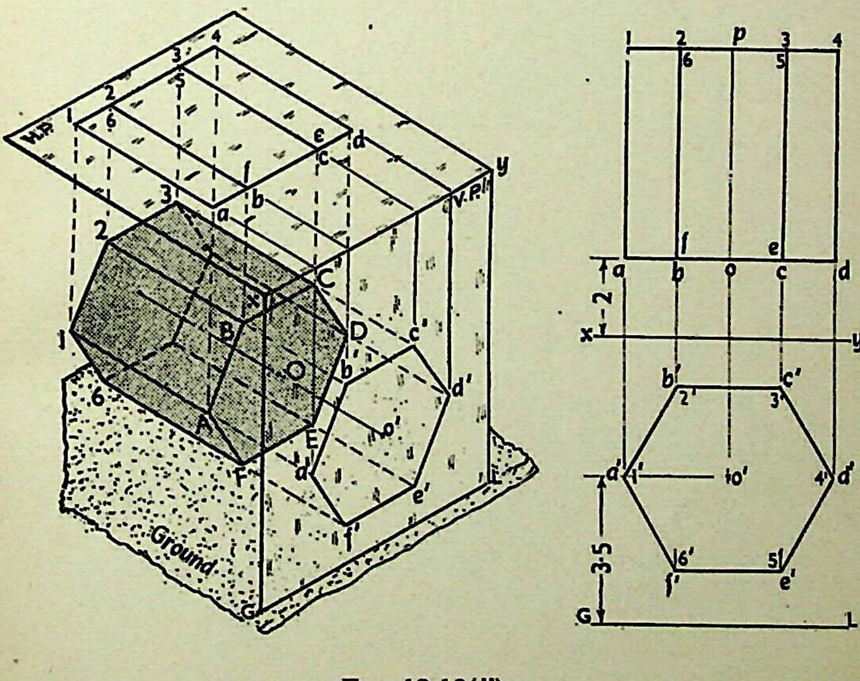


FIG. 12-18(*ii*)

Project the plan as shown, the line 1-4 being 2 cm ($\frac{3}{4}''$) below *xy*.

(b) Third-angle projection [fig. 12-18(*ii*)]:

Construct the regular hexagon with its centre 3·5 cm ($1\frac{3}{8}''$) above *GL* and one side parallel to it. Project up the plan, keeping the nearer end, viz. the line *ad*, 2 cm ($\frac{3}{4}''$) above *xy*.

Problem 8:

A square pyramid, base 4 cm ($1\frac{1}{2}''$) side, axis 6·5 cm ($2\frac{1}{2}''$) long, has its base in the V.P., an edge of the base inclined at 30° to the ground and a corner contained by that edge on the ground. Draw its projections.

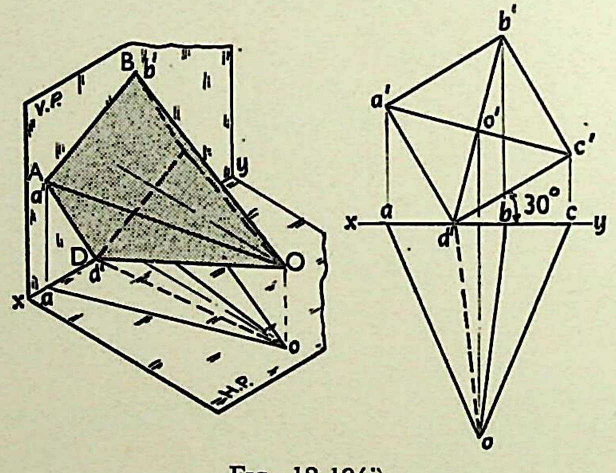


FIG. 12-19(*i*)

(a) First-angle projection [fig. 12-19(*i*)]:

Draw a square in the elevation, with corner *d'* in *xy* and side *d'c'* inclined at 30° to it. Locate the centre *o'* and join it with the corners of the square.

Project down all the corners in *xy* (because the base is in the V.P.). Mark the apex *o* on a projector through *o'*. Draw the slant edges and complete the plan.

(b) Third-angle projection [fig. 12-19(*ii*)]:

The corner d' of the square in elevation will be in *GL*. The slant edges will not be visible; hence, they will be dotted. Project up the plan with all the corners of the base in *xy* and the apex *o* above *xy*.

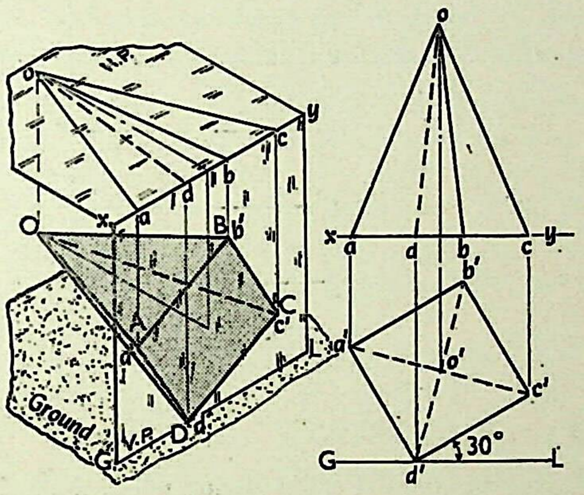


FIG. 12-19(*ii*)

AXIS PARALLEL TO BOTH THE H.P. AND THE V.P.:

Problem 9 (fig. 12-20):

A triangular prism, base 4 cm (1½") side, height 6·5 cm (2½"), is resting on the ground on one of its rectangular faces with the axis parallel to the V.P. Draw its plan and elevation.

As the axis is parallel to both the planes, begin with the end elevation.

Draw an equilateral triangle representing the end elevation, with one side in *xy*. Project the elevation horizontally from this triangle. Project down the plan from the elevation and the end elevation, as shown.

In third-angle projection, the base of the triangle in the end view will be in *GL*. The elevation will be projected to the left of the end view (if the naming of the corner is kept the same). The plan will come above the elevation.

This problem can also be solved in two stages as explained in the next article.

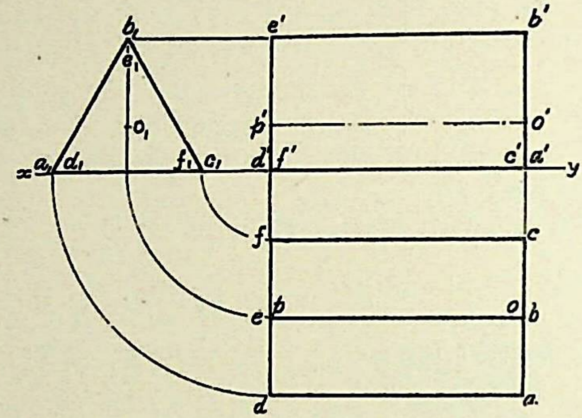


FIG. 12-20

EXERCISES XII(i)

Draw the projections of the following solids, situated in their respective positions, taking a side of base equal to 4 cm ($1\frac{1}{2}''$) or diameter of base equal to 5 cm ($2''$) and axis 6·5 cm ($2\frac{1}{2}''$) long. Use any one method of projection.

(1) A hexagonal pyramid, base on the ground and a side of the base parallel to and 2·5 cm ($1''$) away from the V.P.

(2) A square prism, base on the ground, a side of the base inclined at 30° to the V.P. and the axis 5 cm ($2''$) away from the V.P.

(3) A triangular pyramid, base on the ground and an edge of the base inclined at 45° to the V.P.; the apex 4 cm ($1\frac{1}{2}''$) from the V.P.

(4) A cylinder, axis perpendicular to the V.P. and 4 cm ($1\frac{1}{2}''$) above the ground; one end 2 cm ($\frac{3}{4}''$) away from the V.P.

(5) A pentagonal prism, a rectangular face on the ground, axis perpendicular to the V.P. and one base in the V.P.

(6) A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 5 cm ($2''$) from both the H.P. and the V.P.

(7) A cone, apex on the ground and 4 cm ($1\frac{1}{2}''$) from the V.P.; axis perpendicular to the ground.

(8) A pentagonal pyramid, base in the V.P. and an edge of the base on the ground.

Projections of solids with axes inclined to one of the reference planes and parallel to the other:

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages. In the initial stage, the solid is assumed to be in simple position, i.e. its axis perpendicular to one of the planes.

If the axis is to be inclined to the ground, i e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage.

Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Also, when the solid is to rest on an edge of the base, that edge should be kept perpendicular to the V.P.

Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following methods:

I. *Alteration of position:* The position of one of the views is altered as required and the other view projected from it.

II. *Alteration of ground line:* A new ground line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective ground lines.

After determining the positions of all the corners or points in the final view, difficulty is often felt in comple-

ting the same correctly. The following sequence for joining the corners may be adopted:

(i) Draw the lines for the edges of the visible base. In first-angle projection, the base, which (compared to the other base) is further away from *xy* in one view, will be fully visible in the other view.

In third-angle projection, it is just the reverse. The base or end, which (compared to the other base) is nearer to *xy* in one view will be fully visible in the other view.

(ii) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.

(iii) Draw the lines for the edges of the other base. It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.:

Problem 10 (fig. 12-21):

Draw the projections of a pentagonal prism, base 2·5 cm (1″) side and axis 5 cm (2″) long, resting on one of its rectangular faces on the ground, with the axis inclined at 45° to the V.P.

In the simple position assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

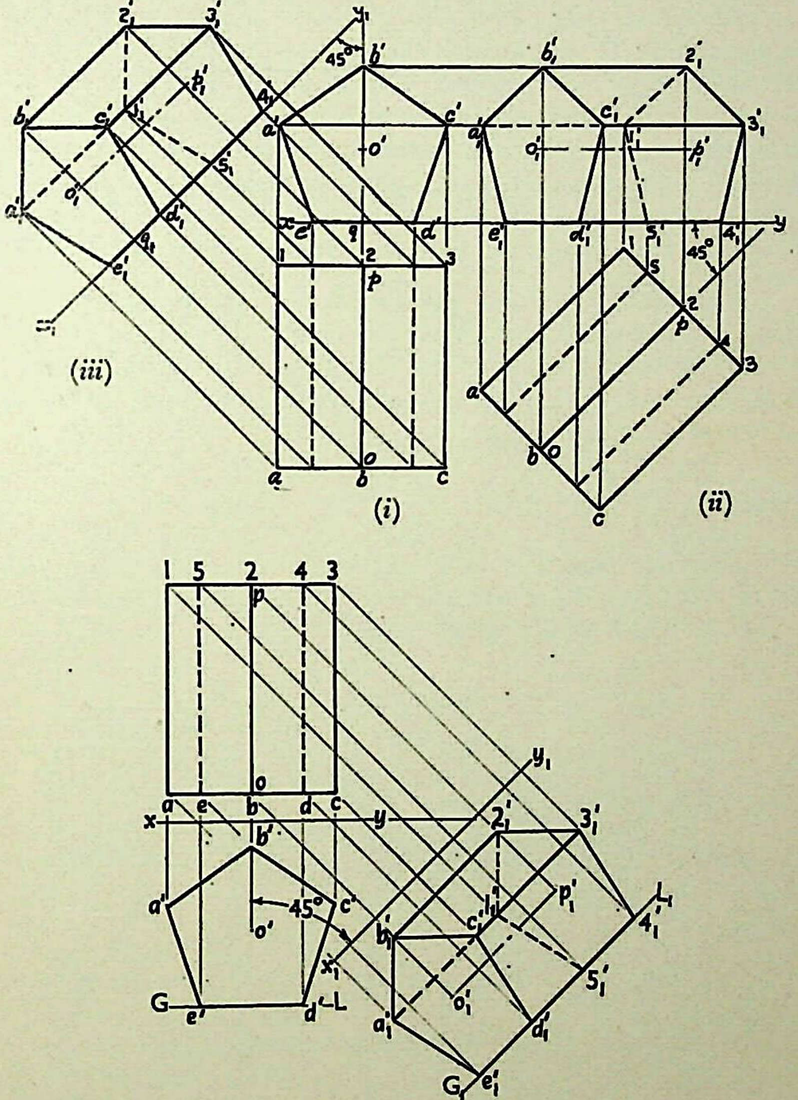
Draw the pentagon in the elevation with one side in *xy* and project the plan [fig. 12-21(*i*)].

The shape and size of the figure in the plan will not change, so long as the prism has its face on the ground. The respective distances of all the corners in the elevation from *xy* will also remain constant.

Final projections:

Method I [fig. 12-21(*ii*)]: Alter the position of the plan, i.e. reproduce it with the axis inclined at 45° to *xy*. Project all the corners upwards from this plan and horizontally from the first elevation, e.g. a vertical from *a* intersecting a horizontal from a' at a point a_1'.

Complete the pentagon $a_1'b_1' \ldots . e_1'$ for the fully visible end of the prism. Next, draw the lines for the longer edges



(iv)

FIG. 12-21

and finally, draw the lines for the edges of the other end. Note carefully that the lines $a_1'1_1'$, $1_1'2_1'$ and $1_1'5_1'$ are

dashed lines. $e_1'5_1'$ is hidden but it coincides with other visible lines.

Method II [fig. 12-21(*iii*)]: Draw a new ground line x_1y_1, making 45° angle with the plan of the axis, to represent an auxiliary vertical plane. Draw projectors from all the points in the plan, perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from xy in the elevation, e.g. $b_1'q_1$ is equal to $b'q$, etc. Join the points as already explained. The auxiliary elevation and the plan are the required projections.

Fig. 12-21(*iv*) shows the solution in third-angle projection, by auxiliary plane method. Note that $e'd'$ is in *GL*. xy and the plan are above the elevation. In auxiliary elevation, the end $a_1'b_1'\ldots e_1'$ is fully visible, because that end is nearer xy in the plan.

Problem 11 (fig. 12-22):

Draw the projections of a cylinder 7·5 cm (3") diameter and 10 cm (4") long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.

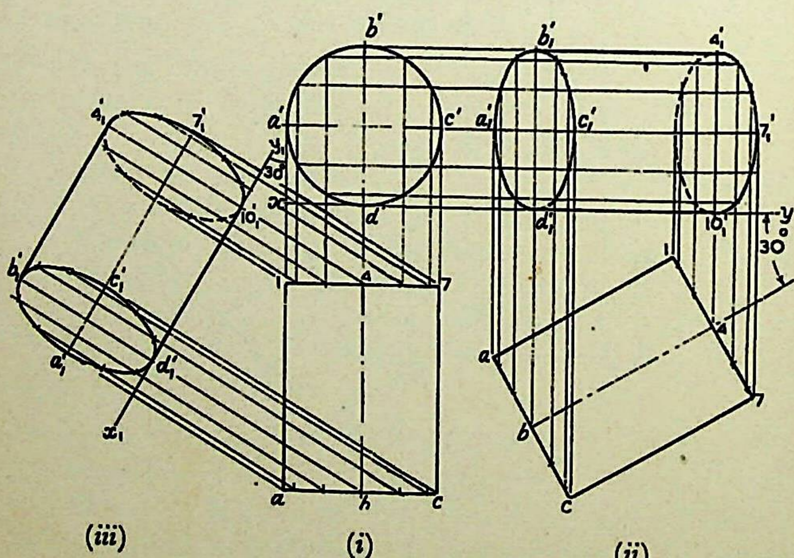


FIG. 12-22

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be dotted.

Fig. 12-22(*iii*) shows the elevation obtained by method II.

In third-angle projection, the method will be the same as in the previous problem.

AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.:

Problem 12 (fig. 12-23):

A hexagonal pyramid, base 2·5 cm (1″) side, axis 5 cm (2″) long, has an edge of its base on the ground. Its axis is inclined a 30° to the ground and parallel to the V.P. Draw its projections.

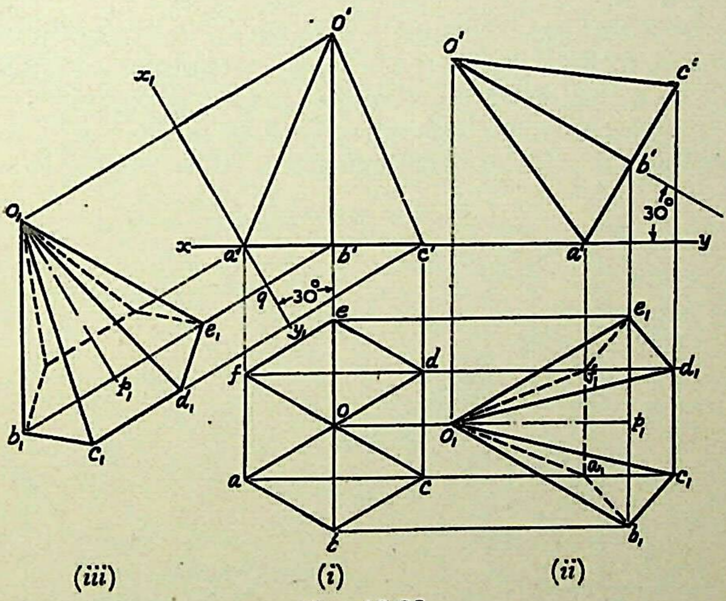


FIG. 12-23

In the initial position assume the axis to be perpendicular to the ground.

(i) Draw the projections with one edge of the base perpendicular to *xy*.

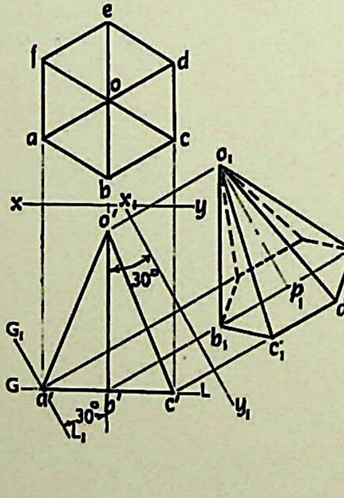
If the pyramid is now tilted about the edge *AF*, the axis will become inclined to the ground but will remain parallel

to the V.P. The distances of all the corners from the V.P. will remain constant.

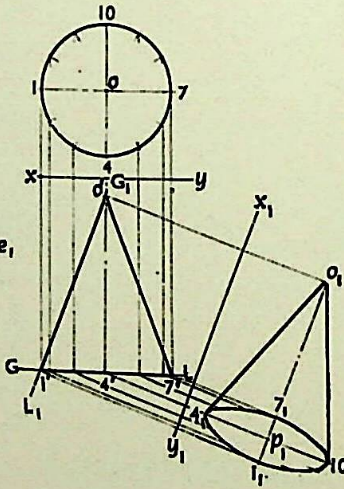
The elevation will not be affected except in its position in relation to xy. The new plan will have its corners at same distances from xy, as before.

Method I [fig. 12-23(*ii*)]: Reproduce the elevation so that the axis makes 30° angle with xy and the edge $a'f'$ remains in xy. Project all the points downwards from this elevation and horizontally from the first plan. Complete the new plan by drawing (i) lines joining the apex o_1 with the corners of the base and (ii) the lines for the edges of the base. The base will be partly hidden as shown by dashed lines a_1b_1, e_1f_1 and f_1a_1. Lines o_1f_1 and o_1a_1 are also not visible.

Method II [fig. 12-23(*iii*)]: Through a', draw a new ground line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane. From the elevation project the required plan on x_1y_1, keeping the distance of each point from x_1y_1 equal to the distance of its first plan from xy, viz. $b_1q = bb'$ etc.



(*iv*)



(*v*)

FIG. 12-23

Fig. 12-23 (*iv*) shows the solution in third-angle projection by method II. G_1L_1 (passing through a'), and x_1y_1

make 30° angle with the axis. The distances of all the corners in auxiliary plan from x_1y_1 are equal to their respective distances from xy in plan. The base is not fully visible.

Problem 13 (fig. 12-24):

Draw the plan and the elevation of a cone, base 7·5 cm (3") diameter, axis 10 cm (4") long, lying on the ground on one of its generators with the axis parallel to the V.P.

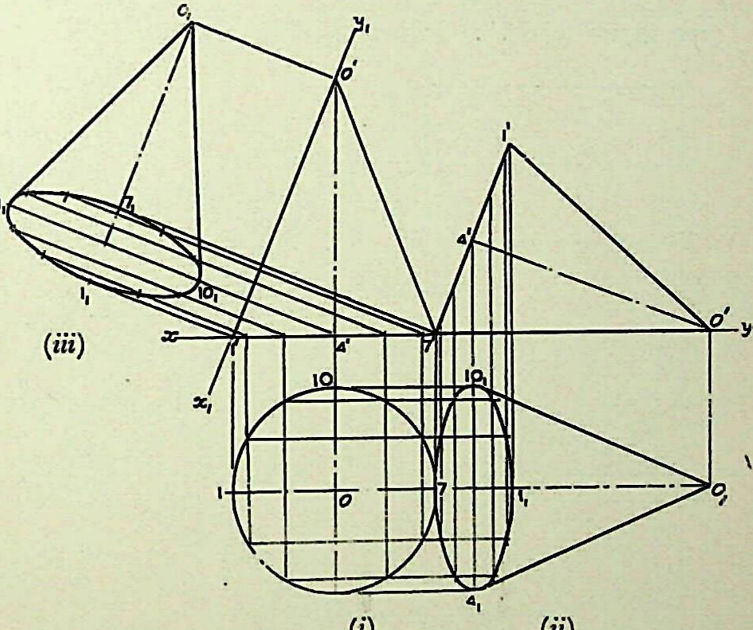


FIG. 12-24

(i) Assuming the cone to be resting on its base on the ground, draw its projections.

(ii) Redraw the elevation so that the line $o'7'$ is in xy. Project the required plan as shown. The lines from o_1 should be tangents to the ellipse.

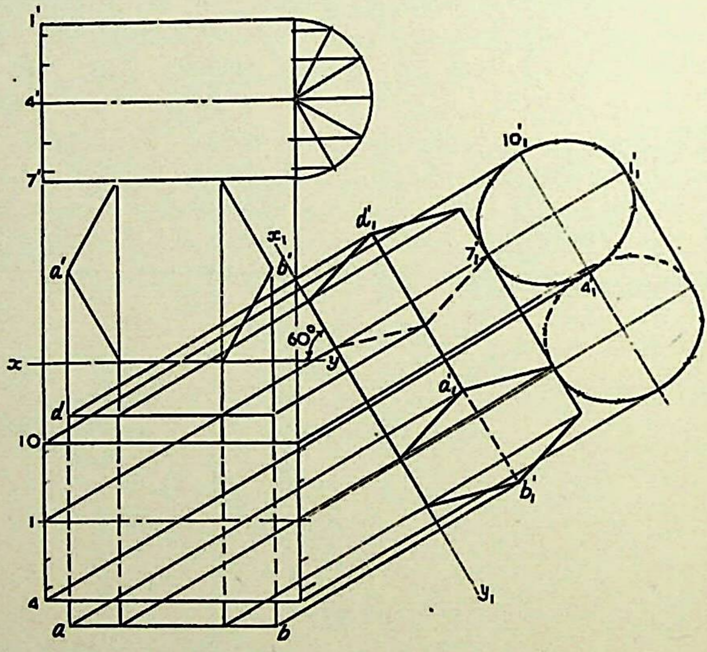
The plan obtained by auxiliary-plane method is shown in fig. 12-24(*iii*). The new ground line is drawn so as to contain the generator $o'1'$ instead of $o'7'$ (for sake of convenience). The cone is thus lying on the generator $o'1'$. Note that $1'1_1 = 1'1$, $o'o_1 = 4'o$ etc.

Fig. 12-23(v) shows the solution in third-angle projection. G_1L_1 coincides with $0'1'$ and x_1y_1 is parallel to it.

Problem 14:

The projections of a cylinder resting centrally on a hexagonal prism are given in fig. 12-25(i). Draw its auxiliary elevation on a ground line inclined at 60° to xy.

See figure 12-25(ii) which is self-explanatory.



(i) (ii)

FIG. 12-25

Problem 15 (fig. 12-26):

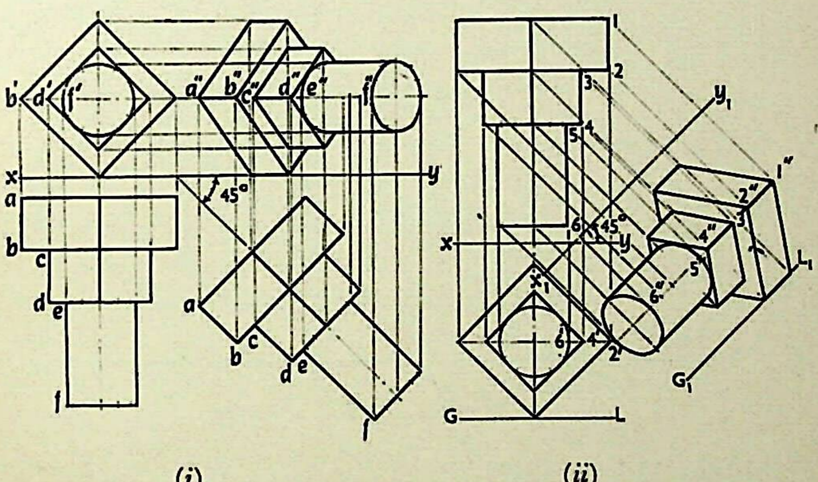
A square-headed bolt 2·5 cm (1") diameter, 12·5 cm (5") long and having a square neck, has its axis parallel to the ground and inclined at 45° to the V.P. All the faces of the square head are equally inclined to the ground. Draw its elevation neglecting the threads and chamfer.

(a) First-angle projection [fig. 12-26(*i*)]:

(i) Assuming the axis to be perpendicular to the V.P. draw the elevation. It will consist of two squares and a circle. The sides of the squares will make 45° angle with *xy*.

Project the plan. (In the figure, the length of the bolt is taken shorter.)

(ii) Reproduce the plan with the axis making 45° angle with *xy* and project the required elevation as shown.



(*i*) (*ii*)

FIG. 12-26

(b) Fig. 12-26(*ii*) shows the elevation in third-angle projection by auxiliary plane method.

Problem 16 (fig. 12-27):

A hexagonal prism, base 4 cm (1½") side, and height 4 cm (1½"), has a hole of 4 cm (1½") diameter drilled centrally through its ends. Draw its plan when it is resting on one of its corners in the ground with its axis inclined at 60° to the ground and two of its faces parallel to the V.P.

(i) Begin with the plan and project up the elevation, assuming the axis to be vertical. (ii) Tilt the elevation and

project the required plan. Note that a part of the ellipse for the lower end of the hole will be visible.

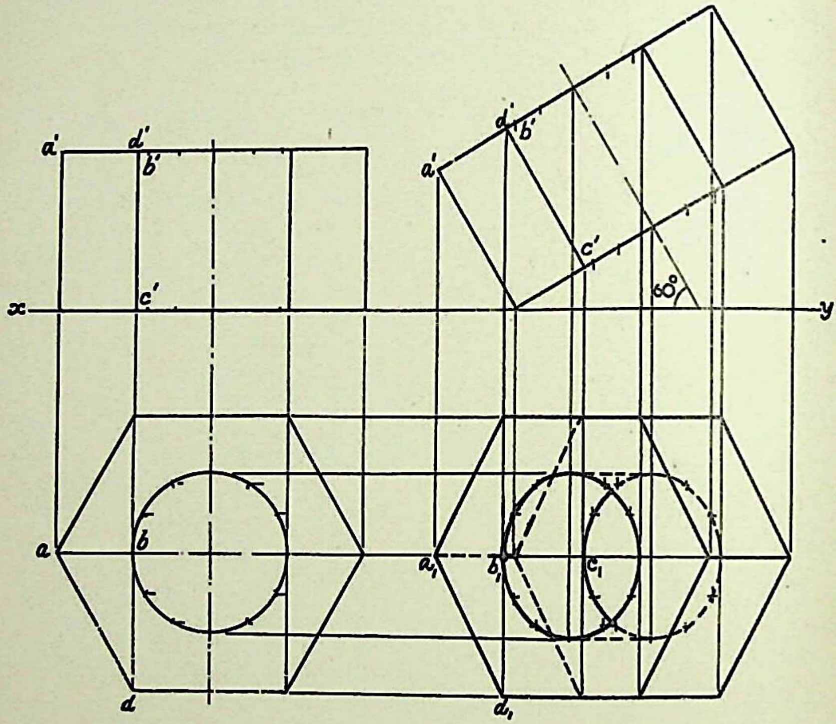


FIG. 12-27

Problem 17 (fig. 12-28):

The projections of a hopper made of tin sheet are given. Project another plan on an auxiliary inclined plane making 45° angle with the ground.

Draw a new ground line x_1y_1 inclined at 45° to xy and project the required plan on it, from the elevation. Show carefully, the visible ellipses for the outer as well as the inner parts of the hopper rings.

Projections of solids with axes inclined to both the H.P. and the V.P.:

The projections of a solid with its axis inclined to both the planes are drawn in three stages: (i) Simple position.

(ii) Axis inclined to one plane and parallel to the other. (iii) Final position. The second and final positions may be obtained either by the alteration of the positions of the solid, i.e. the views, or by the alteration of ground lines.

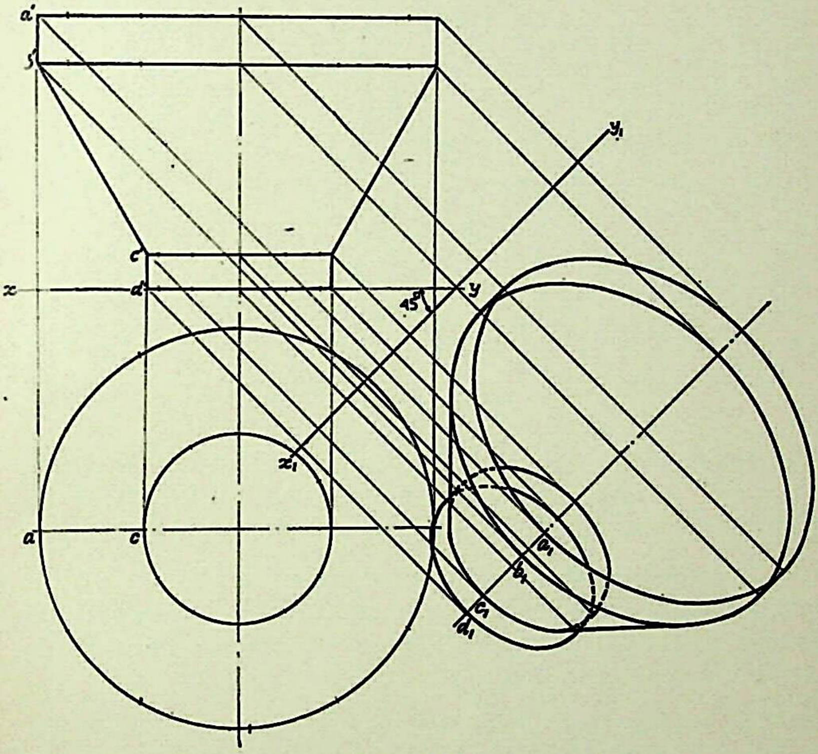


FIG. 12-28

Problem 18:

A square prism, base 2·5 cm (1″) side, height 5 cm (2″), has its axis inclined at 45° to the ground and has an edge of its base on the ground and inclined at 30° to the V.P. Draw its projections.

Method I (fig. 12-29): (i) Assuming the prism to be resting on its base on the ground with an edge of the base, say *fg*, perpendicular to the V.P., draw its projections.

Assume the prism to be tilted about the edge which is perpendicular to the V.P., so that the axis makes 45° angle with the ground.

(ii) Hence, change the position of the elevation, so that the axis is inclined at 45° to *xy*. Project the second plan.

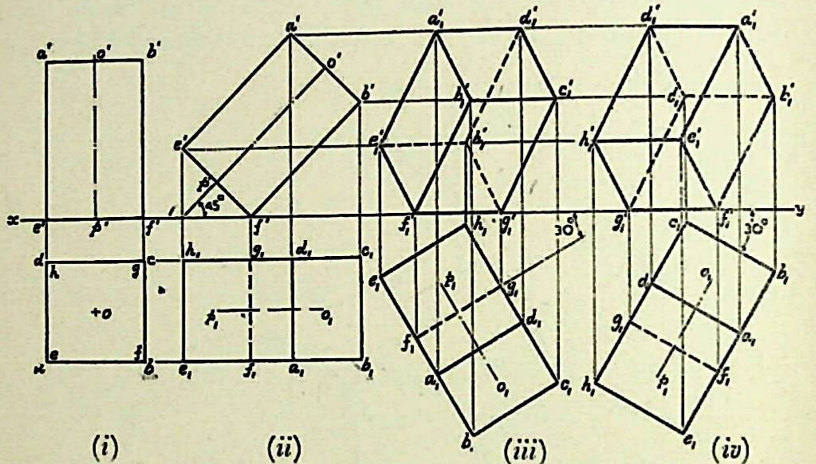


FIG. 12-29

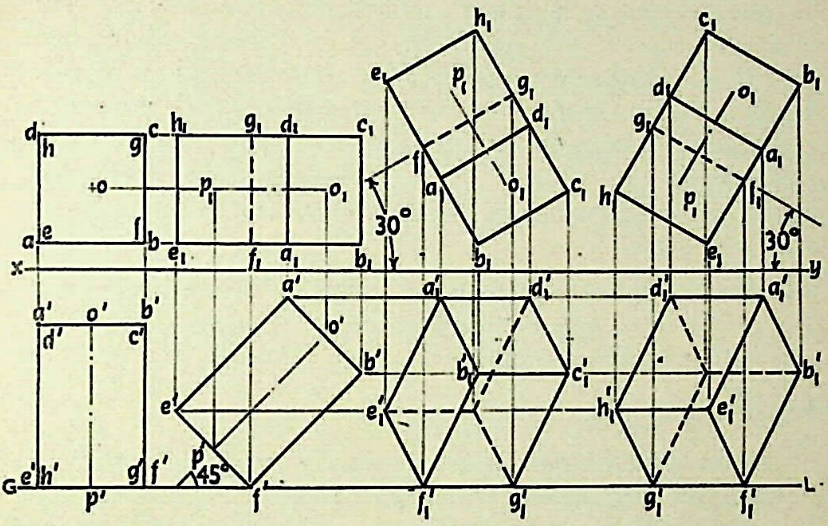
Again, assume the prism to be turned so that the edge on which it rests, makes an angle of 30° with the V.P., keeping the inclination of the axis with the ground constant.

The shape and size of the second plan will remain the same; only its position will change. In the elevation, the distances of all the corners from *xy* will remain the same as in the second elevation.

(iii) Therefore, reproduce the second plan with f_1g_1 inclined at 30° to *xy*. Project the final elevation upwards from this plan and horizontally from the second elevation, e.g. a vertical from a_1 and a horizontal from a', intersecting at a_1'. As the top end is away from *xy* in the plan, it will be fully visible in the elevation. Complete the elevation showing the hidden edges by dashed lines.

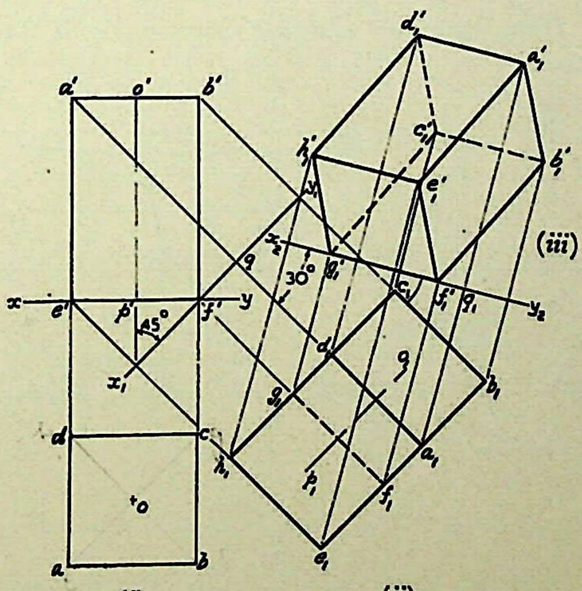
(iv) The second plan may be turned in the opposite direction as shown. In this position, the lower end of the prism, *viz.* $e_1'f_1'g_1'h_1'$ will be fully visible in the elevation.

Fig. 12-29(*a*) shows the solution in third-angle projection. Note that the end which is nearer *xy* in one view is fully visible in the other view.



(*i*) (*ii*) (*iii*) (*iv*)

FIG. 12-29(*a*)



(*i*) (*ii*)

FIG. 12-30

Method II (fig. 12-30): (i) Draw the plan and elevation inthe simple position. (ii) Draw a ground line x_1y_1 through

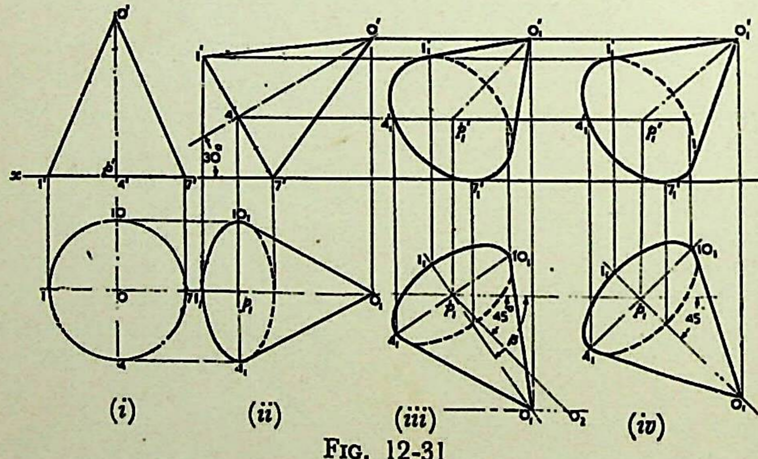
f' (to represent an auxiliary inclined plane) making 45° angle with the axis in the elevation. Project the auxiliary plan. (iii) Draw another ground line x_2y_2 (representing an auxiliary vertical plane), inclined at 30° to the line f_1g_1. Project the required elevation from the auxiliary plan, keeping the distance of each point (in the new elevation) from x_2y_2, equal to its distance (in the first elevation) from x_1y_1, e.g. $a_1'q_1 = a'q$ etc.

Note: The new ground line satisfying the required conditions may be drawn in various positions, as explained in Chapter XI.

Problem 19 (fig. 12-31):

Draw the projections of a cone, base 5 cm (2") diameter, axis 7·5 cm (3") long, when it is resting on the ground on a point on its base-circle with (a) the axis making an angle of 30° with the ground and 45° with the V.P.; (b) the axis making an angle of 30° with the ground and its plan *making 45° with the V.P.*

(i) Draw the plan and the elevation with the base on the ground.



(i) (ii) (iii) (iv)

FIG. 12-31

(ii) Tilt the elevation so that the axis makes 30° angle with xy. Project the second plan.

(a) In order that the axis may make an angle of 45° with the V.P., let us determine the apparent angle of incli-

nation which the plan of the axis, viz. o_1p_1 should make with xy and which will be greater than 45°.

(iii) Mark any point p_1 below xy. Draw a line p_1o_2 equal to the true length of the axis, viz. $o'p'$, and inclined at 45° to xy. With p_1 as centre and radius equal to p_1o_1 (the length of the plan of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the apparent angle of inclination and is greater than 45°.

Around p_1o_1 as axis, reproduce the second plan and project the final elevation as shown.

Note that the base of the cone is not visible in the elevation.

(b) When the *plan* of the axis is to make 45° angle with the V.P., it is evident that p_1o_1 should be inclined at 45° to xy. Hence, reproduce the plan accordingly and project the required elevation [fig. 12-31(*iv*)].

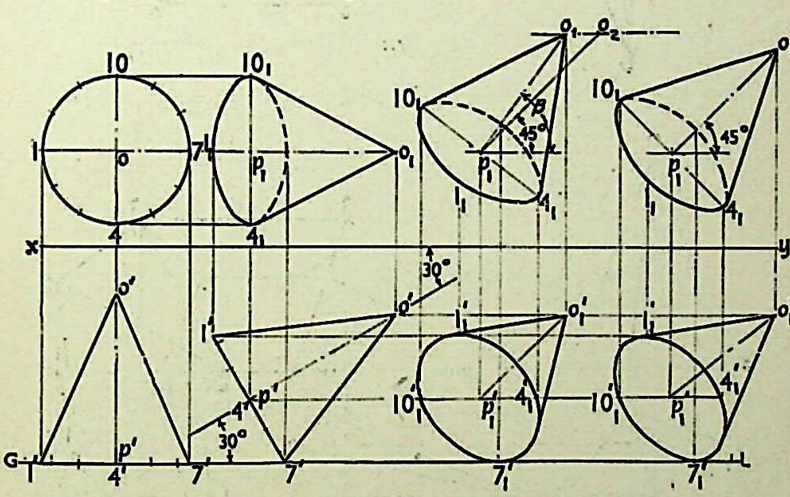


FIG. 12-31(*a*)

Fig. 12-31(*a*) shows the projections in third-angle projection.

Problem 20 (fig. 12-32):

A pentagonal pyramid, base 2·5 cm (1") side, axis 5 cm (2") long has one of its triangular faces in the V.P. and the edge of the base contained by that face makes an angle of 30° with the ground. Draw its projections.

(i) In the initial position, assume the pyramid as having its base in the V.P. and an edge of the base perpendicular to the H.P. The elevation will have to be drawn first and the plan projected from it.

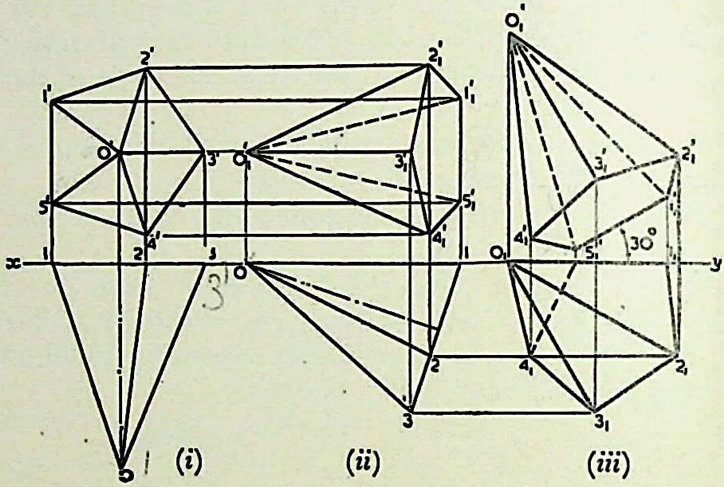


FIG. 12-32

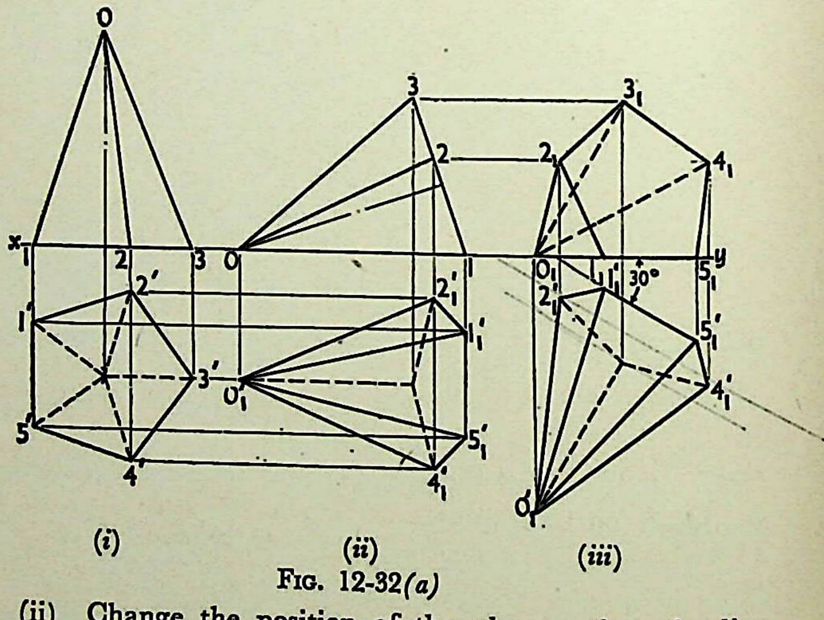


FIG. 12-32*(a)*

(ii) Change the position of the plan so that the line *o*1 (for the face *o* 1 5) is in *xy*. Project the second elevation.

(iii) Tilt this elevation so that the line $1_1'5_1'$ makes 30° angle with *xy*. Project the final plan.

In third-angle projection [fig. 12-32(*a*)] in the first position, the slant edges in the elevation are hidden and hence, shown dotted. In position (ii), the face $0_1'1_1'5_1'$ is visible while the base is hidden. In position (iii), the base in the plan is fully visible as it is nearer *xy* in the elevation.

Problem 21 (fig. 12-33):

A square pyramid, base 4 cm (1½") side, axis 5 cm (2") long, is freely suspended from one of the corners of its base. Draw its projections, when the axis as a vertical plane makes an angle of 45° with the V.P.

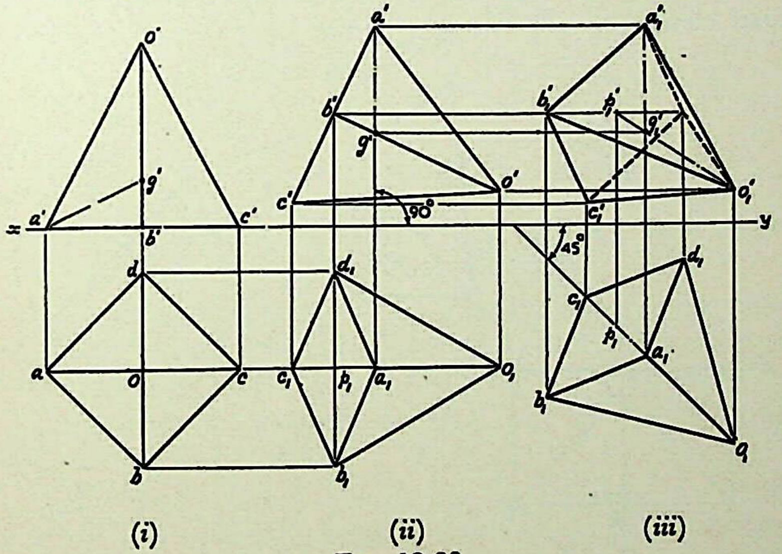


FIG. 12-33

When a pyramid is suspended freely from a corner of its base, the imaginary line joining that corner with the centre of gravity of the pyramid will be vertical.

The centre of gravity of a pyramid lies on its axis and at a distance equal to ¼ of the length of the axis, from the base.

Assume the pyramid to be suspended from a corner *A* of the base.

In the initial position, the pyramid should be kept with its base on the ground and the line joining *A* with the centre of gravity *G*, parallel to the V.P. In the plan, *G* will coincide with the plan of the axis.

(i) Therefore, draw a square *abcd* (in the plan) with *ag*, i.e. *ao* parallel to *xy*. Project the elevation. Mark g' at a distance equal to $\frac{1}{4}$ of the axis from *xy*. Join a' with g'.

(ii) Tilt the elevation so that $a'g'$ is perpendicular to *xy* and project the plan. The axis will still remain parallel to the V.P.

(iii) Reproduce this plan, so that o_1p_1 (the plan of the axis) is inclined at 45° to *xy*. The axis as a vertical plane will thus be making 45° angle with the V.P. Project the final elevation.

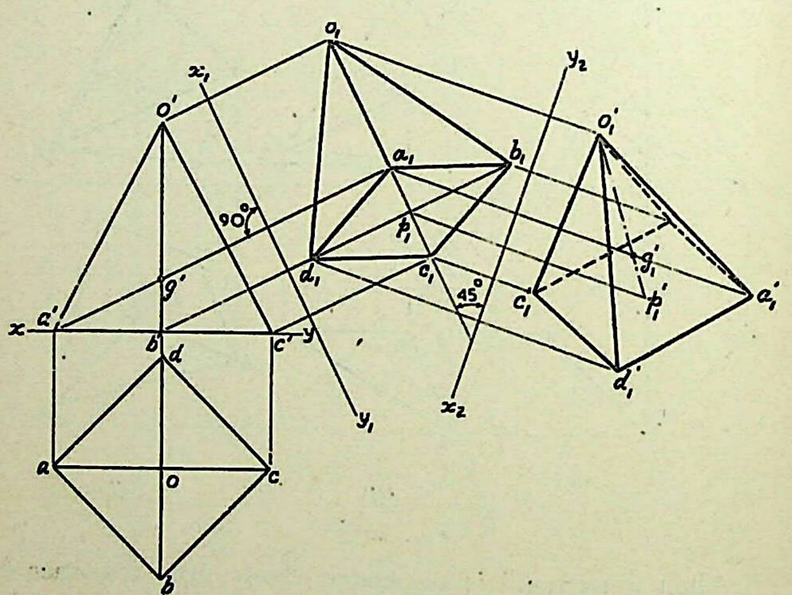


FIG. 12-34

Fig. 12-34 shows the projections obtained by the change of ground line method.

Problem 22 (fig. 12-35):

A hexagonal pyramid, base 2·5 cm (1") side, axis 5 cm (2)" long, has one of its slant edges on the ground. A plane containing

that edge and the axis is perpendicular to the ground and inclined at 45° to the V.P. Draw its projections when the apex is nearer the V.P. than the base.

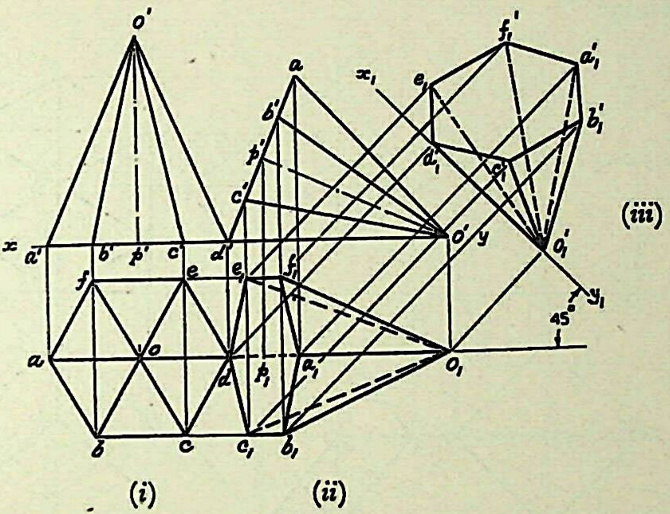


FIG. 12-35

Assume the pyramid to be resting on the ground on its base with a slant edge parallel to the V.P.

(i) Draw a hexagon of 2·5 cm (1″) side (in the plan) with a side parallel to *xy*. The lines *ao* and *do* for the slant edges will also be parallel to *xy*. Project the elevation.

(ii) Tilt this elevation so that *a'o'* or *d'o'* is in *xy*. Project the second plan.

(iii) Draw a new ground line x_1y_1 making 45° angle with o_1p_1 (the plan of the axis) and project the final elevation.

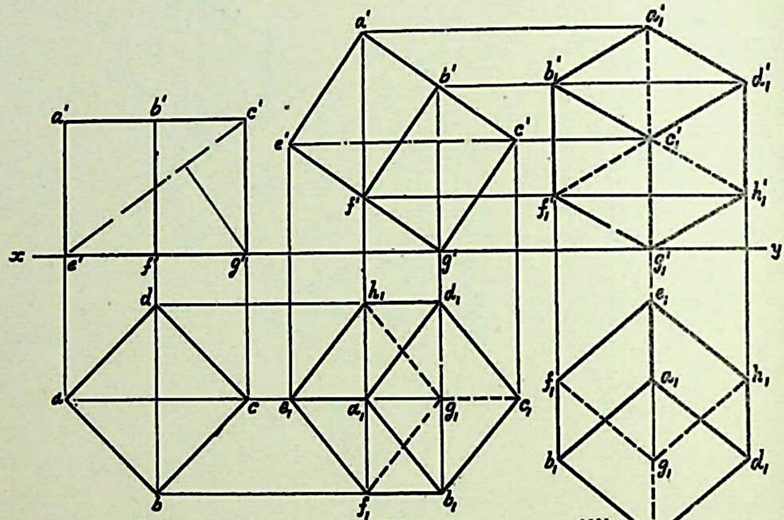
The problem can thus be solved by combination of the change of position and change of ground line methods.

Problem 23 (fig. 12-36): *x*

Draw the projections of a cube of 2·5 cm (1″) edge, resting on the ground on one of its corners with a solid diagonal perpendicular to the V.P.

Assume the cube to be resting on one of its faces on the ground with a solid diagonal parallel to the V.P.

(i) Draw a square *abcd* in the plan, with its sides inclined at 45° to *xy*. The lines *ag* and *ce* representing the solid diagonals *AG* and *CE* are parallel to *xy*. Project the elevation.



(i) (ii) (iii)

FIG. 12-36

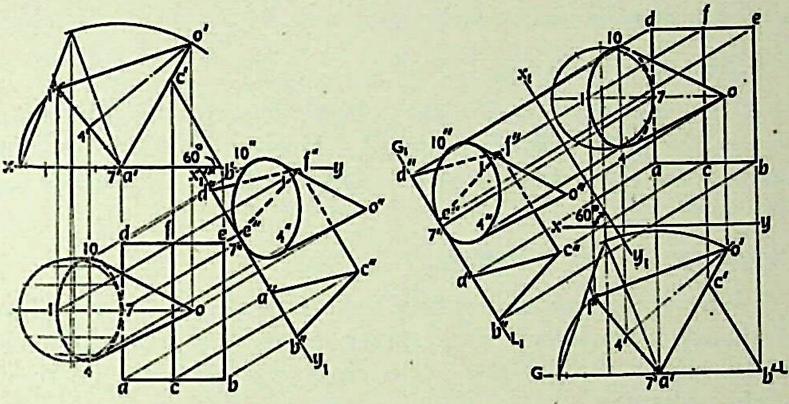
(ii) Tilt the elevation about the corner g' so that the line $e'c'$ becomes parallel to *xy*. Project the second plan. The solid diagonal *CE* is now parallel to both the ground and the V.P.

(iii) Reproduce the second plan so that the plan of the solid diagonal viz. e_1c_1 is perpendicular to *xy*. Project the required elevation.

Problem 24 (fig. 12-37):

A triangular prism, base 4 cm (1½″) side and axis 5 cm (2″) long is lying on the ground on one of its rectangular faces with the axis perpendicular to the V.P. A cone, base 4 cm (1½″) diameter, axis 5 cm (2″) long, is resting on the ground and is leaning centrally on a face of the prism, with its axis parallel to the V.P. Draw the elevation and plan of the solids and project another elevation on a ground line making 60° angle with xy.

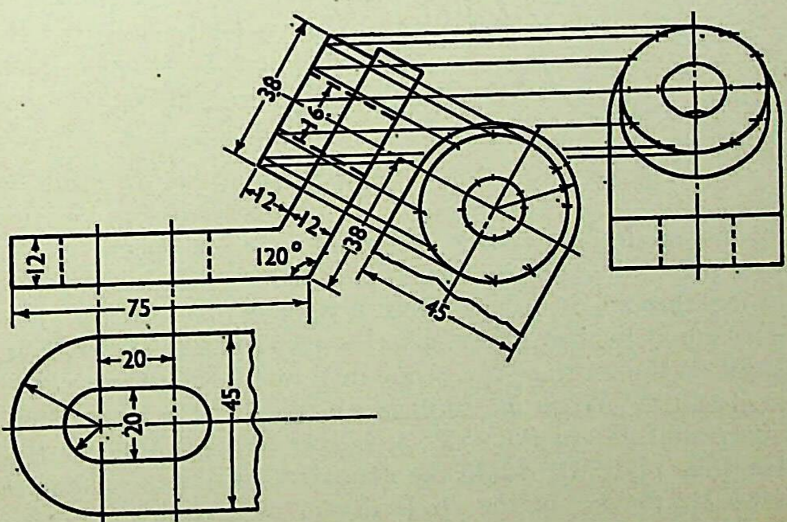
It will first be necessary to draw the cone, with its base on the ground to determine the length of its generator and to project the plan.



First-angle projection Third-angle projection

FIG. 12-37

Next, draw a triangle $a'b'c'$ for the prism and a triangle $o'1'7'$ for the cone as shown by the construction lines. Project the plan. Draw a ground line x_1y_1 and project the required elevation as shown.



(i) FIG. 12-38 (ii)

Problem 25:

The elevation, part plan and part auxiliary view of a casting are given in fig. 12-38(i). Project its end elevation. (All dimensions are given in millimetres).

See fig. 12-38(*ii*).

The construction for the ellipse for 38 mm diameter circle has been shown in detail Horizontal distances are taken from the auxiliary view. Other ellipses are drawn in the same manner.

PROJECTIONS OF SPHERES:

The projection of a sphere in any position on any plane is always a circle whose diameter is equal to the diameter of the sphere (fig. 12-39). This circle represents the contour of the sphere.

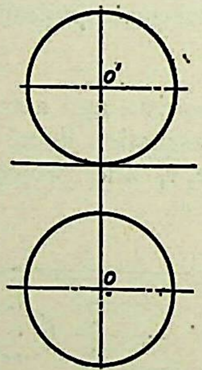
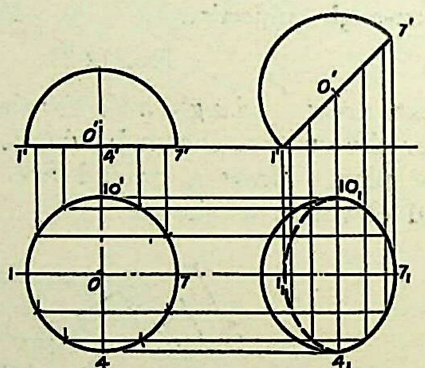


Fig. 12-39



(*i*) (*ii*)

Fig. 12-40

A flat circular surface is formed when a sphere is cut by a plane. A hemisphere (i.e. a sphere cut by a plane passing through its centre) has a flat circular face of diameter equal to that of the sphere. Its elevation, when it is placed on the flat face is a semi-circle and its plan is a circle [fig. 12-40(*i*)]. When the flat face is inclined to the ground and is perpendicular to the V.P., it is seen as an ellipse in the plan [fig. 12-40(*ii*)] while the contour of the hemisphere is shown by the arc of the circle drawn with radius equal to that of the sphere.

Fig. 12-41 shows the plan and elevation of a sphere, a small portion of which is cut off by a plane. Its flat face is perpendicular to the ground and inclined to the V.P. An ellipse is seen in the elevation within the circle for the sphere.

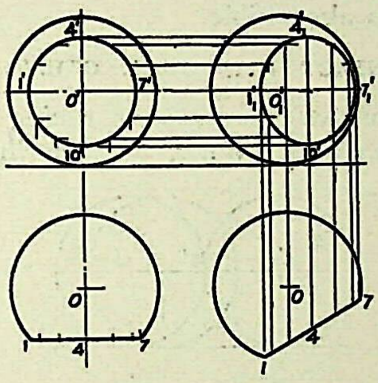


FIG. 12-41

Problem 26:

A brass flower-vase is spherical in shape with flat, circular top 35 cm (14") diameter and bottom 25 cm (10") diameter and parallel to each other. The greatest diameter is 40 cm (16"). Draw the projections of the vase when its axis makes an angle of 60° with the ground (fig. 12-42).

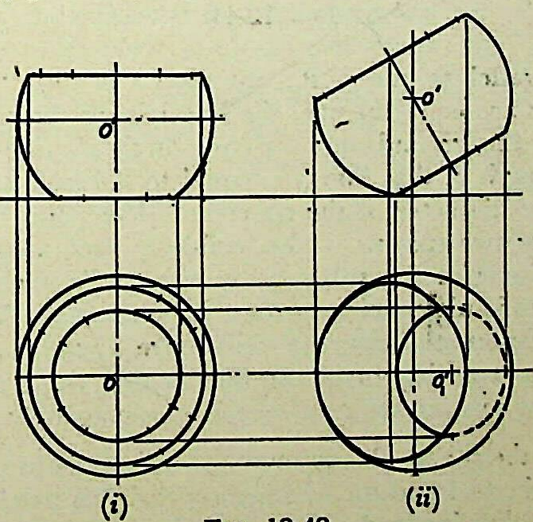


FIG. 12-42

(i) Draw the elevation of the vase resting on its bottom with its axis vertical. Project the plan.

(ii) Tilt the elevation so that the axis makes 60° angle with *xy* and project the plan. Note that a part of the ellipse for the bottom is also visible.

SPHERES IN CONTACT WITH EACH OTHER:

Projections of two equal spheres resting on the ground and in contact with each other, with the line joining their

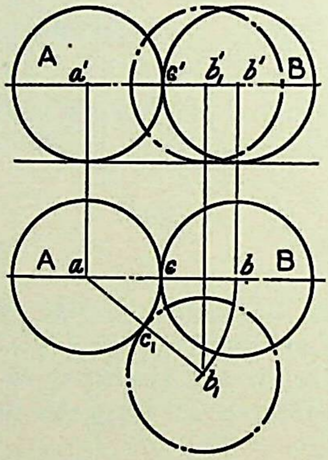


FIG. 12-43

centres parallel to the V.P., are shown in fig. 12-43. As the spheres are equal in size, the line joining their centres is parallel to the ground also. Hence, both *ab* and *a′b′* show the true length of that line (i.e. equal to the sum of the two radii or the diameter of the spheres). The point of contact between the two spheres is also visible in each view. If the position of one of the spheres, say sphere *B*, is changed so that the line joining their centres is inclined to the V.P., then in the elevation, the centre *b′* will move along the line *a′b′* to b_1'. The true length of the line joining the centres and the point of contact are now seen in the plan only.

When the sphere *B* is so moved that it remains in contact with the sphere *A* and the line joining their centres is parallel to the V.P., but inclined to the ground (fig. 12-44), the true

length of that line and the point of contact are visible in the elevation only.

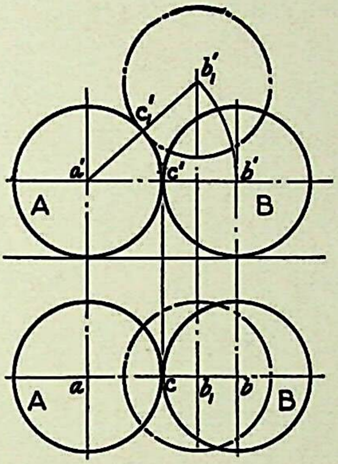


Fig. 12-44

Problem 27:

Three equal spheres of 4 cm ($1\frac{1}{2}''$) diameter are resting on the ground, so that each touches the other two and the line joining the centres of two of them is parallel to the V.P. A fourth sphere 5 cm ($2''$) diameter is placed on top of the three spheres so as to form a pile. Draw three views of the arrangement and find the distance of the centre of the fourth sphere above the ground (fig. 12-45).

As the spheres are resting on the ground and are equal in size, the lines joining their centres will be parallel to the ground. In the plan, the centres will lie at the corners of an equilateral triangle of side equal to the sum of two radii, i.e. 4 cm ($1\frac{1}{2}''$).

Draw (in the plan) an equilateral triangle *abc* of 4 cm ($1\frac{1}{2}''$) side with one side, say *ab*, parallel to *xy*. At its corners, draw three circles of 4 cm ($1\frac{1}{2}''$) diameter. Project the elevation. The centres will lie on a line parallel to and 2 cm ($\frac{3}{4}''$) above *xy*.

When the fourth sphere is placed on top, its centre *d*, in the plan, will be in the centre of the triangle. In the elevation, it will lie on a projector through *d*.

The true distance between the centre of the top sphere and that of any one of the bottom spheres, will be equal to the sum of the two radii viz. 2 cm ($\frac{3}{4}''$) + 2·5 cm ($1''$) or 4·5 cm ($1\frac{3}{4}''$).

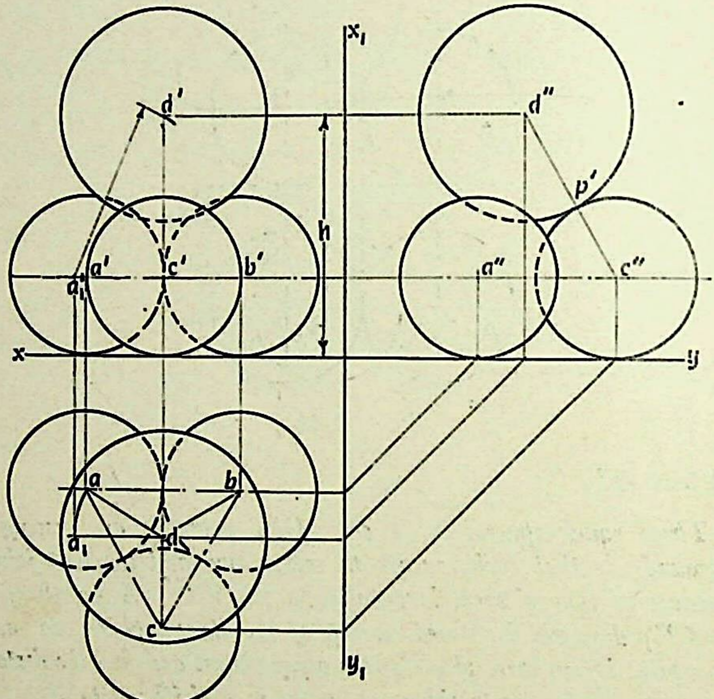


FIG. 12-45

But as the lines *da*, *db* or *dc* and not parallel to *xy*, their elevations will not show their true lengths. Therefore, to locate the position of the centre of top sphere in elevation, (i) make one of the lines, say *da*, parallel to *xy*; (ii) project a_1 to a_1' on the path of a' and (iii) with a_1' as centre and radius equal to 4·5 cm ($1\frac{3}{4}''$), draw an arc cutting the projector through *d* at the required point *d'*. With *d'* as centre and radius equal to 2·5 cm ($1''$), draw the required circle which will be partly hidden as shown. *h* is the distance of the centre of the top sphere from the ground.

Project an end elevation. As *c'd'* is parallel to the new ground line, *c''d''* will be equal to 4·5 cm ($1\frac{3}{4}''$) and the point

of contact p' between the spheres having centres c and d will be visible.

Fig. 12-45(a) shows the views in third-angle projection.

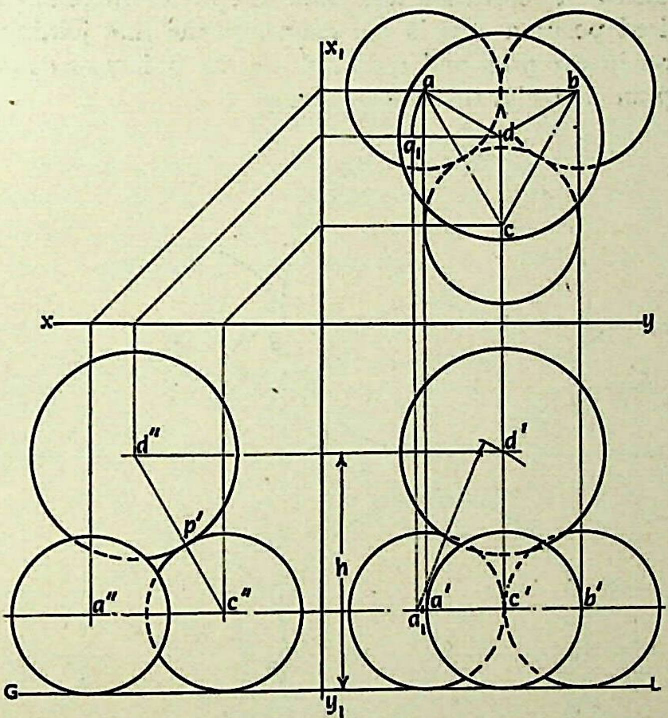


FIG. 12-45(a)

UNEQUAL SPHERES:

When two unequal spheres are on the ground and are in contact with each other, their point of contact and the true length of the line joining their centres will be seen in the elevation if that line is parallel to the V.P. In the plan, the length of the line will be shorter, but will remain constant even when it is inclined to the V.P.

Problem 28:

Three spheres, A, B and C of 7·5 cm (3"), 5 cm (2") and 3 cm ($1\frac{1}{4}$") diameters respectively, rest on the ground each touching

the other two. Draw their projections and show the three points of contact, when the line joining the centres of the spheres A and B is parallel to V.P. (fig. 12-46).

Draw the elevation and plan of spheres A and B in the required position. ab is the length of the line joining their centres in the plan and p and p' are the points of contact in the plan and elevation respectively.

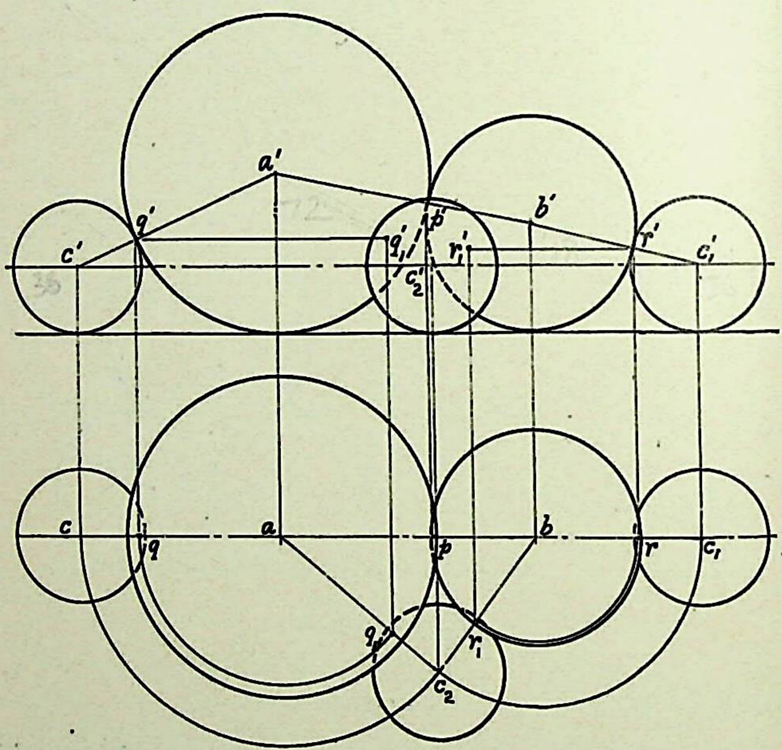


Fig. 12-46

Similarly, draw the views of (i) spheres A and C and (ii) spheres B and C in contact with each other and determine the lengths of the lines joining their respective centres in the plan, viz. ac and bc_1.

With a as centre and radius equal to ac, and with b as centre and radius equal to bc_1, draw arcs intersecting each other at c_2. With c_2 as centre, draw the plan of the sphere C.

Draw the projector through c_2 to cut the path of c' at c_2'. Then c_2' is the required centre of the sphere C in the elevation.

p, q_1 and r_1, and p', q_1' and r_1' are the points of contact in plan and elevation respectively.

Problem 29:

A square prism, base 2 cm ($\frac{3}{4}''$) side, axis 5 cm ($2''$) long, is resting on its base on the ground with two faces perpendicular to the V.P. Determine the radius of four equal spheres resting on the ground, each touching a face of the prism and other two spheres. Draw the plan and elevation of the arrangement (fig. 12-47).

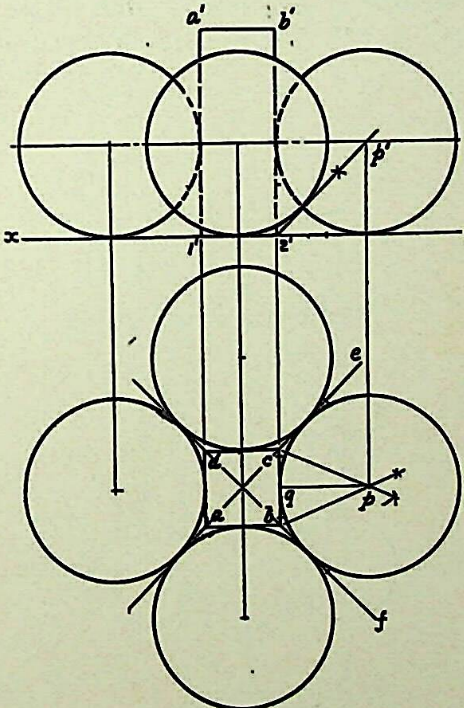


FIG. 12-47

Draw the plan and elevation of the prism. Draw the diagonals and produce them on both sides. Draw the bisectors of the angles bce and cbf intersecting at p. From p,

draw a perpendicular pq to bc. Then pq is the required radius of the sphere.

Draw a bisector of angle $b'2'y$ intersecting the projector through p at p'. Then p' is the centre of the sphere in the elevation.

Draw the other circles in a similar manner.

Problem 30:

Six equal spheres are resting on the ground, each touching other two spheres and a triangular face of a hexagonal pyramid resting on its base on the ground. Draw the elevation and plan of the solids when a side of the base of the pyramid is perpendicular to the V.P. Determine the diameter of each sphere. Base of the pyramid 2 cm ($\frac{3}{4}''$) side; axis 5 cm ($2''$) long (fig. 12-48).

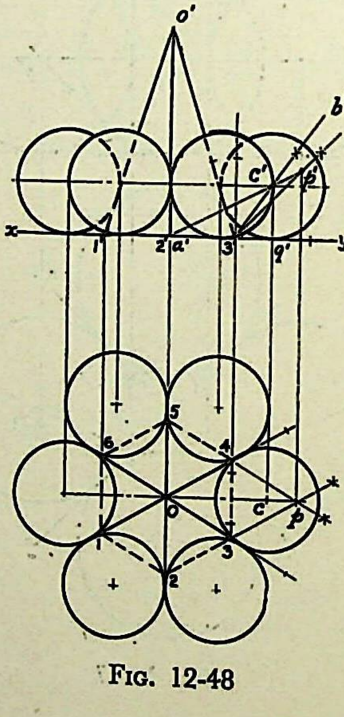
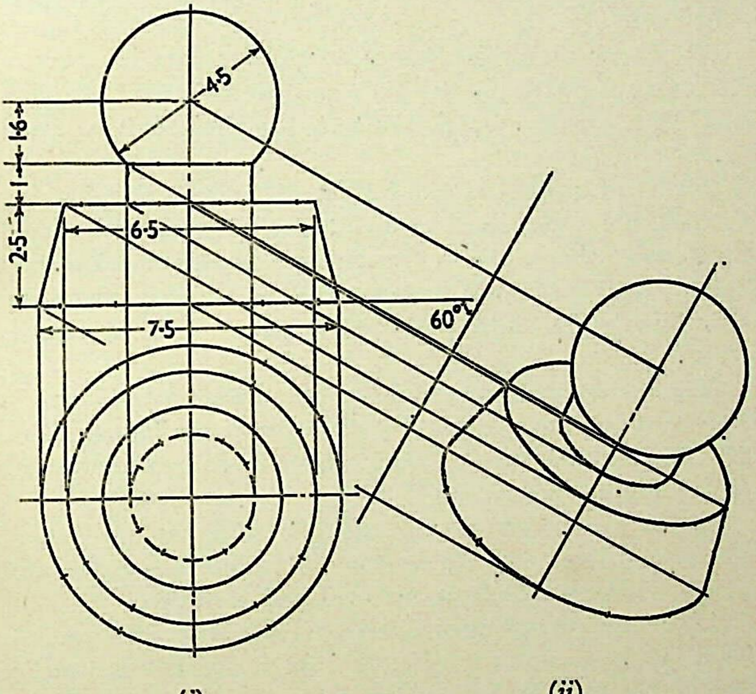


FIG. 12-48

Draw the plan and elevation of the pyramid in the required position. Assuming the solid to be a prism, locate the positions of the centre, (viz. p and p') in the two views.

Draw a line joining p' with a' (the centre of the base) which coincides with $2'$. The centre of the required sphere will lie on this line. Draw a bisector of angle $o'3'y$ cutting $a'p'$ at c'. Draw a line $c'q'$ perpendicular to xy. With c' as centre and radius $c'q'$, draw one of the required circles. Project c' to c on op in the plan. Then c is the centre of the circle in the plan. Other centres may be located in the plan as shown and projected up in the elevation.



(*i*) (*ii*)

FIG. 12-49

Problem 31:

The elevation and plan of a paper-weight with a spherical knob are given in fig. 12-49(i). Draw the two views and project another plan when its flat base makes an angle 60° with the H.P.

See fig. 12-49(*ii*) which is self-explanatory.

EXERCISES XII(ii)

Note: For solving the exercises by third-angle projection method, assume the edge or corner to be on the ground instead of in the H.P.

(1) A rectangular block 7·5 cm × 5 cm × 2·5 cm thick (3″ × 2″ × 1″ thick) has a hole of 3 cm ($1\frac{1}{4}$″) diameter drilled centrally through its largest faces. Draw the elevation and plan when the block has its 5 cm (2″) long edge in the H.P. and perpendicular to the V.P. and has the axis of the hole inclined at 60° to the H.P.

(2) Draw the projections of a square pyramid having one of its triangular faces in the V.P. and the axis parallel to and 4 cm ($1\frac{1}{2}$″) above the ground. Base 3 cm ($1\frac{1}{4}$″) side; axis 7·5 cm (3″) long.

(3) A cylindrical block, 7·5 cm (3″). diameter and 2·5 cm (1″) thick, has a hexagonal hole of 2·5 cm (1″) side, cut centrally through its flat faces. Draw the elevation, plan and end elevation of the block when it is resting in the H.P. with its flat faces vertical and inclined at 30° to the V.P. and two faces of the hole parallel to the H.P.

(4) Draw three views of an earthen flower pot, 25 cm (10″) diameter at the top, 15 cm (6″) diameter at the bottom, 30 cm (12″) high and 2·5 cm (1″) thick, when its axis makes an angle of 30° with the vertical.

(5) A tetrahedron of 7·5 cm (3″) edge has one edge in the H.P. and inclined at 45° to the V.P., while a face containing that edge is vertical. Draw its plan and elevation.

(6) A hexagonal prism, base 3 cm ($1\frac{1}{4}$″) side and axis 7·5 cm (3″) long, has an edge of the base in the H.P. and inclined at 45° to the V.P. Its axis makes an angle of 60° with the H.P. Draw its projections.

(7) A pentagonal prism is resting on a corner of its base in the H.P., with a longer edge containing that corner inclined at 45° to the H.P. and the vertical plane containing that edge and the axis inclined at 30° to the V.P. Draw its projections. Base 4 cm ($1\frac{1}{2}$″) side; height 6·5 cm ($2\frac{1}{2}$″).

(8) Draw three views of a cone, base 5 cm (2″) diameter and axis 7·5 cm (3″) long, having one of its generators in the V.P. and inclined at 30° to the H.P., the apex being in the H.P.

(9) A square pyramid, base 4 cm ($1\frac{1}{2}''$) side and axis 9 cm ($3\frac{1}{2}''$) long, has a triangular face in the H.P. and the vertical plane containing the axis makes an angle of 45° with the V.P. Draw its projections.

(10) A frustum of a pentagonal pyramid, base 5 cm (2″) side, top 2·5 cm (1″) side and axis 7·5 cm (3″) long, is resting on its base in the H.P. with an edge of the base perpendicular to the V.P. Draw its projections. Project another plan on a ground line parallel to the line which shows the true length of the slant edge. From this plan, project an elevation on an auxiliary vertical plane inclined at 45° to the plan of the axis.

(11) Draw the projections of a cone, base 5 cm (2″) diameter and axis 7·5 cm (3″) long, lying on a generator in the H.P. with the plan of the axis making an angle of 45° with the V.P.

(12) The elevation, incomplete plan and incomplete auxiliary plan of a casting are given in fig. 12-38. Draw all the three views completely. All dimensions are in millimetres.

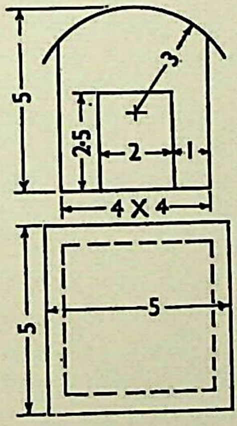


FIG. 12-50

(13) A line sketch (in two views) of a shed with a curved roof is given in fig. 12-50. Draw its elevation on an auxiliary vertical plane inclined at 60° to the V.P. All dimensions are in metres. Scale 2 cm = 1 m.

(14) A cube of 5 cm (2″) edge is resting in the H.P. with its vertical faces equally inclined to the V.P. A hexagonal pyramid, base 2·5 cm (1″) side and axis 5 cm (2″) long, is placed centrally

on top of the cube so that their axes are in a straight line and two edges of its base are parallel to the V.P. Draw the elevation and plan of the solids. Project another plan on an A.I.P. making an angle of 45° with the H.P. From this plan, project another elevation on a vertical plane inclined at 30° to the plan of the combined axis.

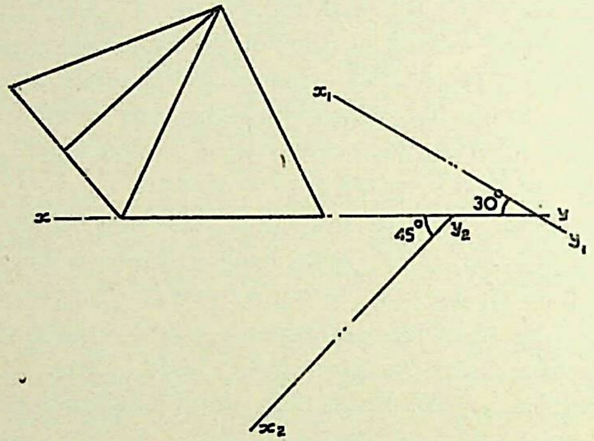


FIG. 12-51

(15) An elevation of a hexagonal pyramid [base 2·5 cm (1″) side], having one of its triangular faces resting centrally on a triangular face of a square pyramid [base 5 cm (2″) side and axis 5 cm (2″) long] is given in fig. 12-51. The plane containing the two axes is parallel to the V.P. Draw the plan of the solids. From this plan, project an elevation on a ground line x_1y_1 inclined at 30° to xy; (ii) from the given elevation, project another plan on a ground line x_2y_2 inclined at 45° to xy.

(16) Four equal spheres of 2·5 cm (1″) diameter are resting on the ground, each touching the other two spheres, so that a line joining the centres of two touching spheres is inclined at 30° to the V.P. A fifth sphere of 3 cm ($1\frac{1}{4}$″) diameter is placed centrally on top of the four spheres, thus forming a pile. Draw the projections of the spheres and measure the height of the centre of the top sphere above the ground.

(17) Three spheres of 2·5 cm (1″), 5 cm (2″) and 7·5 cm (3″) diameters respectively are resting on the ground so that each touches the other two. Draw their projections, when the plan of the line joining centres of any two of them is perpendicular to the V.P.

(18) Three equal cones, base 5 cm (2") diameter and axis 7·5 cm (3") long, are placed on the ground on their bases, each touching the other two. A sphere of 4 cm ($1\frac{1}{2}$") diameter is placed centrally between them. Draw three views of the arrangement and determine the height of the centre of the sphere above the ground.

(19) Five equal spheres are resting on the ground each touching the other two spheres and a vertical face of a pentagonal prism of 2·5 cm (1") side. Determine the diameter of the spheres and draw the projections, when a side of the base of the prism is perpendicular to the V.P.

(20) Four equal spheres are resting on the ground, each touching the other two spheres and a triangular face of a square pyramid, having base 2·5 cm (1") side and axis 5 cm (2") long. Draw their projections and find the diameter of the spheres.

(21) A pentagonal pyramid, base 4 cm ($1\frac{1}{2}$") side, height 7·5 cm (3") rests on one edge of its base in the H.P., so that the highest point in the base is 2·5 cm (1") above the H.P. Draw its elevation and plan when the axis is parallel to the V.P. Draw another elevation on a ground line inclined at 30° to the edge on which it is resting, and so that the base is visible.

(22) A thin lamp shade in the form of a frustum of a cone has its larger end 20 cm (8") diameter smaller end 7·5 cm (3") diameter and height 15 cm (6"). Draw its three views when it is lying on its side in the H.P. and the axis parallel to the V.P.

(23) A bucket made of tin sheet has its top 20 cm (8") diameter, bottom 12·5 cm (5") diameter with a circular ring 4 cm ($1\frac{1}{2}$") wide attached at the bottom. The total height of the bucket is 25 cm (10"). Draw its projections when its axis makes an angle of 60° with the vertical.

CHAPTER XIII

DEVELOPMENT OF SURFACES

When a surface is laid out on a plane, the figure obtained is called its development. Imagine that a solid is enclosed in a wrapper of thin material, such as paper. If this covering is opened out and laid on a flat plane, the flattened-out paper is the development of the solid.

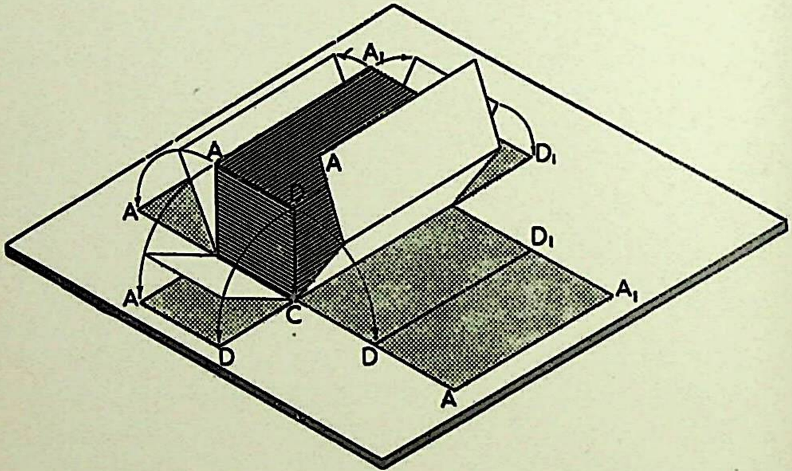


FIG. 13-1

Fig. 13-1 shows a square prism covered with paper, in process of being opened out. Its development (fig. 13-2) consists of four equal rectangles for the faces and two similar squares for its ends. Each figure shows the true size and shape of the corresponding surface of the prism. The development of a solid, thus represents the actual shape of all its surfaces which, when bent or folded at the edges, would form the solid.

Hence, it is very important to note that *every line on the development must be the true length of the corresponding edge on the surface.*

The knowledge of development of surfaces is essential, particularly in sheet-metal work. In construction of boilers, funnels, chimneys etc., the plates are marked and cut according to the developments which, when folded, form the desired objects.

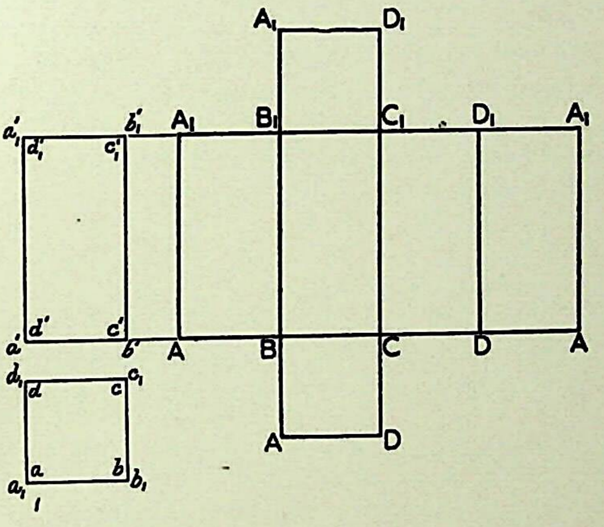


Fig. 13-2

Only the surfaces of polyhedra (such as prisms and pyramids) and single-curved surfaces (as of cones and cylinders) can be accurately developed. Warped and double-curved surfaces are undevelopable These can however be approximately developed by dividing them up into a number of parts.

The principal methods of development are:

(i) *Parallel-line development:* It is employed in case of prisms and cylinders in which stretch-out-line principle is used. Lines A-A and A_1-A_1 in fig. 13-3, are called the stretch-out lines.

(ii) *Radial-line development:* It is used for pyramids and cones in which the true length of the slant edge or the generator is used as radius.

In what follows, the ends or bases of the solids have been omitted from their developments. They can however be easily incorporated if required.

Note: *First-angle projection method has been used in all the problems. In third-angle projection, the plan will be above the elevation. That will be the only difference.*

RIGHT AND REGULAR PRISMS:

Development of a prism consists of a number of equal rectangles in contact, one side of the rectangle being equal to the length of a side of the base and the other equal to the length of the axis of the prism.

Problem 1:

(*a*) *To draw the development of the surface of a truncated prism shown at* (*i*) *in fig. 13-3;* (*b*) *also, to draw an elevation of the line joining the points P and Q (whose projections p, p′ and q, q′ are given), along the surface of the prism by the shortest distance.*

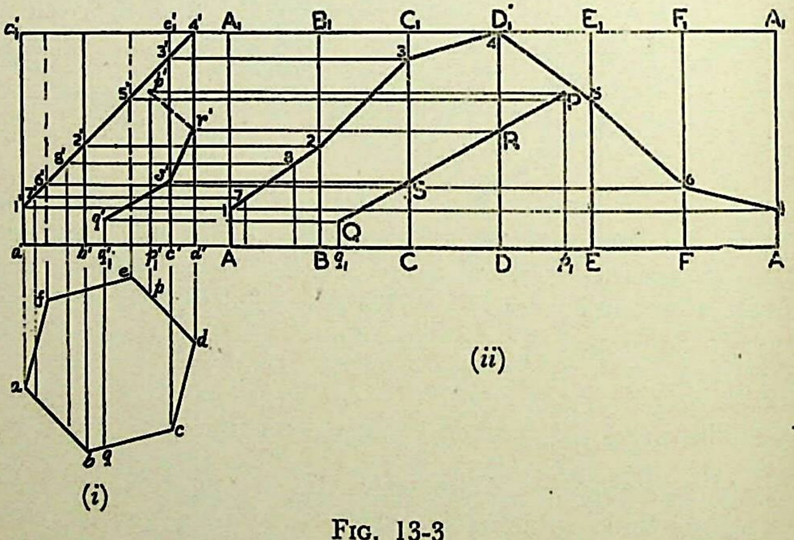


Fig. 13-3

(*a*) Draw a stretch-out line *AA* [fig. 13-3(*ii*)] directly in line with the base in the elevation and on it, set off six equal divisions *AB*, *BC* etc. each equal to the length of the side of the base (obtained from the plan). The length of *AA* will thus be equal to the perimeter of the base. Construct a rectangle

on AA as one side and maximum length of the prism as the other side. Draw perpendiculars BB_1 CC_1 etc., thus completing the development of the whole prism. Mark points 1, 2 etc. on the verticals AA_1, BB_1 etc. such that $A1 = a'1'$, $B2 = b'2'$ etc. These lengths can be directly obtained by drawing horizontal lines from the corresponding points in the elevation. As the prism has all plane surfaces and is truncated by a plane, lines joining the upper corners will be straight lines. This can however be verified by marking some intermediate points 7, 8 etc. as shown. Draw lines joining 1 and 2, 2 and 3 etc. Make these lines and the lines in the lower portion dark, thus showing the development of the truncated prism clearly. The other lines may be kept thin and fainter.

(*b*) Mark a point p_1 on DE such that $Dp_1 = dp$. Erect a perpendicular at p_1 and mark on it a point P such that $Pp_1 = p'p_1'$. Similarly, obtain the point Q. P and Q are the positions of the given points in the development. Draw a straight line joining P and Q. Then PQ will show the shortest distance between them. To draw this line in the elevation, the process must be reversed. Let the line PQ cut the edge DD_1 at R and CC_1 at S. Draw horizontals through R and S, cutting $d'd_1'$ at r' and $c'c_1'$ at s'. Draw lines $p'r'$, $r's'$ and $s'q'$ which show the elevation of the line PQ. Note that $p'r'$ is hidden and hence, should be dotted.

Problem 2:

The elevation and plan of a square prism with a hole drilled in it, are given in fig. 13-4(i). Draw the development of the lateral surface of the prism.

Mark a number of points on the circle for the hole. Draw the development of the whole prism [fig. 13-4(*ii*)] and locate the positions of these points on it. For example, to locate points p_1' and p_2' and points q_1' and q_2' which coincide with them in the elevation, draw a perpendicular through them cutting the base at p'. Project p' to p on ab and to q on ad. Mark points P and Q on AB and DA respectively, such that $AP = ap$ and $AQ = aq$. Erect verticals at points P and Q. Draw horizontal lines through p_1' and p_2' cutting these verti-

cals at P_1, Q_1, P_2 and Q_2. Locate all points in a similar way and draw smooth curves through them, thus completing the development.

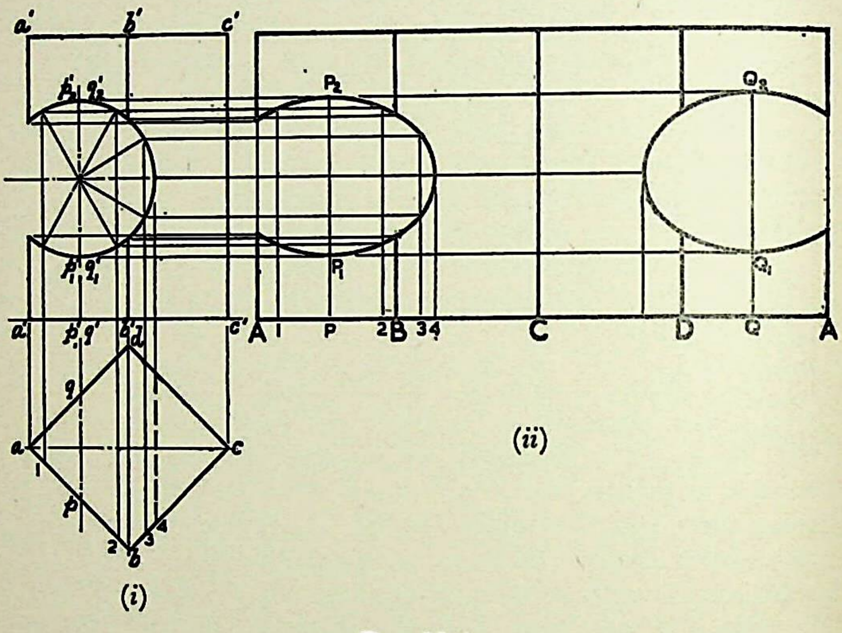


FIG. 13-4

RIGHT CIRCULAR CYLINDERS:

The development of the curved surface of a cylinder is a rectangle, having one side equal to the circumference of its base-circle and the other equal to its length.

Problem 3:

To develop the surface of a truncated cylinder shown in fig. 13-5(i).

Divide the base-circle into twelve equal parts.

Project the division-points in the elevation and draw generators $2'b'$, $3'c'$ etc. Note that the generators on the back side of the cylinder coincide with these front generators e.g. $12'p'$ coincides with $2'b'$ etc.

Draw a stretch-out line 1-1 [fig. 13-5(*ii*)] equal to the circumference of the base-circle (i.e. πD). This length can be marked approximately as shown below. With a bow-divider step-off along the line 1-1, twelve divisions, each equal to the chord-length *ab*. By this method the length obtained is about 1% shorter than the exact length.

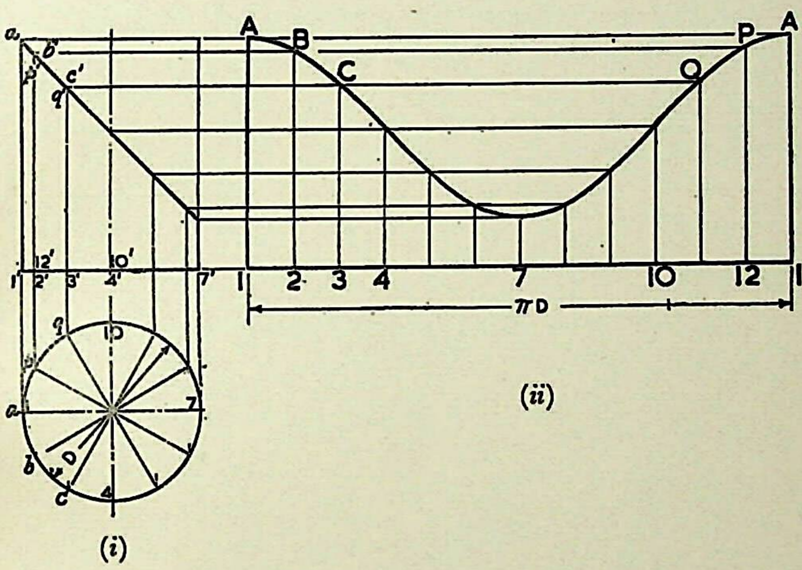


FIG. 13-5

Erect perpendiculars at the division-points and make them equal to their corresponding lengths in the elevation by drawing horizontal lines through respective points, viz. $B2 = P12 = b'2'$ etc. Complete the development by drawing a curve through *A*, *B*, *C* etc.

Note that the rectangle *A* 1 1 *A* is the development of the whole cylinder of length equal to $a1'$.

Problem 4:

Draw the development of the lateral surface of the cylinder cut as shown in fig. 13-6(*i*).

Using the same method as described in problem 3, draw the development as shown in fig. 13-6(*ii*).

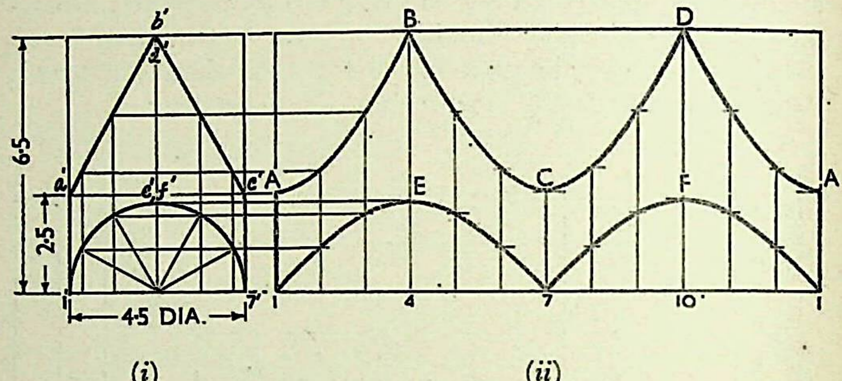


Fig. 13-6

Problem 5:

To develop the surface of the three-piece cylindrical pipe elbow shown in fig. 13-7(i).

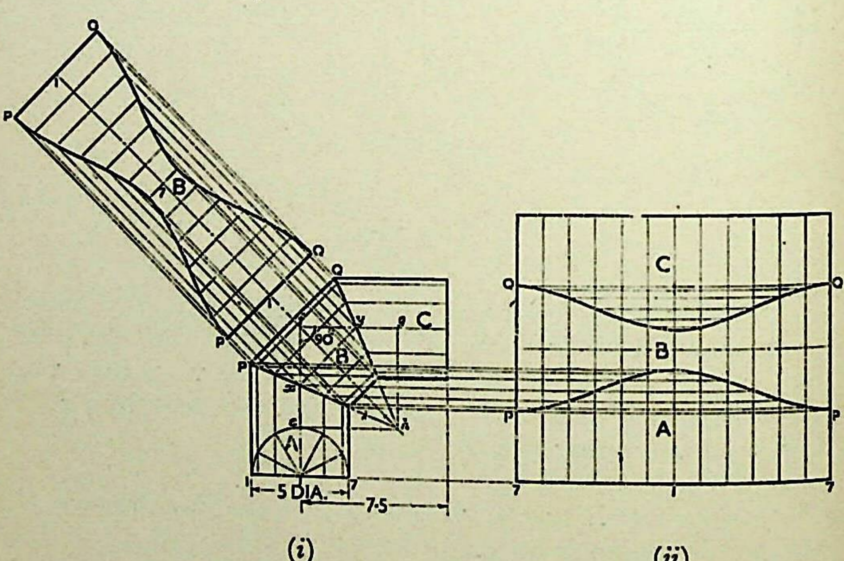


Fig. 13-7

The elevation is drawn as shown below:

Draw a square *efgh* of a side equal to the diameter of pipes (i.e. 5 cm). Bisect angles *ehf* and *ghf* by lines *Ph* and

Qh to intersect *ef* and *fg* at *x* and *y* respectively. Then *xy* is the axis of the middle piece *B*.

Parts *A* and *C* are similar and are in the form of cylinders truncated at one end only. *A* is shown developed on a stretch-out line drawn directly in line with its base. The twelve divisions are obtained from the semi-circle drawn on the base as diameter.

The part *B* is truncated at both the ends. It is developed on a stretch-out line drawn through *h* and at right angles to its axis *xy*.

As the curves of the part *B* are exactly similar to those of *A* and *C*, the three developments may also be shown combined as in fig. 13-7(*ii*).

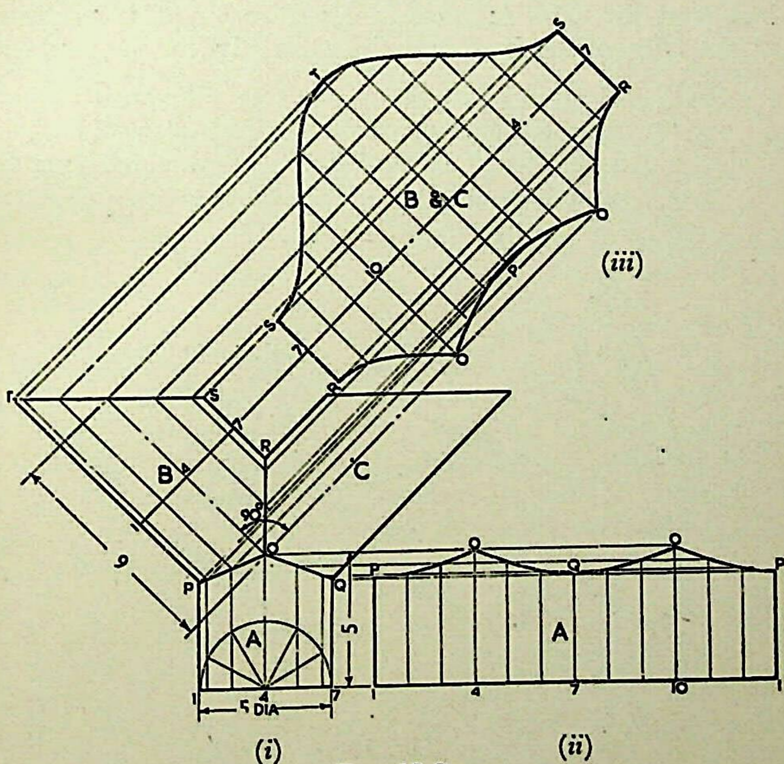


FIG. 13-8

Problem 6:

Three cylindrical pipes of 5 cm (2") diameter form a Y-piece as

shown in elevation in fig. 13-8. Draw the development of the surface of each pipe.

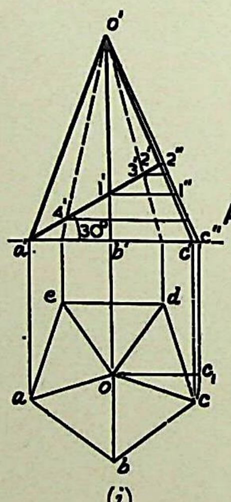
Draw a semi-circle on the base of pipe *A* as diameter and obtain twelve divisions. Draw the development [fig. 13-8(*ii*)] as in the previous problem.

Draw any convenient line at right angles to the axis of the pipe *B* [fig. 13-8(*iii*)]. On this line as a stretch-out line, draw the development as shown. Pipes *B* and *C* are similar and hence, their developments will also be similar.

RIGHT AND REGULAR PYRAMIDS:

The development of the surface of a pyramid consists of a number of equal isosceles triangles in contact. The base and the sides of each triangle are equal, respectively, to the edge of the base and the slant edge of the pyramid.

Note: The true length of a slant edge of a pyramid can be measured from the elevation, if the plan of that edge is parallel to *xy*; and it can be measured from the plan, if the slant edge is parallel to *xy* in the elevation.



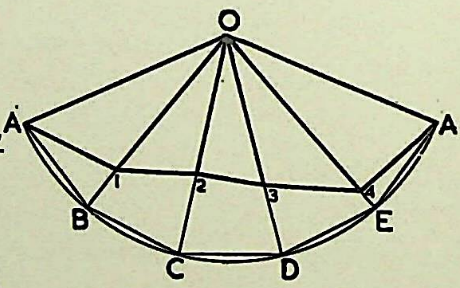


FIG. 13-9

Problem 7:

To develop the surface of a pentagonal pyramid with its base removed, as shown in elevation in fig. 13-9(i).

As all slant edges are inclined to the V.P., their true length is not seen in the elevation. Hence, determine the true length $o'c''$ by making oc (in the plan) parallel to xy.

With a point O as centre and radius equal to $o'c''$, draw an arc and on it, step-off five divisions AB, BC etc. each equal to ab [fig. 13-9(*ii*)]. Complete triangles OAB, OBC etc. which will give the development of the whole pyramid. Determine the true lengths of lines $o'1'$, $o'2'$ etc. by drawing lines through $1'$, $2'$ etc. parallel to the base and intersecting $o'c''$ at $1''$, $2''$ etc.

Mark points 1, 2 etc. on OB, OC etc. such that $O1 = o'1''$, $O2 = o'2''$ etc. Draw lines $A1$, 1-2, 2-3 etc. thus completing the development.

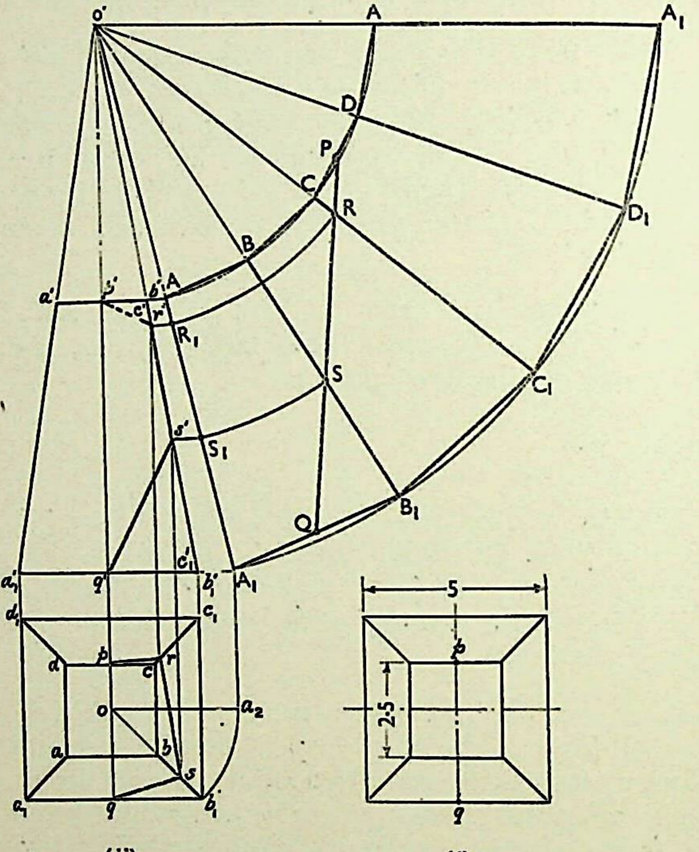
Problem 8:

The plan of a frustum of a square pyramid, 7·5 cm (3") high, is given at (i) in fig. 13-10. Draw (a) the development of the surface of the frustum and (b) the projections of the frustum showing the line joining points p and q by the shortest distance along the surface of the frustum.

Draw the plan and project the elevation of the frustum [fig. 13-10(*ii*)]. Find o', the apex of the pyramid, by producing the slant edges. Determine the true length AA_1 of the slant edges. With o' as centre and radii $o'A$ and $o'A_1$, draw arcs of circles. On the outer arc, step-off six divisions, each equal to a_1b_1. Join these points with o' cutting the inner arc at A, B etc. Draw the inner and outer chords. The enclosed figure will be the development of the frustum.

Mark the mid-point P of CD and Q of A_1B_1. Draw a line joining P and Q and cutting CC_1 at R and BB_1 at S. Transfer these points to the elevation and the plan. For example, with o' as centre and radius $o'R$, draw an arc cutting $o'A_1$ at R_1. Through R_1, draw a line parallel to the base and cutting $c'c_1'$ at r'. Project r' to r on cc_1 in the plan. r' and r are the projections of R. Similarly, obtain s' and s on $b'b_1'$ and bb_1 respectively. Draw lines pr, rs and sq which

will show the plan of the line PQ. $p'r's'q'$ will be the path of the line PQ in the elevation.



(ii) (i)

FIG. 13-10

RIGHT CIRCULAR CONES:

The development of the curved surface of a cone is a sector of a circle, the radius and the length of the arc of which are respectively equal to the slant height and the circumference of the base-circle of the cone.

Problem 9:

To draw the development of the surface of a truncated cone, shown in elevation in fig. 13-11(i).

Assuming the cone to be whole, let us draw its development.

Draw the base-circle in the plan and divide it into twelve equal parts.

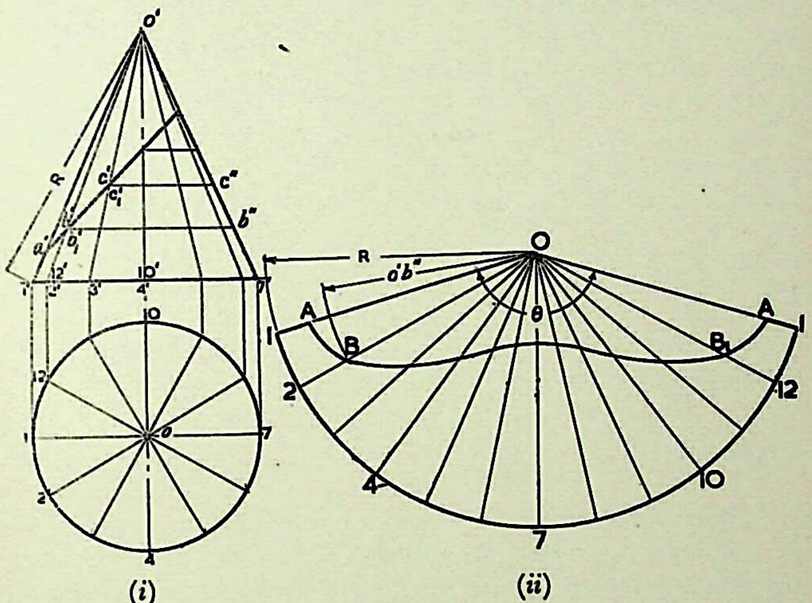


FIG. 13-11

With any point O as centre and radius equal to $o'1'$ or $o'7'$, draw an arc of the circle [fig. 13-11(*ii*)]. The length of this arc should be equal to the circumference of the base-circle. This can be determined in two ways.

I. Calculate the subtended angle θ by the formula,

$$\theta = 360° \times \frac{\text{radius of the base-circle}}{\text{slant height}}.$$

Cut-off the arc so that it subtends the angle θ at the centre and divide it into twelve equal parts.

II. Step-off with a bow-divider, twelve equal divisions on the arc, each equal to one of the divisions of the base-circle. (This will give an approximate length of the circumference).

Join the division-points with O, thus completing the development of the whole cone with twelve generators shown in it [fig. 13-11(*ii*)].

The truncated portion of the cone may be deducted from this development by marking the positions of points at which generators are cut, and then drawing a curve through them. For example, generators $o'2'$ and $o'12'$ in the elevation are cut at points b' and b_1' which coincide with each other. The true length of $o'b'$ may be obtained by drawing a line through b', parallel to the base and cutting $o'7'$ at b''. Then $o'b''$ is the true length of $o'b'$.

Mark points B and B_1 on generators $O2$ and O-12 respectively, such that $OB = OB_1 = o'b''$. Locate all points in similar way and draw a smooth curve through them. The figure enclosed between this curve and the arc is the development of the truncated cone.

Problem 10:

Draw the elevation and plan of a cone resting on the ground on its base and show on them, the shortest path by which a point P, starting from a point on the circumference of the base and moving around the cone will return to the same point. Base of cone 6·5 cm ($2\frac{1}{2}''$) diameter; axis 7·5 cm ($3''$) long (fig. 13-12).

Draw the plan, elevation and the development of the cone showing all twelve generators. The development may be drawn attached to $o'1'$.

Assume that P starts from a point 1 (i.e. point $1'$ in the elevation). Draw a straight line $1'1'$ on the development. This line shows the required shortest path.

To transfer this line to the elevation, the process adopted in problem 9 must be reversed. Let us take a point P_4 at which the path cuts the generator $o'4$. Mark a point p_4'' on $o'1'$ such that $o'p_4'' = o'P_4$. This can be done by drawing an arc with o' as centre and radius equal to $o'P_4$ cutting $o'1'$ at p_4''. Through p_4'', draw a line parallel to the base and cutting $o'4'$ at p_4'.

Similarly, transfer all the points to the elevation and draw the required curve through them. The curve at the back will coincide with the front curve.

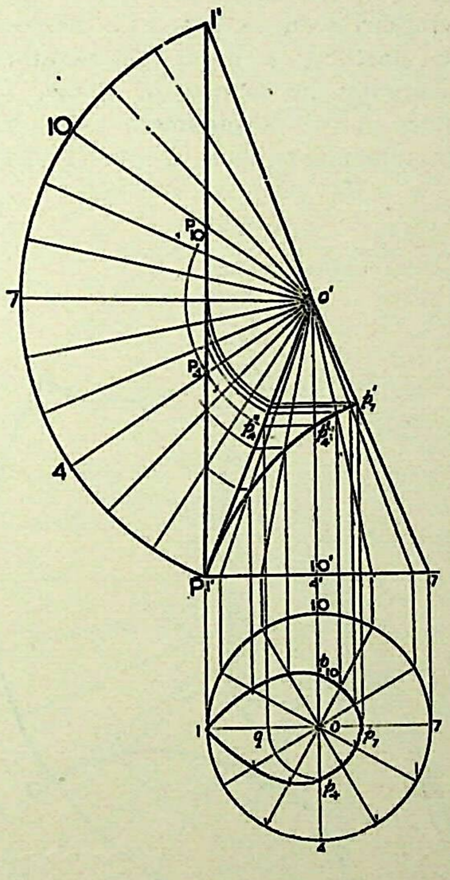


FIG. 13-12

Project these points to the plan on the respective generators. p_4' and p_{10}' cannot be projected directly. Hence, project p_4'' to a point q on $o1$. With o as centre and radius equal to oq, draw an arc cutting $o4$ at p_4 and o-10 at p_{10}. Thus, $op_4 = op_{10} = oq$. A curve drawn through the points thus obtained will show the path in the plan.

Problem 11:

Draw the development of the surface of the part of a cone, the elevation of which is shown in fig. 13-13.

Draw a semi-circle on the base as a diameter and divide it into six equal parts for positions of generators. Draw the development assuming the cone to be whole. Obtain points on the generators in the development as explained in problem 9. Additional points such as a', may also be marked

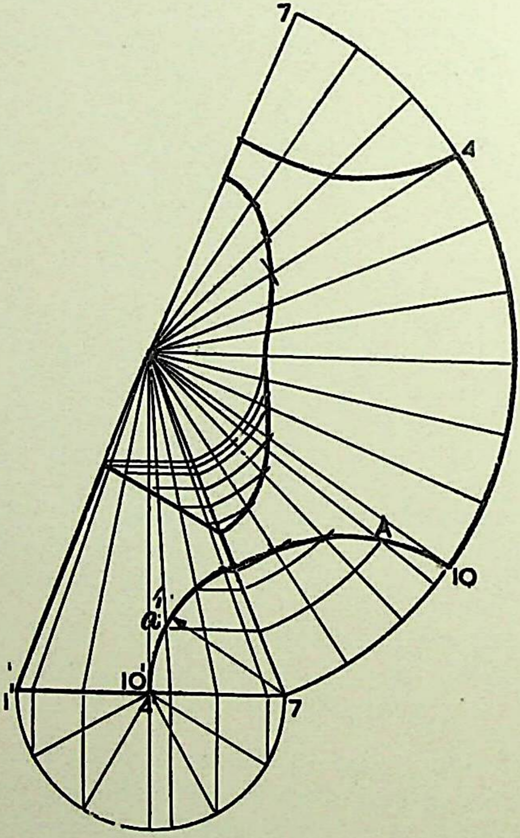


Fig. 13-13

to determine the correct shape of the curve. Draw curves through these points and complete the development as shown.

Problem 12:

Draw the development of the funnel shown in fig. 13-14.

The funnel consists of a conical part C and two cylindrical pipes A and B — all truncated.

Developments of the pipes may be drawn as shown in problem 3, while for that of the conical piece, the method shown in problem 9 should be applied.

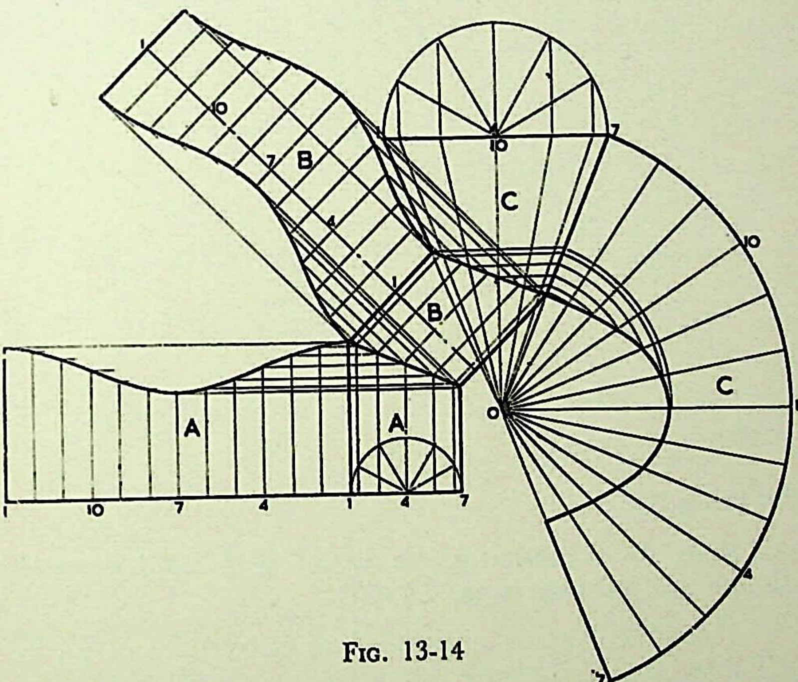


FIG. 13-14

Problem 13:

The elevation and plan of a solid composed of a truncated half-cylinder and a cut half-prism, are given at (i) in fig. 13-15. Draw the development of its lateral surface.

Assuming the solid to be whole, draw its development [fig. 13-15(*ii*)]. Draw a stretch-out line and on it, step-off (i) 1A equal to the arc 1a, (ii) AB, BC and CD, each equal to the edge ab of the base and (iii) D1 equal to the arc d1. Complete the rectangle.

Draw perpendiculars at points A, B etc. and at other intermediate points. Locate on them, positions of points

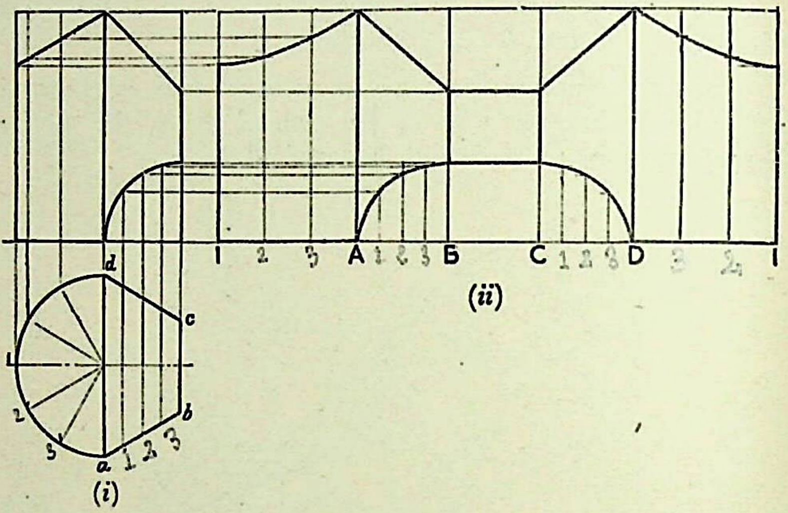


FIG. 13-15

at which they are cut and draw the curves and straight lines as shown.

Problem 14:

Draw the development of the surface of the solid shown in elevation and full plan in fig. 13-16(i).

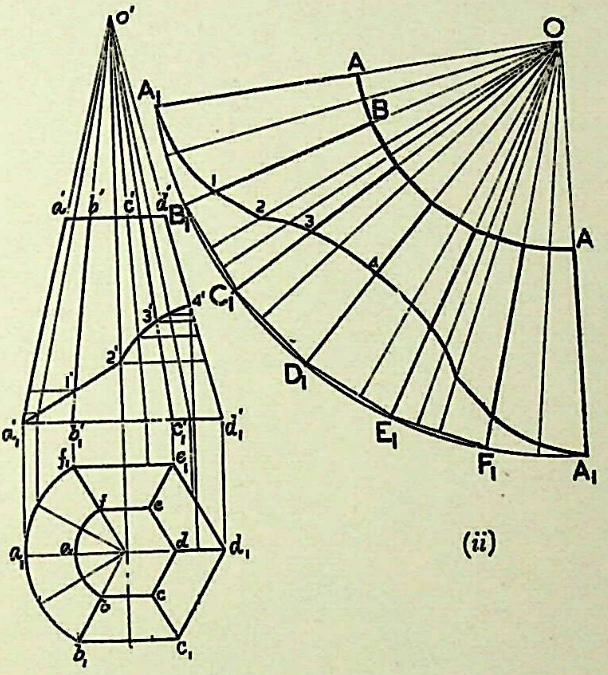
The solid is made-up of portions of frustums of a cone and a hexagonal pyramid.

The slant height of the cone is equal to the slant edge of the pyramid.

Therefore, with any point O as centre and radii $o'a'$ and $o'a_1'$, draw arcs [fig. 13-16(*ii*)]. On the outer arc, step-off distances (i) A_1B_1 equal to the arc a_1b_1; (ii) B_1C_1, C_1D_1, D_1E_1 and E_1F_1, each equal to the side of the base, viz. b_1c_1 and (iii) F_1A_1 equal to the arc f_1a_1.

Join these points with O, cutting the inner arc at points A, B etc. Locate positions of various points as explained

in problems 7 and 9 and complete the development as shown.



(*i*)

Fig. 13-16

SPHERES:

The surface of a sphere can be approximately developed by dividing it into a number of parts. The divisions may be made in two different ways: (i) in zones and (ii) in lunes. A *zone* is a portion of the sphere enclosed between two planes perpendicular to the axis. A *lune* is the portion between two planes which contain the axis of the sphere.

Zone method: Fig. 13-17 shows the top half of a sphere divided into four zones of equal width. By joining points *PQ*, *QR* etc. by straight lines, each zone becomes a cone frustum, except the upper-most zone which becomes a cone of small altitude.

Developments of these cone frusta and the upper cone will give the development of the half sphere. For example, take the zone *C*. It is a frustum of a cone whose vertex is

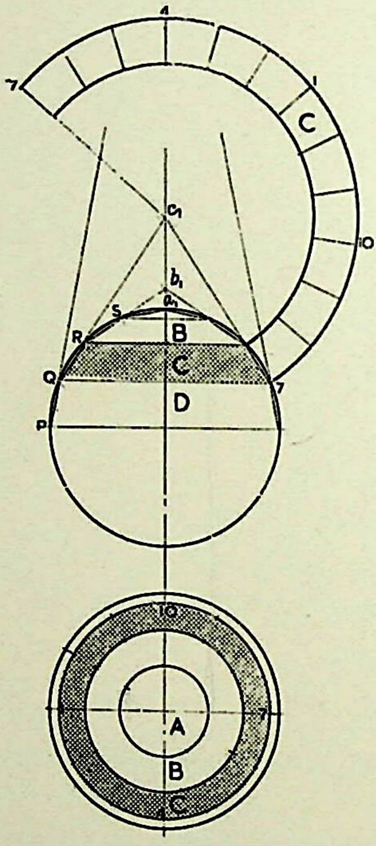


FIG. 13-17

at c_1. The surface of this frustum is shown developed in the elevation. The length of the divisions on the arc is obtained from the plan.

All the zones can be developed in the same manner.

Lune method: A sphere may be divided into twelve lunes, one of which is shown in elevation in fig. 13-18. The semi-circle *QR* is the plan of the centre line of that lune.

It is evident that the length of the lune is equal to the length of the arc *QR* and its maximum width is equal to *gh*.

Divide the semi-circle into a number of equal parts, say 8 and project the division-points on the elevation, to points $1'$, $2'$ etc. With r' as centre and radii equal to $r'1'$, $r'2'$ and $r'3'$, draw arcs *ab*, *cd* and *ef* which will show the widths of the lune at points 1 and 7, 2 and 6, and 3 and 5 respectively.

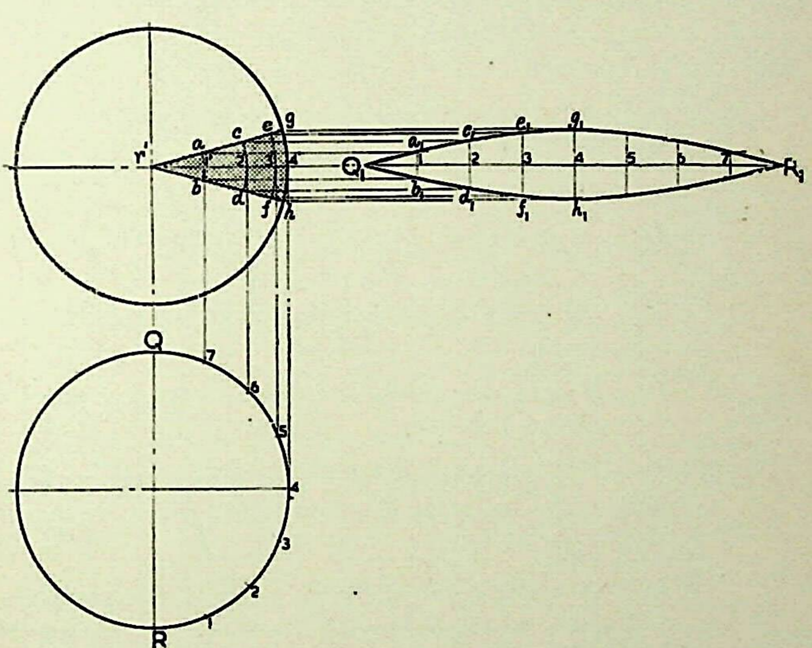


FIG. 13-18

Draw a line Q_1R_1 equal to the length of the arc QR. This may be obtained by stepping off eight divisions, each equal to the chord-length $R1$.

Draw perpendiculars at each division-point and make a_1b_1 equal to *ab* at points 1 and 7, c_1d_1 equal to *cd* at points 2 and 6 etc. Draw smooth curves through points Q_1, a_1, c_1 etc. The figure thus obtained will be the approximate development of one-twelfth of the surface of the sphere.

Development of some more solids cut by different planes, and solids with holes cut or drilled through them are treated in Chapters XVI and XVII.

EXERCISES XIII

Refer to fig. 13-19 for the following exercises and for dimensions, assume each square to be of 1·2 cm ($\frac{1}{2}''$) side.

(1) Draw the development of the surfaces of the portions of the following prisms, elevations of which are shown in the top row:

(a) A hexagonal prism having a face parallel to the V.P.

(b) A square prism having all faces equally inclined to the V.P.

(c) A pentagonal prism having a vertical face parallel to the V.P.

(d) A triangular prism having a vertical face parallel to the V.P.

(e) A hexagonal prism having two faces perpendicular to the V.P.

(2) Draw the development of the surfaces of the portions of the following pyramids, elevations of which are shown in the second row:

(a) A square pyramid having a side of base perpendicular to the V.P.

(b) A hexagonal pyramid having a side of base parallel to the V.P.

(c) A pentagonal pyramid having a side of base parallel to the V.P.

(d) A triangular pyramid having a side of base parallel to the V.P.

(e) A square pyramid having all sides of the base equally inclined to the V.P.

(3) Draw the development of the surfaces of the portions of the cylinders shown in the third row.

(4) Draw the development of the surfaces of the portions of the cones shown in the fourth row.

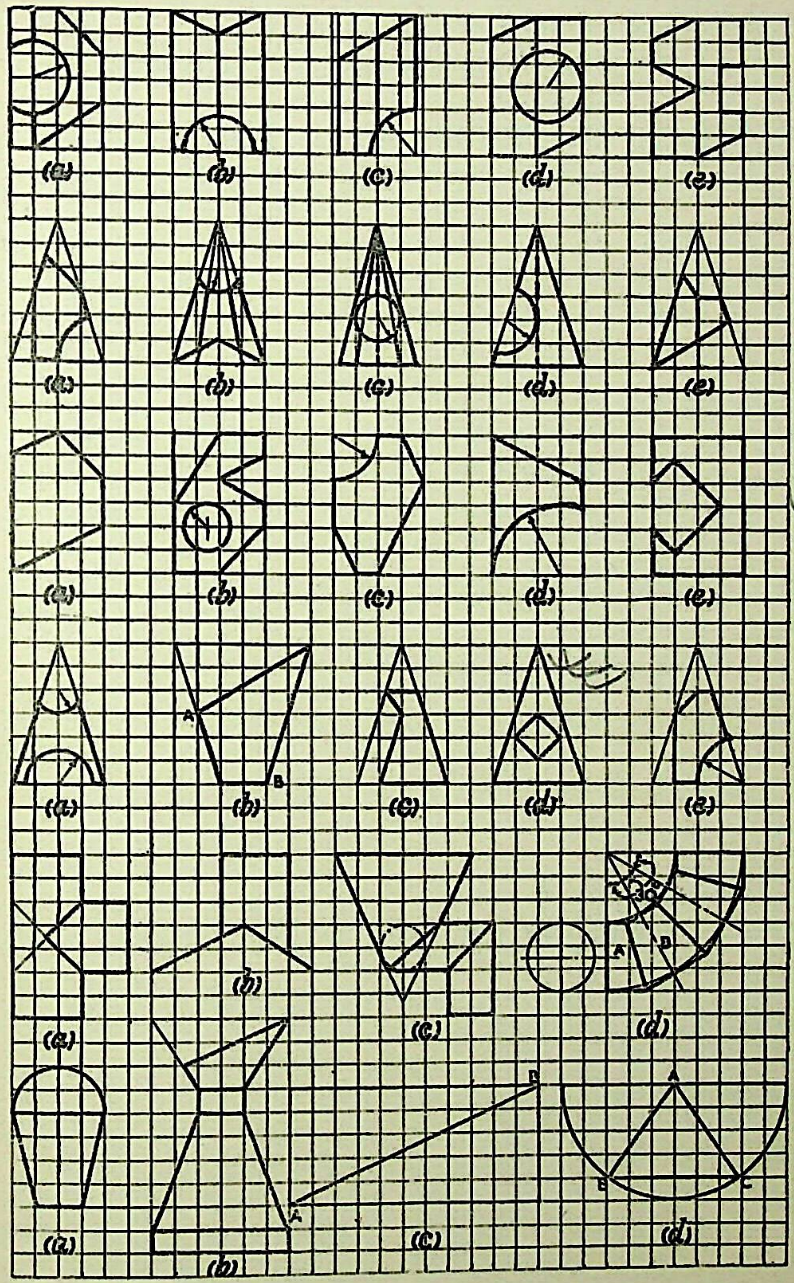


FIG. 13-19

(5) Refer to the fifth row and

- (i) Draw the development of the pipes forming a Tee shown at (*a*).
- (ii) Draw the development of the cylindrical steel chimney erected on a roof [shown at (*b*)], assuming the squares to be of 30 cm (1 ft) side.
- (iii) Draw the development of the three parts of the funnel shown at (*c*).
- (iv) Develop parts *A* and *B* of the transition piece shown at (*d*).

(6) Refer to the last row and

- (i) Develop the surface of the conical buoy with a hemispherical top, shown at (*a*).
- (ii) Determine the shape of the tin sheet required to prepare the can shown at (*b*).
- (iii) The development of the surface of a cylinder is given at (*c*). Draw the elevation of the cylinder showing the line *AB* in it.
- (iv) The development of the surface of a cone is shown at (*d*). Draw the elevation and plan of the cone, showing lines *AB*, *BC* and *CA* in each view.

CHAPTER **XIV**

HELICES AND SCREW THREADS

Helix is defined as a curve, generated by a point, moving around the surface of a right circular cylinder in such a way that, its axial advance, i.e. its movement in the direction of the length of the cylinder, is uniform with its movement around the surface of the cylinder.

The axial advance of the point, during one complete revolution, is called the *pitch* of the helix. If the pitch is say 2 cm ($\frac{3}{4}''$), and the point starts upwards from the base of the cylinder, in one-fourth of a revolution, the point will move up a distance of 0·5 cm ($\frac{3}{16}''$) from the base.

Note: First-angle projection method is used in the following problems. In third-angle projection, the plan will be above the elevation. That will be the only difference.

Problem 1:

To draw a helix of one convolution, given the pitch and the diameter of the cylinder. Also to develop the surface of the cylinder showing the helix in it.

Draw the plan and elevation of the cylinder [fig. 14-1(*i*)]. Divide the circle (in the plan) into any number of equal parts, say 12.

Mark a length P-12′ equal to the given pitch, along a vertical side of the rectangle in the elevation and divide it into the same number of equal parts, viz. 12.

Assume the point P to move upwards and in anti-clockwise direction. When it has moved through 30° around the circle, it will have moved up by one division. To locate this position, draw a vertical line through the point 1 and a horizontal line through the point 1′, both intersecting at a point P_1, which will be on the helix.

Obtain other points in a similar manner and draw the helix through them. The portion of the curve from P_6 to

P_{12} will be on the back side of the cylinder and hence, it will not be visible.

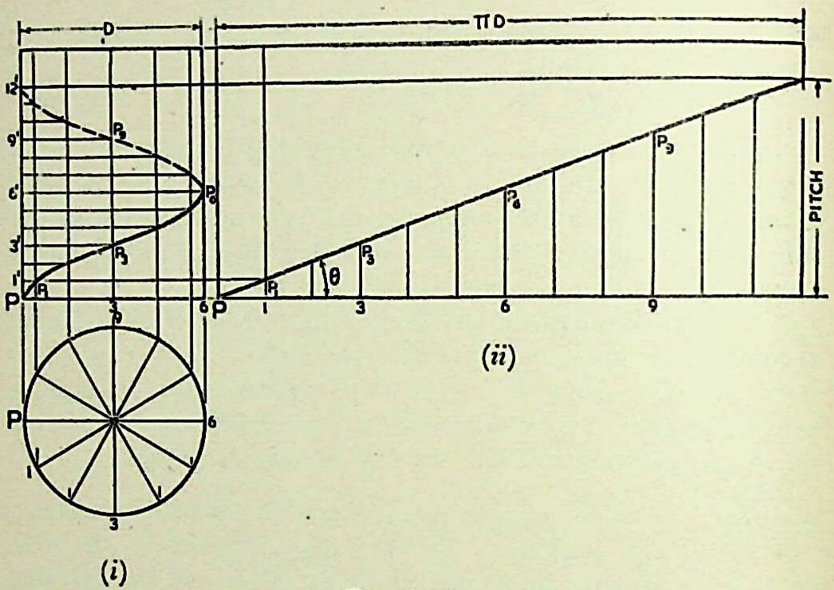


Fig. 14-1

Fig. 14-1(*ii*) shows the development of the helix. It is a straight line and is the hypotenuse of a right-angled triangle, having base equal to the circumference of the circle and the vertical side equal to the pitch of the helix. The angle θ, which it makes with the base, is called the *helix angle*.

Helical springs: In springs having wire of square cross-section, the outer two corners of the section may be assumed to be moving around the axis, on the surface of a cylinder having a diameter, equal to the outside diameter of the spring; the inner two corners of the section will move on the surface of a cylinder having a diameter, equal to the inside diameter of the spring. The pitch in case of each corner will be the same.

Problem 2:

Draw the projections of two complete turns of a spring of a wire of square section of 2 cm side. Outside diameter of the spring = 11 cm; pitch = 6 cm (fig. 14-2).

Instead of full circles, semi-circles of 11 cm and 7 cm diameters, for the outside and inside diameters of the spring, may be drawn in the plan. Divide each semi-circle into $\frac{12}{2}$, i.e. 6 equal parts.

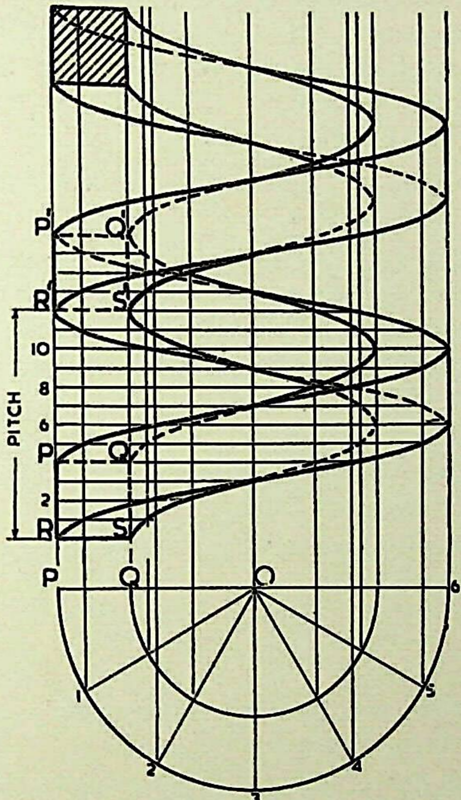


FIG. 14-2

Project up the division-points as lines in elevation. Let $PQSR$ be the square section of the wire. In one revolution, the point P will move to P' so that $PP' = 6$ cm. Similarly, RR', QQ' and SS' will each be equal to 6 cm. Divide these distances into 12 equal parts and plot the helices traced out by P and R on the outer surface and by Q and S on the inner surface, as shown. The outer helices will be parallel to each other. Similarly, the inner curves will also

be parallel to each other. Note carefully the visible and hidden parts of the curves.

Springs of wire of circular cross-section: When the wire is of circular cross-section, a helical curve for the centre of the cross-section is first traced out. A number of circles of diameter equal to that of the cross-section, are then drawn with a number of points on this curve as centres. Curves, tangent to these circles, will give the elevation of the spring.

Screw threads: These also are constructed on the principle of the helix.

In a screw thread, the *pitch* is defined as the distance from a point on a thread, to a corresponding point on the adjacent thread, measured parallel to the axis. The axial advance of a point on a thread, per revolution, is called the *lead* of the screw.

In single-threaded screws, which are most commonly used in practice, pitch is equal to lead. Therefore, pitch of the screw is equal to pitch of the helix.

Unless stated otherwise, screws are always assumed to be single-threaded.

Problem 3:

Project two complete turns of a square thread; outside diameter 12 cm; pitch 4·5 cm (fig. 14-3).

In a square thread, the thickness of the thread = the depth of the thread = $\frac{1}{2}$ pitch.

Hence, the section of the thread will be a square of 2·25 cm side and the diameter at the bottom of the thread will be 7·5 cm.

Project the threads in the same manner as the spring in problem 2. The screw differs from the spring in having a solid cylinder inside, which completely hides the back portions of the curves.

In double-threaded screws, two threads of the same size run parallel to each other. The axial advance per

revolution, viz. the lead, is made twice the lead of the single-threaded screw, the pitch of the thread being kept the same in both cases.

Hence, in double-threaded screws,

pitch of the helix = lead

= twice the pitch of the screw.

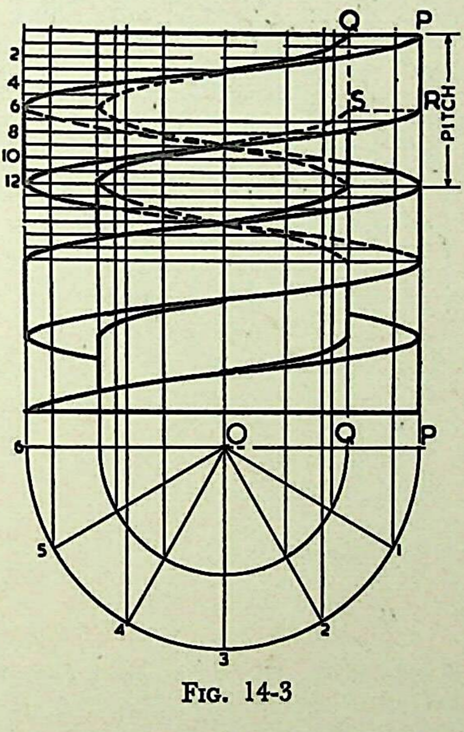


Fig. 14-3

Fig. 14-4 shows double-threaded screw, having the same cross-section and the same pitch of the screw, as in problem 3.

Helix upon a cone: This curve is traced out by a point which, while moving around the axis and on the surface of the cone, approaches the apex. The movement around the axis is uniform with its movement towards the apex, measured parallel to the axis. The pitch of the helix is measured parallel to the axis of the cone.

As the whole surface of the cone is visible in its plan, the helix will be fully seen in it.

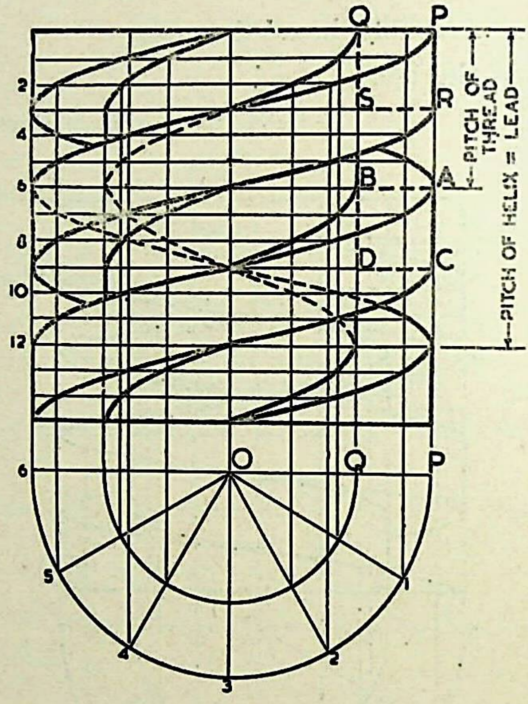


FIG. 14-4

Problem 4:

To draw a helix of one convolution upon a cone, diameter of base 7·5 cm (3"), height 10 cm (4") and pitch 7·5 cm (3"); also to develop the surface of the cone, showing the helix on it (fig. 14-5).

Draw the plan and elevation of the cone as shown. Divide the circle into twelve equal parts and join points 1, 2 etc. with *o*.

Project these points to the base-line in elevation and join them with *o′*.

Mark a point *A* on the axis at a distance of 7·5 cm (3") from the base. Draw a horizontal line through *A* to cut the generator *o′P* at *A′*. Divide *PA′* into twelve equal parts.

Let P be the starting point. When it has moved around through 30°, it will have moved up through one division to a point p_1' on the generator $o'1'$, obtained by drawing

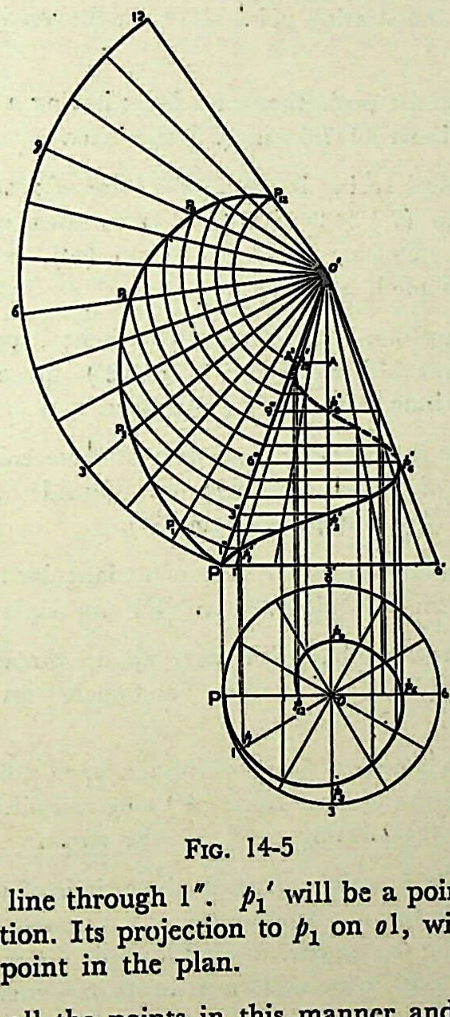


FIG. 14-5

a horizontal line through 1″. p_1' will be a point on the helix in the elevation. Its projection to p_1 on $o1$, will be the position of the point in the plan.

Obtain all the points in this manner and draw smooth curves through them in both the views.

Draw the devlopment of the surface of the cone and locate points p_1, p_2 etc. as explained in prolbem 9 of Chapter X11.

EXERCISES XIV

(1) Draw a helix of pitch equal to 5 cm (2″) upon a cylinder of 7·5 cm (3″) diameter and develop the surface of the cylinder. Assume the starting point to be on the vertical centre line in the plan.

(2) Draw the projections of a helix having a helix-angle of 30°, on a cylinder of 7·5 cm (3″) diameter.

(3) A spiral spring is made of a wire of rectangular cross-section 2·5 cm (1″) × 2 cm ($\frac{3}{4}$″). Draw two complete turns of the spring. Outside diameter 10 cm (4″), inside diameter 5 cm (2″) and pitch 5 cm (2″).

(4) Project one complete turn of a helical spring of outside diameter 7·5 cm (3″) and pitch 5 cm (2″), the cross-section of wire being a circle of 2 cm ($\frac{3}{4}$″) diameter.

(5) Draw the elevation and plan of three coils of a helical spring of steel wire 2 cm ($\frac{3}{4}$″) diameter. Outside diameter of the spring 10 cm (4″) and pitch 5 cm (2″).

(6) Project two complete turns of a triangular thread, outside diameter 12·5 cm (5″), pitch 2·5 cm (1″) and angle 60°.

(7) A screw having triple-start square thread has outside diameter 15 cm (6″), lead 12 cm ($4\frac{1}{2}$″) and pitch 4 cm ($1\frac{1}{2}$″). Draw its projections.

(8) Draw a helix of one convolution upon a cone, diameter of the base 7·5 cm (3″), axis 10 cm (4″) long and pitch 7·5 cm (3″). Take apex as the starting point for the curve.

(9) A point P, starting from the base-circle of a cone, reaches the apex, while moving around the axis through two complete turns. Assuming the movement of *P* towards the apex (measured parallel to the axis) to be uniform with its movement around the axis, draw the elevation, plan and development of the cone showing the path of *P* in each. Diameter of the base of the cone 7·5 cm (3″); axis 10 cm (4″) long.

CHAPTER **XV**

ISOMETRIC PROJECTION

Isometric projection is a type of pictorial projection in which the three dimensions of a solid are not only shown in one view, but their actual sizes can be measured directly from it.

If a cube is placed on one of its corners on the ground, with a solid diagonal perpendicular to the V.P., the elevation is the *Isometric Projection* of the cube. The step-by-step construction is shown in fig. 15-1 and explained in detail in problem 23 of Chapter XII.

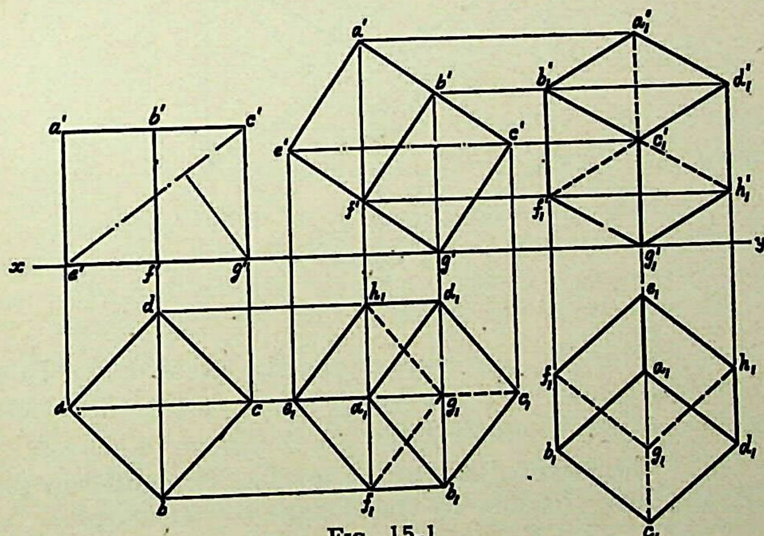


FIG. 15-1

Fig. 15-2 shows the elevation of the cube in the above position, with the corners named in capital letters. Its careful study will show that:

(a) All the faces of the cube are equally inclined to the V.P. and hence, they are seen as similar and equal rhombuses instead of squares.

(b) The three lines *CB*, *CD* and *CG* meeting at *C* and representing the three edges of the solid right-angle are also equally inclined to the V.P. and are, therefore, equally foreshortened. They make equal angles of 120° with each other. The line *CG* being vertical, the other two lines *CB* and *CD* make 30° angle each, with the horizontal.

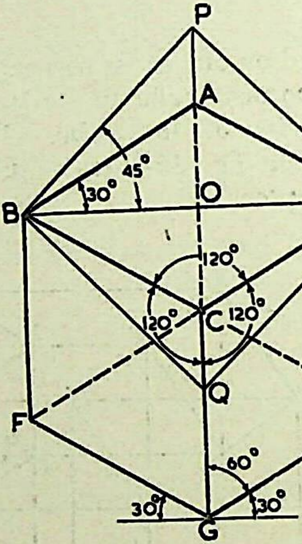


FIG. 15-2

(c) All other lines representing the edges of the cube are parallel to one or the other of the above three lines and are also equally foreshortened.

(d) The diagonal *BD* of the top face is parallel to the V.P. and hence, retains its true length.

Isometric axes, lines and planes: The three lines *CB*, *CD* and *CG* meeting at the point *C* and making 120° angles with each other are termed ***isometric axes***. The lines parallel to these axes are called ***isometric lines***. The planes representing the faces of the cube as well as other planes parallel to these planes are called ***isometric planes***.

Isometric scale: As all the edges of the cube are equally foreshortened, the square faces are seen as rhombuses. The rhombus $ABCD$ (fig. 15-2) shows the isometric projection of the top square face of the cube, in which BD is the true length of the diagonal.

Construct a square $BQDP$ around BD as a diagonal. Then BP shows the true length of BA.

In triangle ABO, $\dfrac{BA}{BO} = \dfrac{1}{\cos 30^\circ} = \dfrac{2}{\sqrt{3}}$

In triangle PBO, $\dfrac{BP}{BO} = \dfrac{1}{\cos 45^\circ} = \dfrac{\sqrt{2}}{1}$

$$\therefore \quad \frac{BA}{BP} = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = 0{\cdot}815.$$

$$\therefore \quad \text{The ratio, } \frac{\text{isometric length}}{\text{true length}} = \frac{BA}{BP} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$= 0{\cdot}815 \text{ or } \frac{9}{11} \text{ (approx).}$$

Thus, the isometric projection is reduced in the ratio $\sqrt{2} : \sqrt{3}$, i.e. the isometric lengths are 0·815 of the true lengths.

Therefore, while drawing an isometric projection, it is necessary to convert true lengths into isometric lengths for measuring and marking the sizes. This is conveniently done by constructing and making use of an isometric scale, as shown below.

(1) Draw a horizontal line BD of any length (fig. 15-3). At the end B, draw lines BA and BP, such that $\angle DBA = 30^\circ$ and $\angle DBP = 45^\circ$. Mark divisions of true length on the line BP and from each division-point, draw verticals to BD, meeting BA at respective points. The divisions thus obtained on BA give lengths on isometric scale.

(2) The same scale may also be drawn with divisions of natural scale on a horizontal line AB (fig. 15-4). At the ends A and B, draw lines AC and BC making 15° and 45° angles with AB respectively, and intersecting each other at C.

From division-points of true lengths on *AB*, draw lines parallel to *BC* and meeting *AC* at respective points. The divisions along *AC* give dimensions to isometric scale.

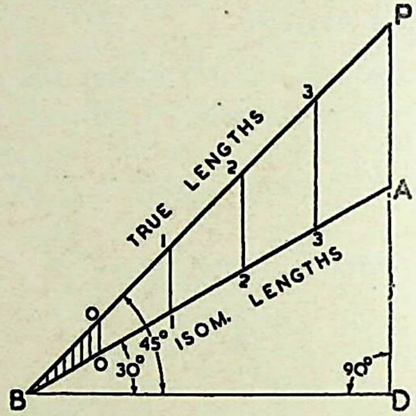


FIG. 15-3

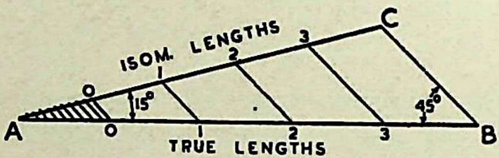


FIG. 15-4

The lines *BD* and *AC* (fig. 15-2) represent equal diagonals of a square face of the cube, but are not equally shortened in isometric projection. *BD* retains its true length, while *AC* is considerably shortened. Thus, it is seen that lines which are not parallel to the isometric axes, are not reduced according to any fixed ratio. Such lines are called non-isometric lines.' The measurements should, therefore, be made on *isometric axes and isometric lines only*. The non-isometric lines are drawn by locating positions of their ends on isometric planes and then joining them.

If the foreshortening of the isometric lines in an isometric projection is disregarded and instead, the true lengths are marked, the view obtained will be exactly of the same shape but larger in proportion than that obtained by the use of the

isometric scale, as shown in fig. 15-5. Due to the ease in construction and the advantage of measuring the dimensions directly from the drawing, it has become a general practice to use the natural scale instead of the isometric scale.

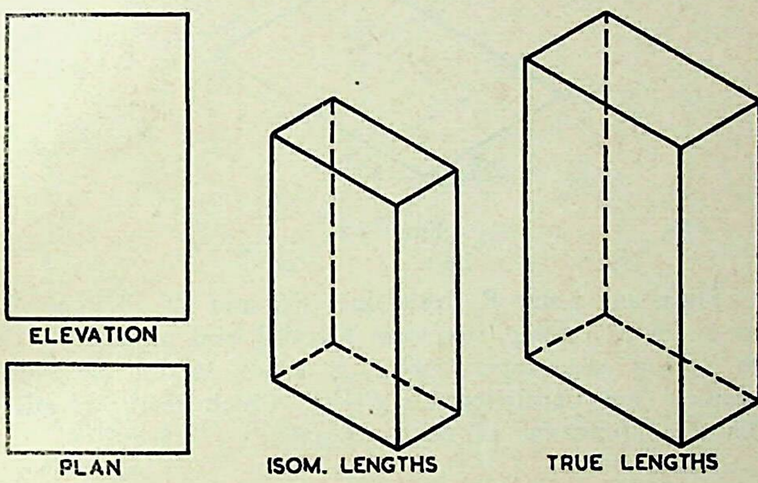


FIG. 15-5

Referring again to fig. 15-2, the axes *BC* and *CD* represent the sides of a right angle in horizontal position. Each of them together with the vertical axis *CG*, represents the right angle in vertical position. Hence, in isometric projection of any rectangular solid resting on a face in the H.P., each horizontal face will have its sides parallel to the two sloping axes; each vertical face will have its vertical sides parallel to the vertical axis and the other sides parallel to one of the sloping axes. In other words, the vertical edges are shown by vertical lines, while the horizontal edges are represented by lines, making 30° angles with the horizontal. These lines are very conveniently drawn with the T-square and a 30°-60° set-square.

Problem 1:

To draw the isometric projections (using isometric scale) of (i) a rectangle of 7·5 cm (3") and 5 cm (2") sides, its plane being horizontal and (ii) a regular pentagon of 2·5 cm (1") side, its plane being vertical and one of its sides horizontal.

(i) As the plane of the rectangle is horizontal, its sides will be parallel to the two sloping axes (fig. 15-6).

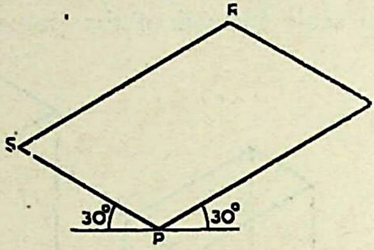


FIG. 15-6

From any point P, draw lines PQ and PS, 7·5 cm (3″) and 5 cm (2″) long (isometric lengths) and parallel to the two sloping axes respectively, i.e. at 30° to the horizontal. Complete the parallelogram $PQRS$, which is the required isometric projection of the rectangle.

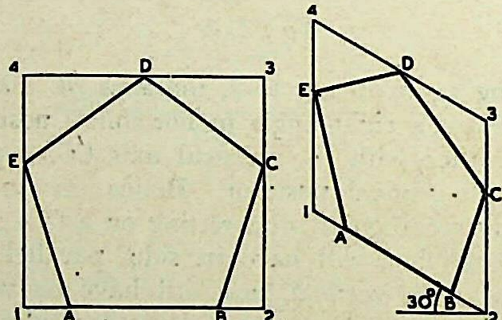


FIG. 15-7

(ii) Draw the given pentagon with one side horizontal (fig. 15-7). As the pentagon has no right angle, enclose it in an oblong. The surface of the pentagon is to be vertical; hence, draw the isometric projection of the oblong (using isometric scale) with one side parallel to the vertical axis and the other parallel to one of the sloping axes. Mark points for the corners of the pentagon on corresponding sides, taking the distances from the corners of the oblong and converting them into isometric lengths. Join these points as shown.

Problem 2 (fig. 15-8):

To draw the isometric projection of a square prism when its axis is (i) vertical (ii) horizontal.

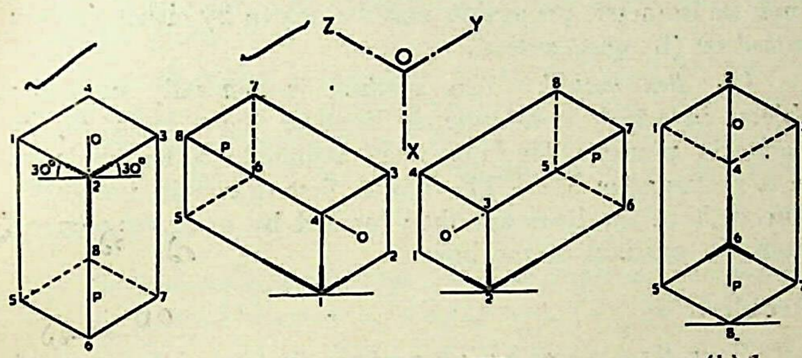


FIG. 15-8

(i) When the axis of the prism is vertical, its ends will be horizontal.

Draw the isometric projection of the top end [fig. 15-8(*i*)]. It will be a rhombus with sides inclined at 30° to the horizontal. The length of the prism must be shown in the third direction, i.e. parallel to the vertical axis. Hence, from the corners of the rhombus, draw vertical lines, each equal to the length of the axis and complete the longer faces. Edges 5-8, 4-8 and 7-8 are hidden.

In fig. 15-8(*iv*) the bottom end of the prism is shown fully visible and therefore, the top is hidden.

(ii) When the axis is horizontal, the ends will be vertical. These can be drawn in two ways as shown in fig. 15-8(*ii*) and fig. 15-8(*iii*). The length is shown in the third direction which is different in each case.

While dealing with isometric projection of solids the following important points should be carefully noted:

(1) At every visible corner of the solid, three lines must converge. Of these, either all the three or any two may be visible. The hidden lines may not be shown but it is advisable to check up every corner so that no visible line is left out.

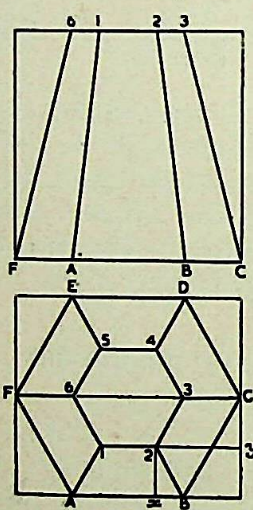
(2) Two outlines (for visible edges) will never cross each other.

When an object contains a number of non-isometric lines, its isometric projection may be drawn by either (a) *box method* or (b) *offset method*.

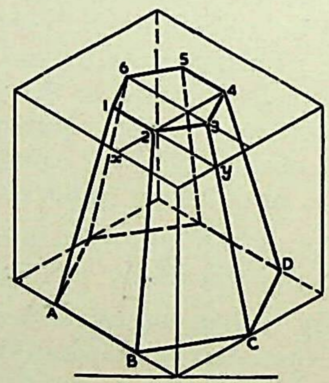
(a) *Box method:* This method is generally used for objects in which non-isometric lines or their ends lie in isometric planes. The object is assumed to be enclosed in a rectangular box. The box is first drawn in isometric. The ends of the lines are then located by measuring on or from the outlines of the box.

Problem 3:

Draw the isometric projection of the frustum of the hexagonal pyramid shown in fig. 15-9(i).



(*i*)



(*ii*)

FIG. 15-9

Enclose the elevation and the plan in rectangles.

Draw the isometric view of the rectangular box [fig. 15-9(*ii*)]. Locate the six points of the base of the frustum on the sides of the bottom of the box. The upper six points on the top surface of the box are located by drawing isometric

lines, e.g. $x2$ and $y2$ intersecting at a point 2. Join the corners and complete the isometric view as shown.

(b) *Offset method:* This method is adopted for objects in which neither non-isometric lines nor their ends lie in isometric planes.

Perpendiculars are dropped from each end of the line to a horizontal or a vertical reference plane. The points at which the perpendiculars meet the plane, are located by drawing co-ordinates or offsets to the edges of the plane.

Problem 4:

To draw the isometric projection of a pentagonal pyramid, the plan and elevation of which are given in fig. 15-10(i).

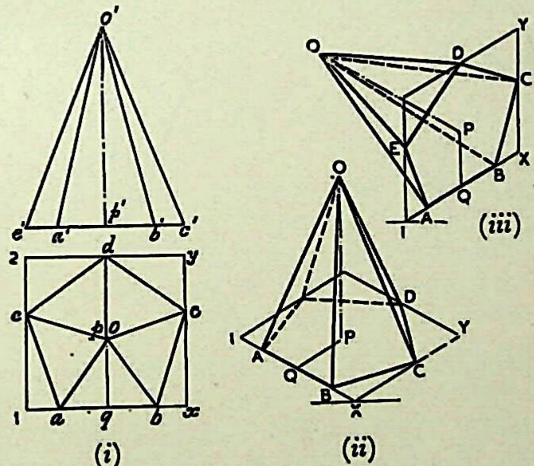


FIG. 15-10

Enclose the base (in the plan), in an oblong. Draw an offset pq on the line $1x$.

Draw the isometric view of the oblong and locate the corners of the base in it [fig. 15-10(*ii*)]. Mark a point Q such that $QX = qx$. From Q, draw a line QP equal to qp and parallel to XY. At P, draw a vertical OP equal to $o'p'$. Join O with the corners of the base, thus completing the isometric view of the pyramid.

Fig. 15-10(*iii*) shows the isometric projection of the same pyramid with its axis horizontal.

Problem 5:

To draw the isometric projection of the truncated triangular pyramid shown in fig. 15-11(i).

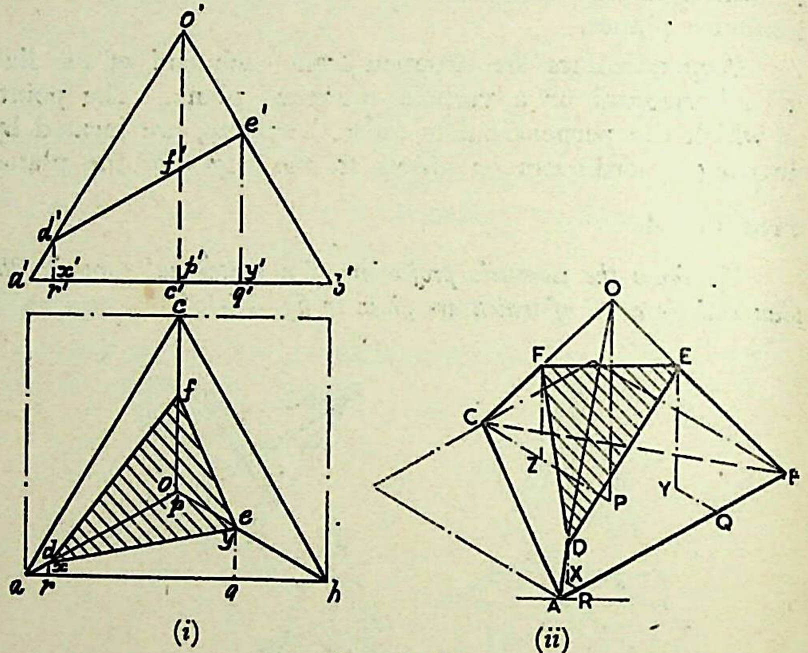
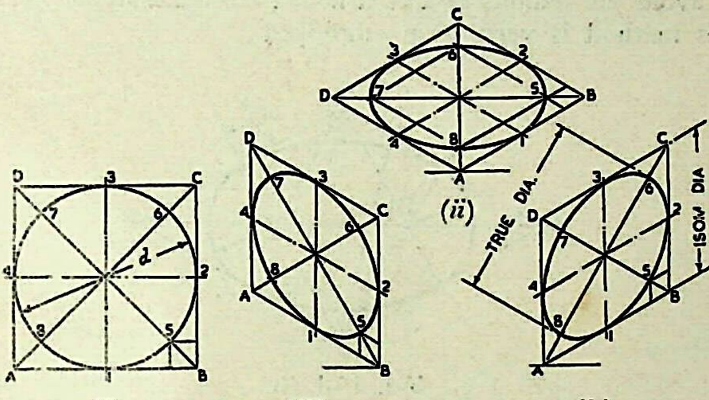


FIG. 15-11

A perpendicular $d'x'$ is drawn from the corner d' to the base, which is enclosed in an oblong and taken as a reference plane. From x which coincides with d in the plan, an offset xr is drawn to the edge ab. This offset is then transferred to the isometric view of the base [fig. 15-11(*ii*)] and the point D is located by drawing a vertical DX equal to $d'x'$. The corners E and F and the apex O are also located in the same manner.

Isometric projection of a circle (fig. 15-12): A circle may be drawn in isometric projection by enclosing it in a square and locating a number of points on it by the offset method. The mid-points of the sides of the square give the positions of four points. Four more points viz. intersection points of the diagonals with the circle may be located by offset method.

In fig. 15-12(*i*) the circle is enclosed in a square. Eight points are marked on it, as stated above.



(*i*) (*iii*) (*iv*)

Fig. 15-12

The isometric view of the square in horizontal position is shown in fig. 15-12(*ii*). A point 5 is shown located by the offset method. The other three points on the diagonals may be found in the same way or by drawing through 5, lines parallel to the sides of the rhombus. A smooth curve drawn through these points and the mid-points of the sides, will be the isometric projection of the circle. *It is an ellipse, the major axis of which is equal to the true diameter of the circle.* The length of the side of the rhombus is equal to the isometric diameter of the circle.

Isometric projections of the circle with its plane vertical are shown in figs. 15-12(*iii*) and (*iv*).

APPROXIMATE METHOD OF CONSTRUCTING THE ELLIPSE:

Draw the rhombus *ABCD* [fig. 15-13(*i*)]. At the mid-points of its sides, erect perpendiculars which will intersect (i) at points *P* and *Q* on the diagonal *BD* and (ii) at corners *A* and *C*. With *A* and *C* as centres and radius equal to *AE*, and with *P* and *Q* as centres and radius equal to *PE*, draw arcs to form the ellipse.

The figure obtained by the approximate method is not a true ellipse and differs considerably in size and shape from the real ellipse. But owing to the ease in construction and to avoid the tedious task of drawing freehand smooth curves, this method is very often employed.

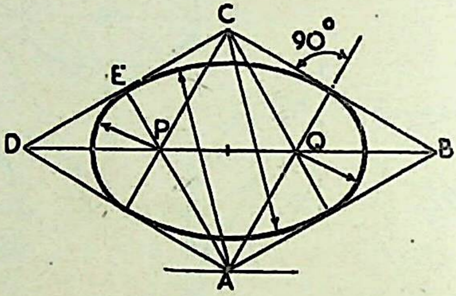


FIG. 15-13(*i*)

Isometric projections of quadrants are very conveniently drawn by the approximate method as shown in fig. 15-13(*ii*).

From the point of intersection of the isometric lines, points are marked on these lines at a distance equal to *R*, the radius of the arc. At these points, perpendiculars to the

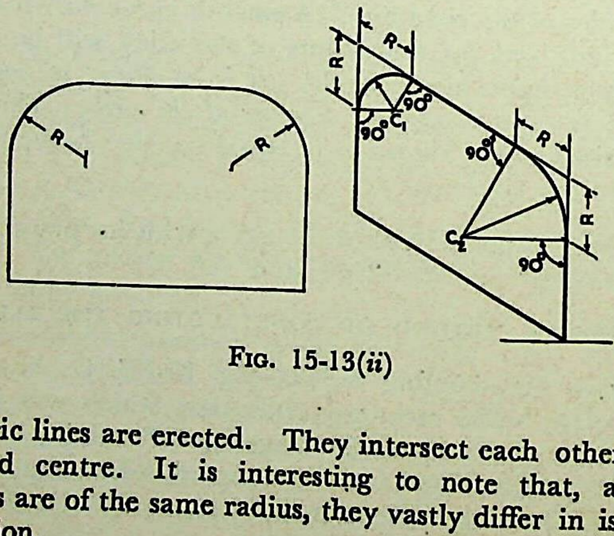


FIG. 15-13(*ii*)

isometric lines are erected. They intersect each other at the required centre. It is interesting to note that, although the arcs are of the same radius, they vastly differ in isometric projection.

Isometric projection of a cylinder (fig. 15-14): This is drawn by first constructing the ellipses for the ends and then joining them by common tangents. The chain-line figure within the cylinder, shows the isometric projection of a cone, having the diameter of the base and the length of the axis same as those of the cylinder.

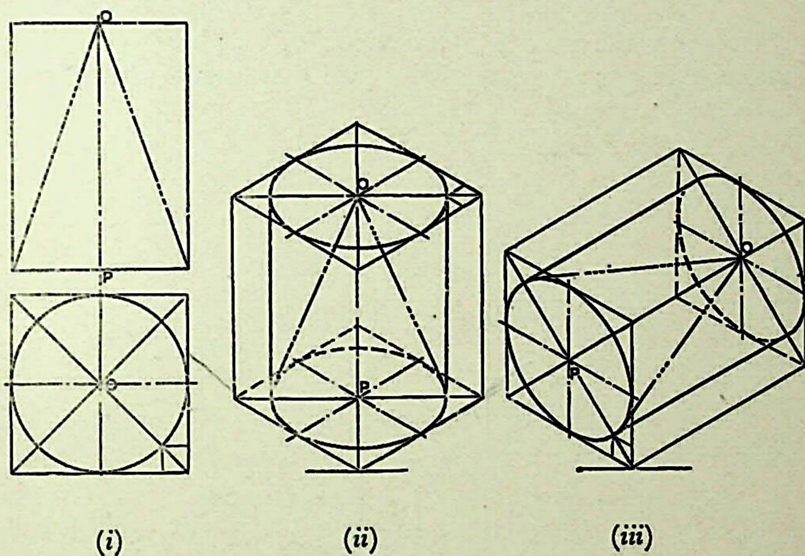


FIG. 15-14

Isometric projection of a sphere: The elevation and plan of a sphere resting in the H.P. is shown in fig. 15-15(*i*). *C* is its centre, *D* is the diameter and *P* is the point of its contact with the H.P.

Assume a vertical section through the centre of the sphere. Its shape will be a circle of diameter *D*. The isometric projection of this circle is shown in fig. 15-15(*ii*) by ellipses 1 and 2, drawn in two different vertical positions around the same centre *C*. The length of the major axis in each case is equal to *D*. The distance of the point *P* from the centre *C* is equal to the isometric radius of the sphere.

Again, assume a horizontal section through the centre of the sphere. The isometric projection of this circle is shown

by an ellipse 3, drawn in a horizontal position around the same centre *C*. In this case also, the distance of the outermost points on the ellipse from the centre *C* is equal to $\frac{1}{2}D$.

Thus, it can be seen that in an isometric projection, the distances of all the points on the surface of a sphere from its centre, are equal to the radius of the sphere.

Hence, the isometric projection of a sphere is a circle whose diameter is equal to the true diameter of the sphere.

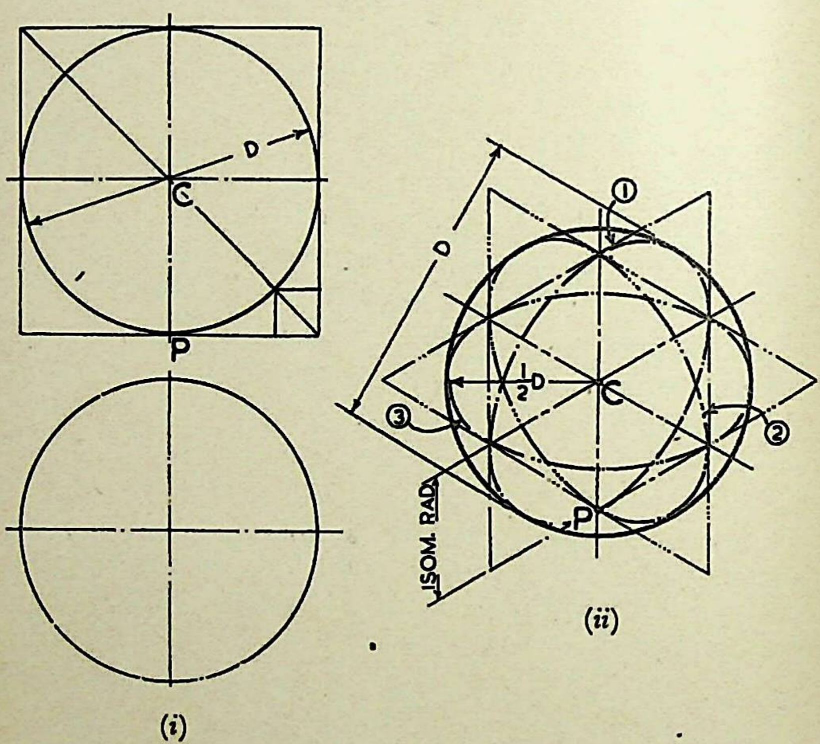


FIG. 15-15

Also, the distance of the centre of the sphere from its point of contact with the ground, is equal to the isometric radius of the sphere, viz. *CP*.

It is, therefore, of the utmost importance to note that, *isometric scale should invariably be used, while drawing isometric*

projections of solids in conjunction with spheres or having spherical parts.

Problem 6:

To draw the isometric projection of a sphere resting centrally on the top-end of a square prism (fig. 15-16).

Draw the isometric projection of the square prism and locate the centre *P* of its top surface. Draw a vertical at

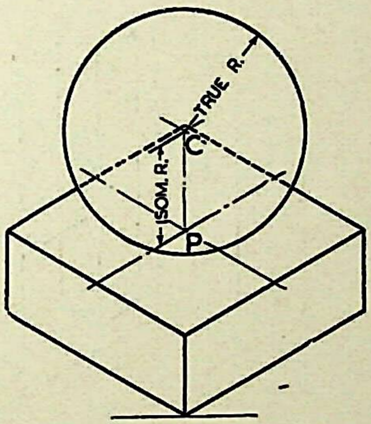


Fig. 15-16

P and mark a point *C* on it, such that, *PC* = *the isometric radius* of the sphere. With *C* as centre and *radius equal to the true radius of the sphere*, draw a circle, which will be the isometric projection of the sphere.

Typical problems:

The solutions given in the following typical problems are mostly self-explanatory. Explanations are however given where deemed necessary. Construction lines are left intact for guidance. Unless otherwise stated, all dimensions are given in millimetres.

Problem 7:

The elevation of a board fitted with a letter H and mounted on a wooden post is given in fig. 15-17. Draw its isometric projection, assuming the thickness of the board and of the letter to be equal to 3 cm. Scale, half full size. (All dimensions are given in centimetres.)

See fig. 15-18.

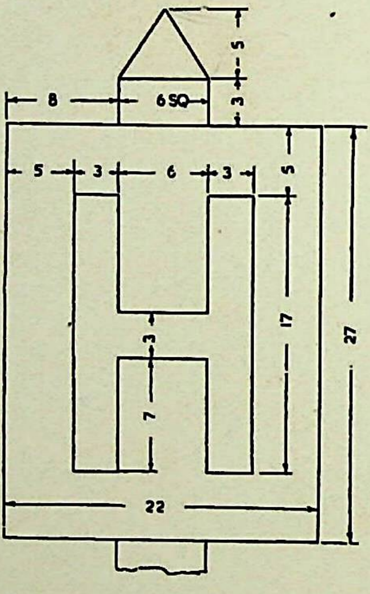


FIG. 15-17

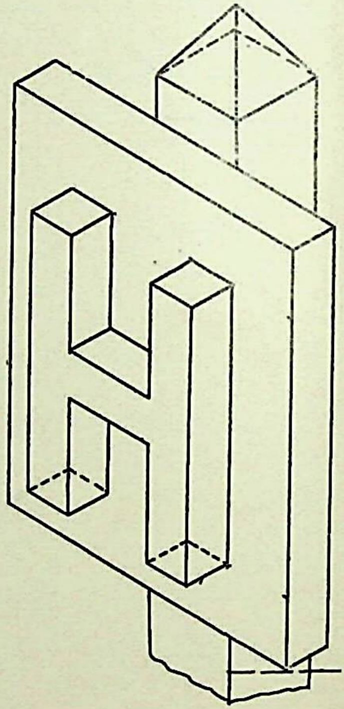


FIG. 15-18

Problem 8:

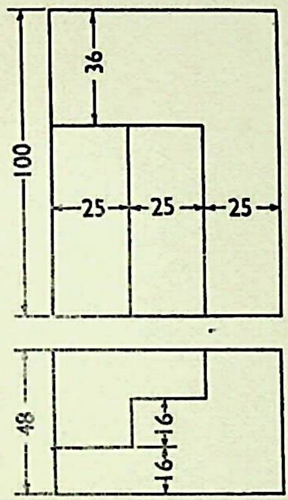
Draw the isometric projection of the model of steps, two views of which are shown in fig. 15-19.

See fig. 15-20.

Problem 9:

Two pieces of wood joined together by a dovetail joint are shown in two views in fig. 15-21. Draw the isometric projection of the two pieces separated but in a position ready for fitting.

See fig. 15-22.



(Third-angle projection)

Fig. 15-19

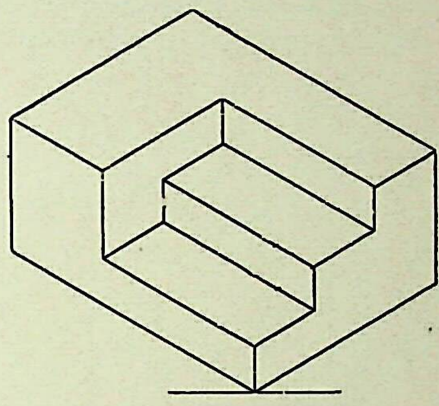


Fig. 15-20

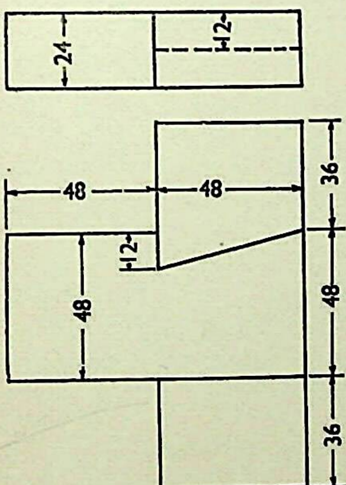


Fig. 15-21

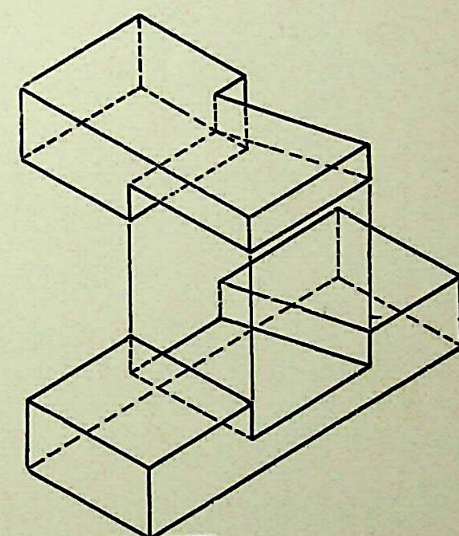


Fig. 15-22

Problem 10:

The outside dimensions of a box made of 4 cm ($1\frac{1}{2}''$) thick planks are 90 cm × 60 cm × 60 cm height ($3' \times 2' \times 2'$ height). The depth of the lid on the outside is 12 cm ($4\frac{1}{2}''$). Draw the isometric projection of the box when the lid is (a) 90° open and (b) 120° open.

Draw the orthographic view of the box with the lid in required positions as shown in fig. 15-23.

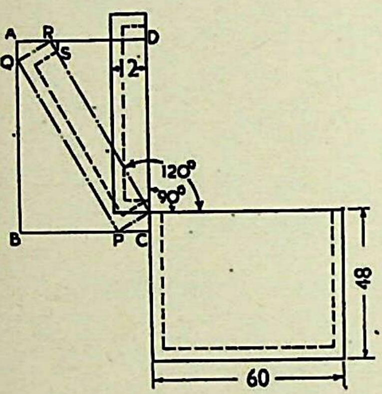


Fig. 15-23

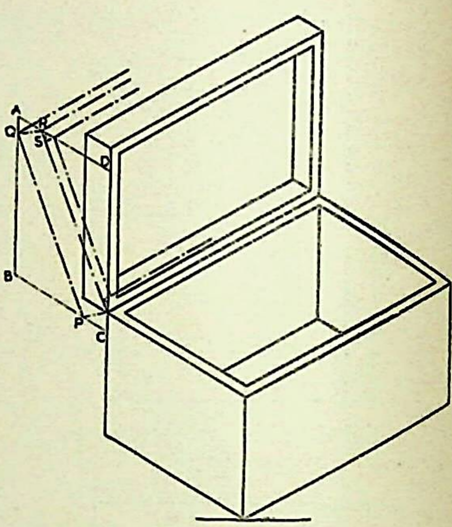


Fig. 15-24

(a) This position is simple to draw in isometric projection. Care must, however, be taken to deduct the thickness of the wood for the bottom and the top, when showing the lines for the inside of the box and the lid (fig. 15-24).

(b) In this position, points *P*, *Q*, *R* etc. for the lid are located by enclosing the lid in the oblong and transferring the same on the isometric view as shown. The view is left incomplete to avoid congestion.

Problem 11:

Two views of a cast-iron block are shown in fig. 15-25. Draw its isometric projection.

See fig. 15-26.

The slope of the lines for the grooves on the outer surface on all the four sides is different and is obtained as shown

by construction lines. The depth is measured along vertical lines.

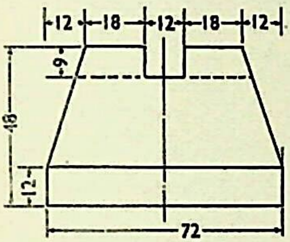


FIG. 15-25

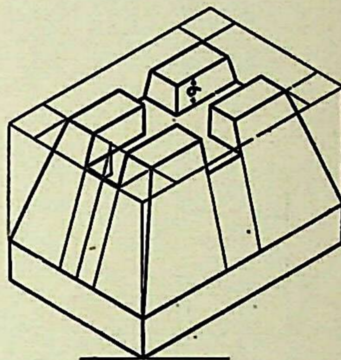


FIG. 15-26

Problem 12:

Draw the isometric projection of the casting shown in two views in fig. 15-27.

See fig. 15-28.

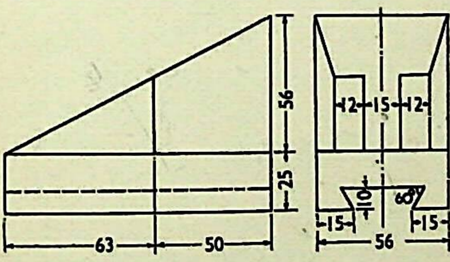


FIG. 15-27

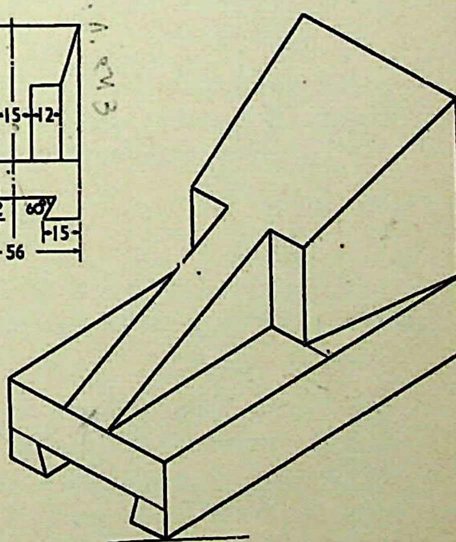
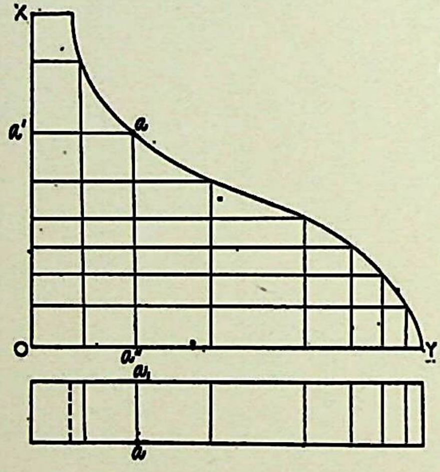


FIG. 15-28

Problem 13:

Draw the isometric projection of the simple moulding shown in fig. 15-29.

See fig. 15-30.



(Third-angle projection)
FIG. 15-29

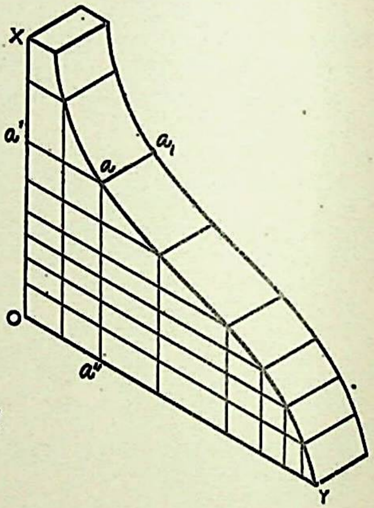


FIG. 15-30

The points on the curve are located by co-ordinate method. The parallel curve is obtained by drawing lines in the third direction and equal to the thickness of the moulding.

Problem 14:

The elevation of three solids placed one above the other, with their axes in a straight line is shown in fig. 15-31. Draw the isometric projection of the arrangement.

See fig. 15-32.

In this problem isometric lengths must be taken for all dimensions except for the radius of the circle for the sphere.

The centre C of the sphere is at a distance equal to the isometric radius from the centre P of the top face of the cone frustum. *The circle for the sphere is drawn with the true radius.* The ellipse for the section of the sphere is drawn within

the rhombus constructed around the point Q on the axis.

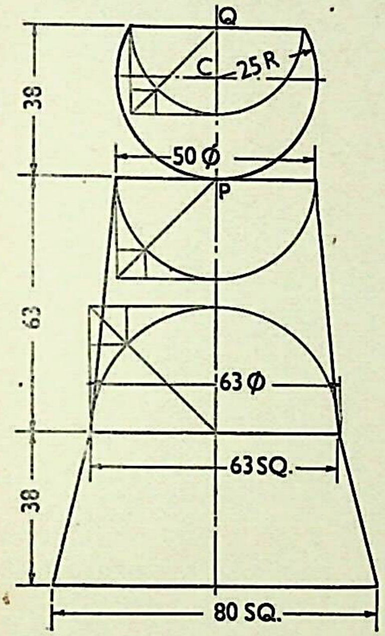


Fig. 15-31

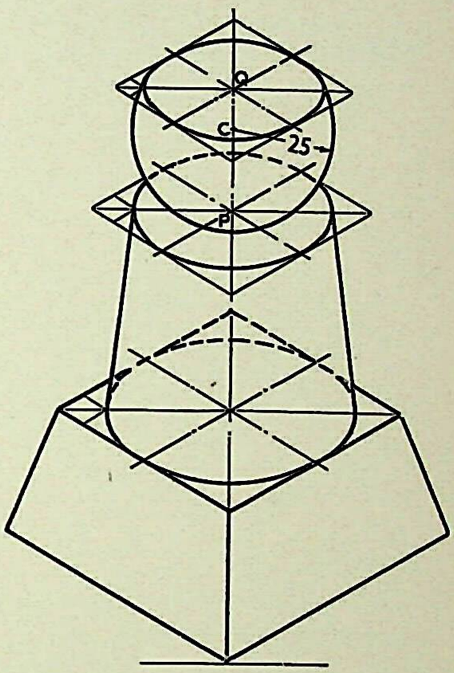


Fig. 15-32

Problem 15:

Draw the isometric projection of the clamping piece shown in fig. 15-33.

See fig. 15-34.

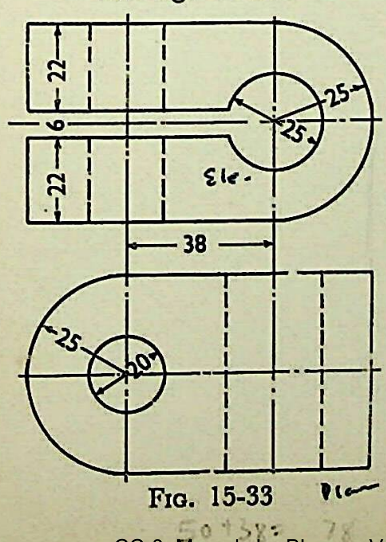


Fig. 15-33 Plan

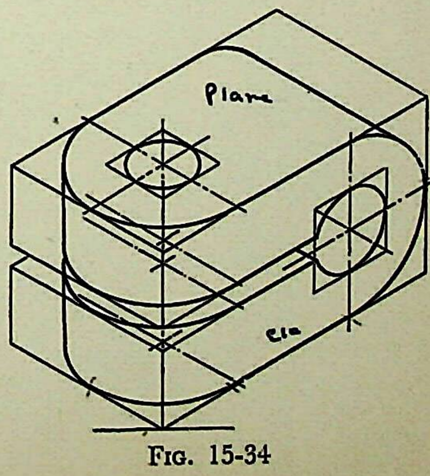


Fig. 15-34

Problem 16:

Draw the isometric projection of a hexagonal nut for a 24 mm diameter bolt, assuming approximate dimensions. The threads may be neglected but chamfer must be shown.

See fig. 15-35.

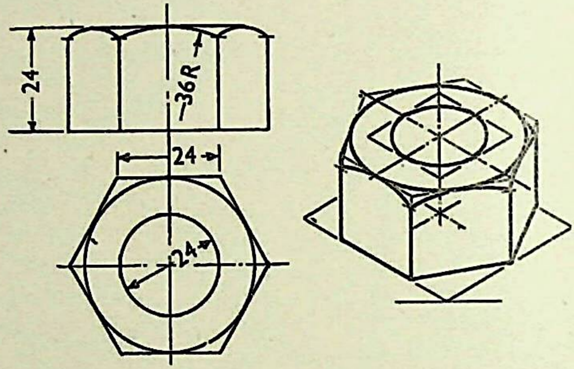


Fig. 15-35

Problem 17:

Draw the isometric projection of a square-headed bolt 24 mm diameter and 70 mm long, with a square neck 18 mm thick and a head, 40 mm square and 18 mm thick.

See fig. 15-36. (The orthographic views are drawn according to third-angle projection method).

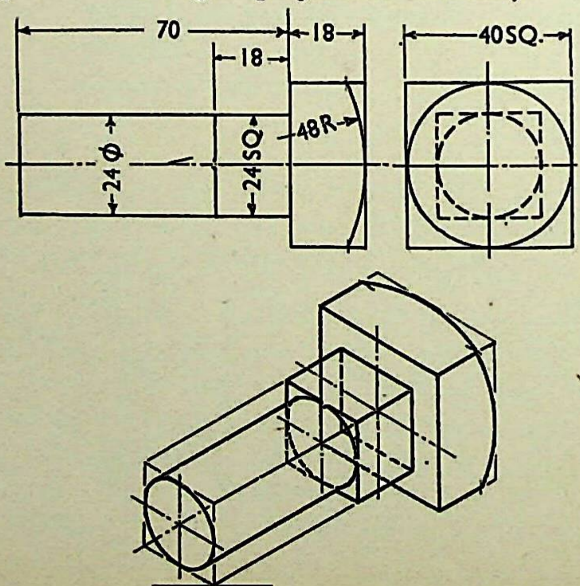


Fig. 15-36

Problem 18:

Draw the isometric projection of a paper-weight with spherical knob shown in fig. 15-37.

See fig. 15-38.

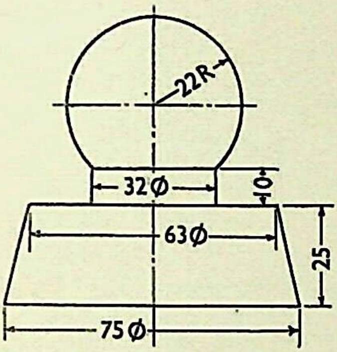


Fig. 15-37

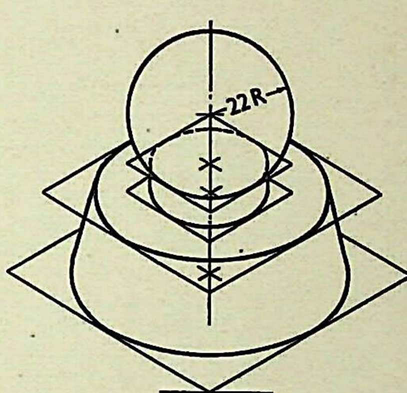


Fig. 15-38

Problem 19:

Draw the isometric projection of the casting shown in fig. 15-39.

See fig. 15-40.

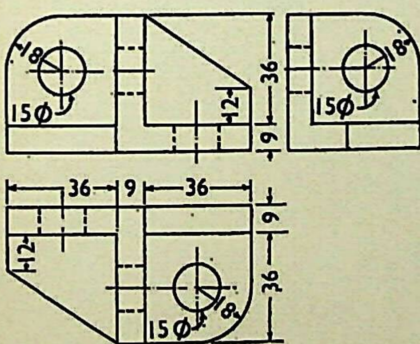


Fig. 15-39

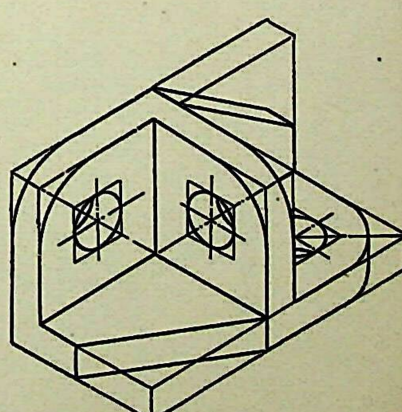
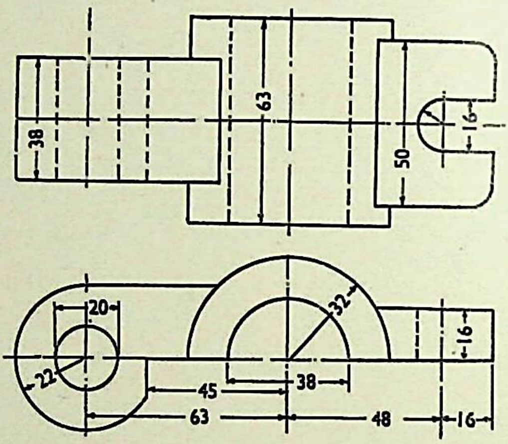


Fig. 15-40

Problem 20:

The elevation and plan of a casting are shown in fig. 15-41. Draw its isometric projection.

See fig. 15-42.



(Third-angle projection)

FIG. 15-41

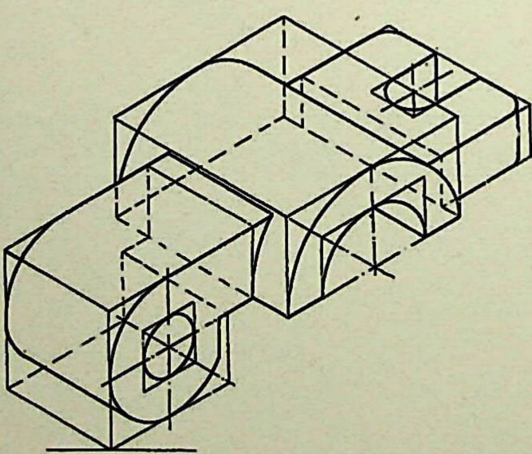


FIG. 15-42

Problem 21:

Draw the isometric projection of the bracket shown in two views in fig. 15-43.

See fig. 15-44.

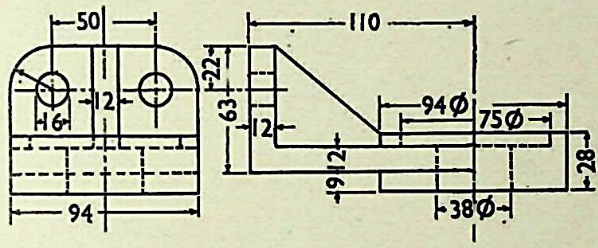


Fig. 15-43

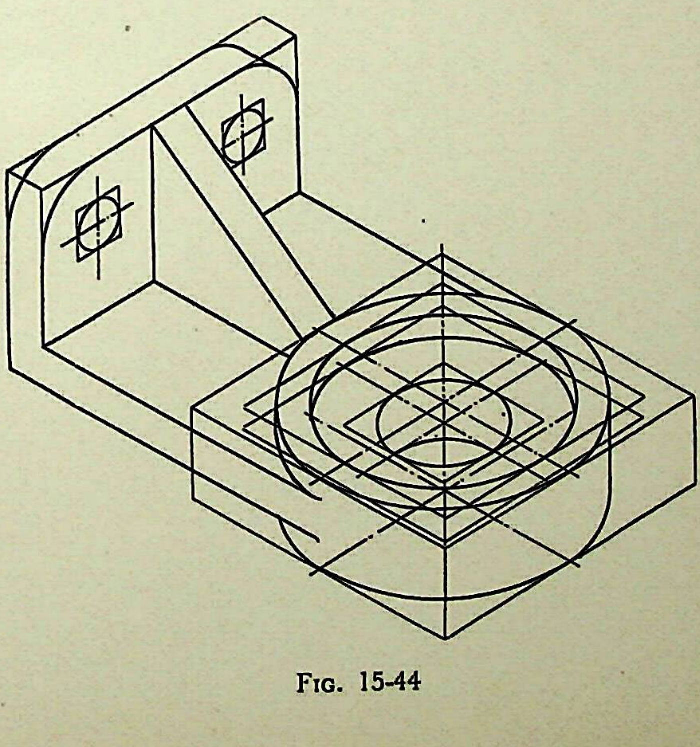


Fig. 15-44

Problem 22:

Draw the isometric projection of the machine-handle shown in fig. 15-45.

See fig. 15-46.

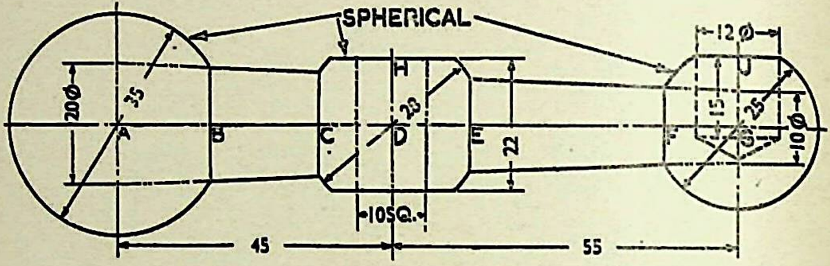


FIG. 15-45

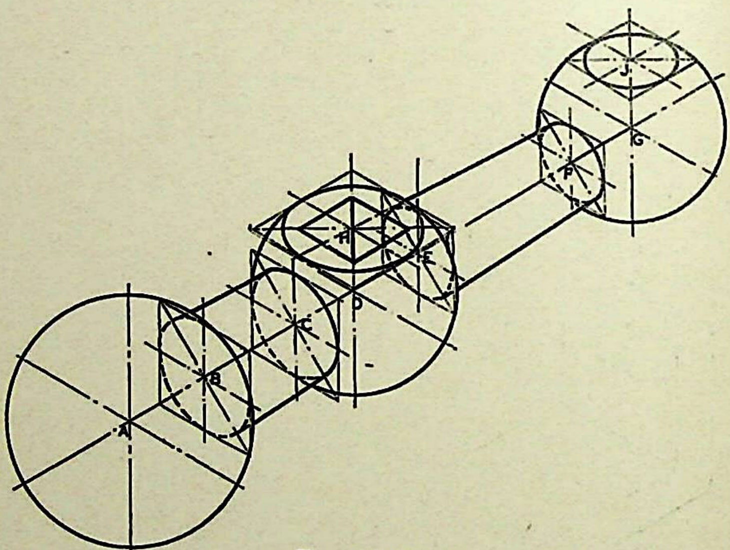


FIG. 15-46

All measurements must be in isometric lengths except those for the diameters of spherical parts.

Draw an axis and mark on it the positions of points *A*, *B* etc. At points *B*, *C*, *E* and *F*, draw ellipses for circular sections of the conical handle. Ellipses at *B* and *E* will be completely hidden. With points *A*, *D* and *G* as centres, draw circles for the spheres, with their respective true radii.

Mark points J and H on the vertical axes through G and D respectively and draw ellipses for the respective sections of the spheres. Around H, draw a rhombus for the square hole. The dotted lines for the depth of the holes are omitted.

Problem 23:

The elevation of a stool having a square top and four legs, is shown in fig. 15-47. Draw its isometric projection. Unit=1 cm.

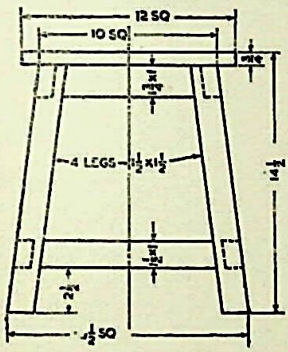


FIG. 15-47

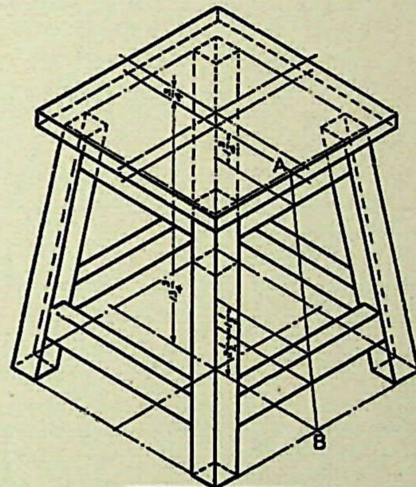


FIG. 15-48

The legs lie along the slant edges of a frustum of a square pyramid (fig. 15-48).

Positions of the connecting horizontal strips between the legs at the top and at the bottom are determined, by marking the heights along the axis and then drawing isometric lines upto the line AB, which shows the slope of the face of the frustum.

EXERCISES XV

Assuming unit length to be equal to 1 cm or ½", draw the isometric views of objects shown in figs. 15-49 to 15-61.

Note: Isometric scale must be used for objects Nos. 4, 5 and 15 in fig. 15-50.

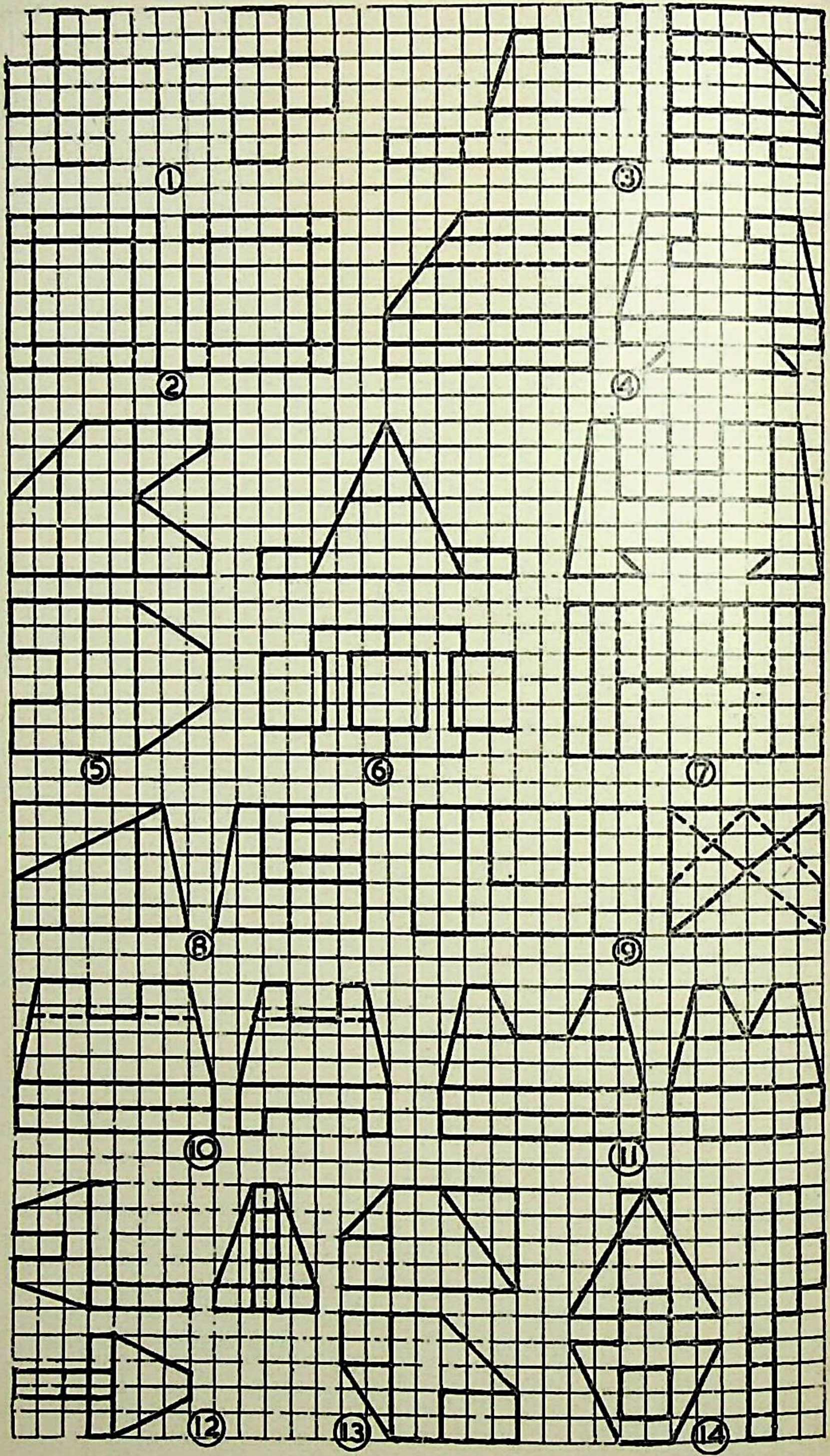


FIG. 15-49

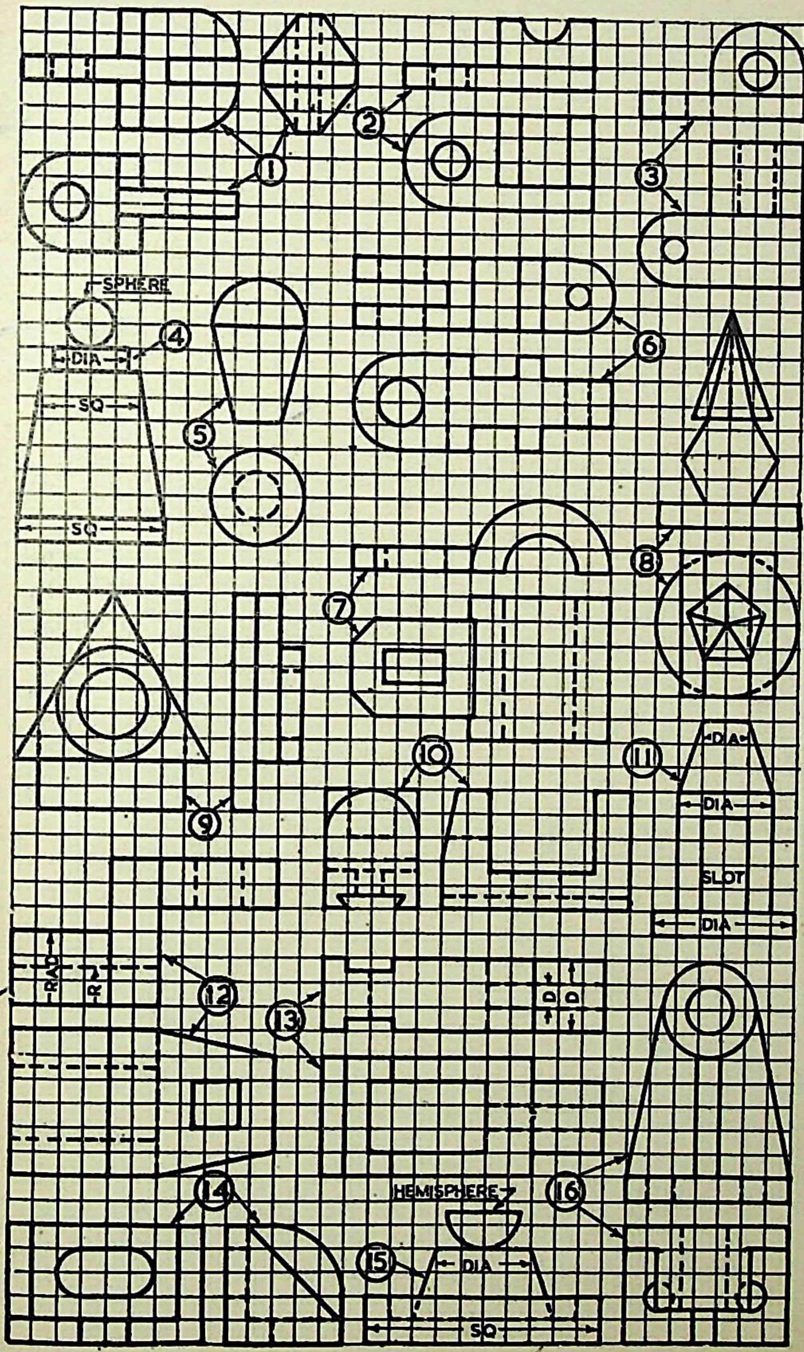

1
2
3
SPHERE
4
DIA
SQ
SQ
5
6
7
8
9
10
11
DIA
DIA
SLOT
DIA
RAD
R
12
13
D
D
14
HEMISPHERE
15
DIA
SQ
16

Fig. 15-50

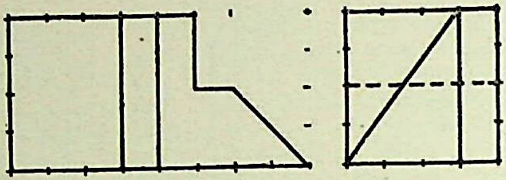


FIG. 15-51

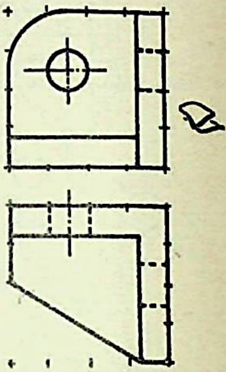
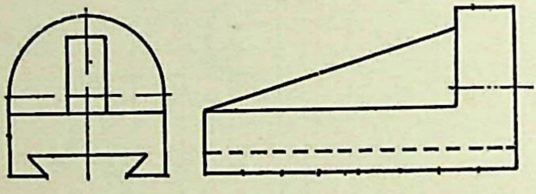


FIG. 15-55



(Third-angle projection)
FIG. 15-52

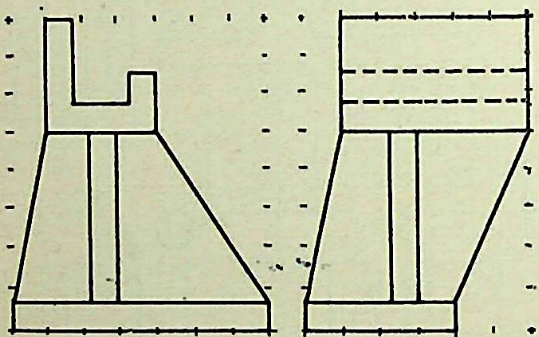


FIG. 15-53

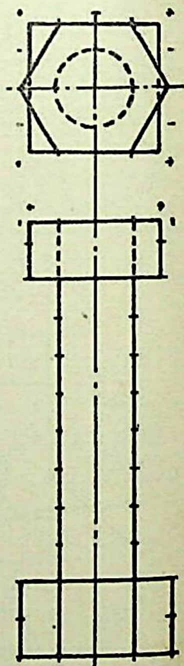


FIG. 15-56

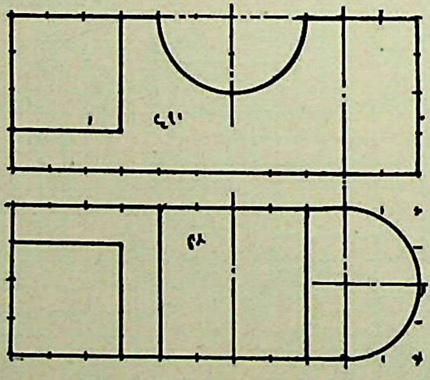
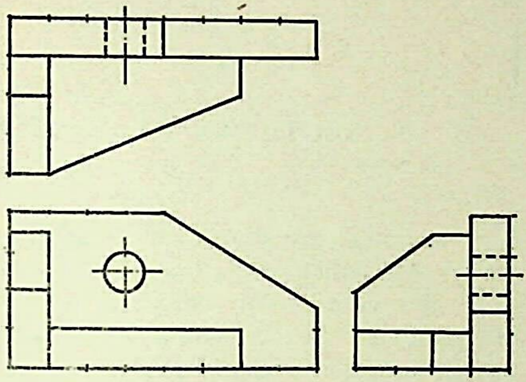


FIG. 15-54



(Third-angle projection)
FIG. 15-57

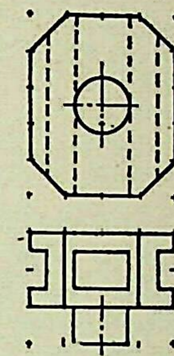


FIG. 15-58

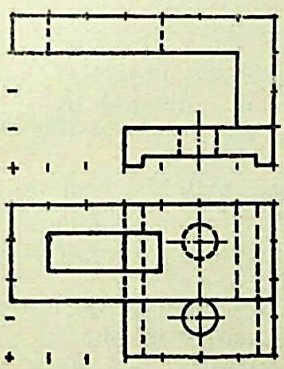


FIG. 15-59

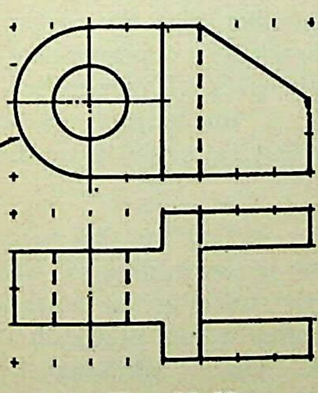


FIG. 15-60

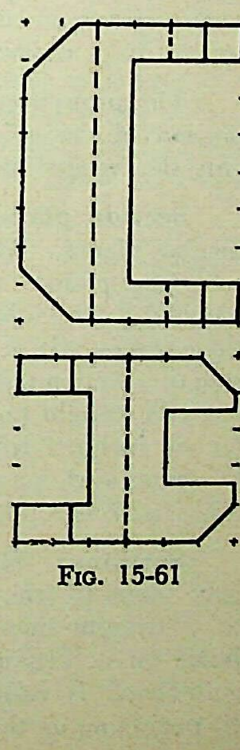


FIG. 15-61

CHAPTER **XVI**

SECTIONS OF SOLIDS

Invisible features of an object are shown by dotted lines in their projected views. But when such features are too many, these lines make the views more complicated and difficult to interpret. In such cases, it is customary to imagine the object as being cut through or *sectioned* by planes. The part of the object between the cutting plane and the observer is assumed to be removed and the view is then shown *in section.*

The imaginary plane is called a *section plane* or a *cutting plane.*

The surface produced by cutting the object by the section plane is called the *section.* It is indicated by thin section lines uniformly spaced and inclined at 45°.

The projection of the section along with the remaining portion of the object is called a *sectional view.* Sometimes, only the word section is also used to denote a sectional view.

Section planes: Section planes are generally perpendicular planes. They may be perpendicular to one of the reference planes and either perpendicular, parallel or inclined to the other plane. They are usually described by their traces. It is important to remember that the projection of a section plane, on the plane to which it is perpendicular, is a straight line. This line will be parallel, perpendicular or inclined to *xy*, depending upon the section plane being parallel, perpendicular or inclined respectively to the other reference plane.

Sections: The projection of the section on the reference plane to which the section plane is perpendicular, will be a straight line coinciding with the trace of the section plane on it. Its projection on the other plane to which it is inclined is called *apparent section.* This is obtained by (i) projecting on the other plane, the points at which the trace

of the section plane intersects the edges of the solid and (ii) joining these points by lines in proper sequence.

True shape of a section: The projection of the section on a plane parallel to the section plane will show the true shape of the section. Thus, when the section plane is parallel to the H.P., the true shape of the section will be seen in sectional plan. When it is parallel to the V.P., the true shape will be visible in the sectional elevation. But when the section plane is inclined, the section has to be projected on an auxiliary plane, parallel to the section plane, to obtain its true shape. When the section plane is perpendicular to both the reference planes, the sectional end elevation will show the true shape of the section.

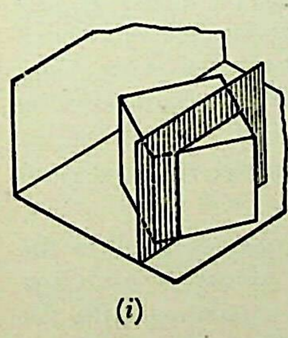
Sections of different solids are explained in stages by means of typical problems.

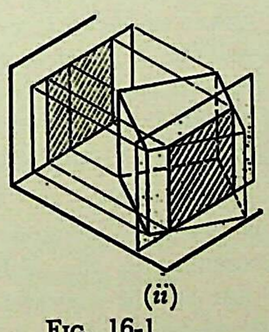
SECTIONS OF PRISMS:

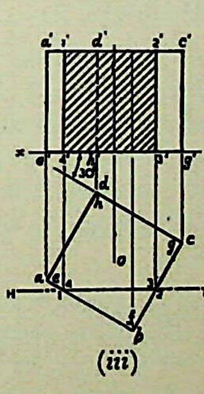
(a) Section plane parallel to the V.P.

Problem 1:

A cube of 4 cm ($1\frac{1}{2}''$) long edges is resting on the ground on one of its faces with a vertical face inclined at 30° to the V.P. It is cut by a section plane parallel to the V.P. and 1 cm ($\frac{7}{16}''$) away from the axis and further away from the V.P. Draw its sectional elevation and plan (fig. 16-1).


(*i*)


(*ii*)



(*iii*)

FIG. 16-1

The section plane parallel to the V.P. and passing through the cube is shown in fig. 16-1(*i*).

When the cut-portion is removed, the section will be visible.

In fig. 16-1(*ii*), the section plane is assumed to be transparent and the cube is shown with the cut-portion removed. It can be seen that four edges of the cube are cut and hence, the section is a figure having four sides.

Draw the projections of the cube in the required position as shown in fig. 16-1(*iii*) and as explained below.

As the section plane is parallel to the V.P., it is perpendicular to the H.P ; hence, the section will be seen as a line in plan, coinciding with the H.T. of the section plane.

Draw a line H.T. in the plan (to represent the section plane) parallel to *xy* and 1 cm ($\frac{7}{16}''$) from *o*. Name the points at which the edges are cut, viz. *ab* at 1, *bc* at 2, *gf* at 3 and *fe* at 4.

Project these points on the corresponding edges in the elevation and join them in proper order.

As the section plane is parallel to the V.P., figure 1′2′3′4′ in the elevation, shows the true shape of the section.

Show the views by dark but thin lines, leaving the lines for the cut-portion fainter. Draw section lines in the rectangle for the section.

(b) Section plane parallel to the H.P.

Problem 2:

A triangular prism, base 3 cm ($1\frac{1}{4}''$) side and axis 5 cm (2″) long, is lying on the ground on one of its rectangular faces with its axis inclined at 30° to the V.P. It is cut by a horizontal section plane, at a distance of 1·2 cm ($\frac{1}{2}''$) above the ground. Draw its elevation and sectional plan (fig. 16-2).

Draw the projections of the prism in the required position.

As the section plane is horizontal, i.e. parallel to the H.P., it is perpendicular to the V.P. Hence, the section will be seen as a line in the elevation, coinciding with the V.T. of the section plane.

Therefore, draw a line V.T. in the elevation, to represent the section plane, parallel to *xy* and 1·2 cm ($\frac{1}{2}''$) above it.

Name in correct sequence, points at which the edges are cut, viz. $a'b'$ at $1'$, $a'c'$ at $2'$, $d'f'$ at $3'$ and $d'e'$ at $4'$. Project these points on the corresponding lines in the plan and complete the sectional plan by joining them in proper order.

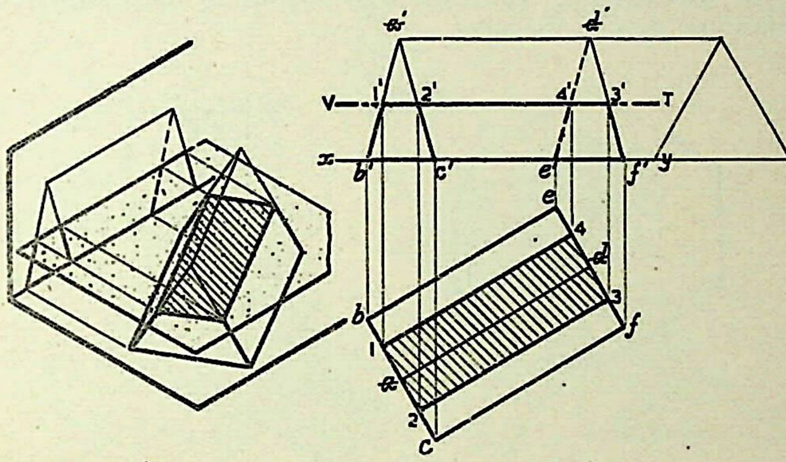


FIG. 16-2

As the section plane is parallel to the H.P., the figure 1 2 3 4 (in the plan) is the true shape of the section.

(c) Section plane perpendicular to the H.P. and inclined to the V.P.

Problem 3:

A cube in the same position as in problem 1, is cut by a section plane, inclined at 60° to the V.P. and perpendicular to the H.P., so that the face which makes 60° angle with the V.P. is cut in two equal halves. Draw the sectional elevation, plan and true shape of the section (fig. 16-3).

The section will be seen as a line in the plan, coinciding with the H.T. of the section plane.

Draw the elevation and the plan of the cube. Draw a line H.T. in the plan, inclined at 60° to *xy* and cutting the line *ad* (or *bc*) at its mid-point.

Name the corners at which the four edges are cut and project them in the elevation. As the section plane is inclined to the V.P., the elevation of the section viz. 1′2′3′4′ does not reveal its true shape. Only the vertical lines show their true lengths, while the true lengths of the horizontal lines are seen in the plan.

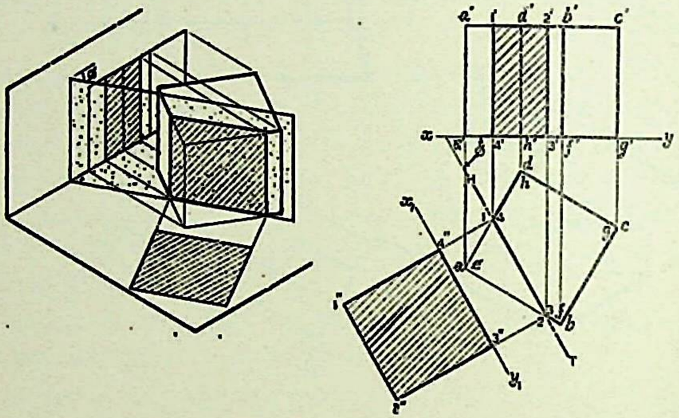


FIG. 16-3

The true shape of the section will be seen when it is projected on an auxiliary vertical plane, parallel to the section plane.

Therefore, draw a new ground line x_1y_1 parallel to the H.T. and project the section on it. The distances of the points from x_1y_1 should be taken equal to their corresponding distances from xy in the elevation. Thus, 4″ and 3″ will be on x_1y_1; 1″4″ and 2″3″ will be equal to 1′4′ and 2′3′ respectively. Complete the rectangle 1″2″3″4″ which is the true shape of the section and draw section lines in it.

(d) Section plane perpendicular to the V.P. and inclined to the H.P.

Problem 4:

A cube in the same position as in problem 1, is cut by a section plane, perpendicular to the V.P., inclind at 45° to the H.P. and

passing through the top end of the axis. (*i*) *Draw its elevation, sectional plan and true shape of the section.* (*ii*) *Project another plan on an auxiliary plane, parallel to the section plane* (fig. 16-4).

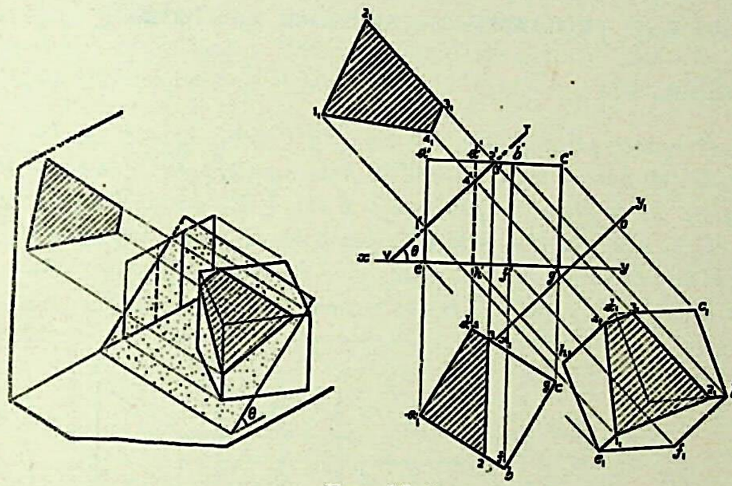


FIG. 16-4

The section will be seen as a line in the elevation.

Draw a line V.T. in the elevation, inclined at 45° to xy and passing through the top end of the axis. It cuts four edges, viz. $a'e'$ at $1'$, $a'b'$ at $2'$, $c'd'$ at $3'$ and $d'h'$ at $4'$. Project the plan of the section, viz. the figure 1 2 3 4. It does not show the true shape of the section, as the section plane is inclined to the H.P.

To determine the true shape, an auxiliary plan should be projected on an A.I.P. parallel to the section plane. Assuming the new ground line for the A.I.P. to coincide with the V.T., the true shape may be projected as shown by the quadrilateral $1_1 2_1 3_1 4_1$. The distances of all the points from the V.T. should be taken equal to their corresponding distances from xy in the plan, e.g. $1_1 1' = e'1$, $4_1 4' = h'4$ etc.

To project an auxiliary plan of the cube, draw $x_1 y_1$ parallel to V.T. The whole cube may first be projected and the points for the section may then be projected on corresponding edges. Join these points in correct sequence.

Draw section lines in the cut-surface, in the views where it is seen. Keep the lines for the removed edges, thin and fainter.

ADDITIONAL PROBLEMS ON SECTIONS OF PRISMS:

Problem 5:

A square prism, base 4·5 cm ($1\frac{3}{4}''$) side, axis 8 cm ($3\frac{3}{16}''$) long, has its base in the H.P. and its faces equally inclined to the V.P. It is cut by a plane, perpendicular to the V.P., inclined at 60° to the H.P. and passing through a point on the axis 5·5 cm ($2\frac{3}{16}''$) above the H.P. Draw its elevation, sectional plan and another plan on an A.I.P. parallel to the section plane (fig. 16-5).

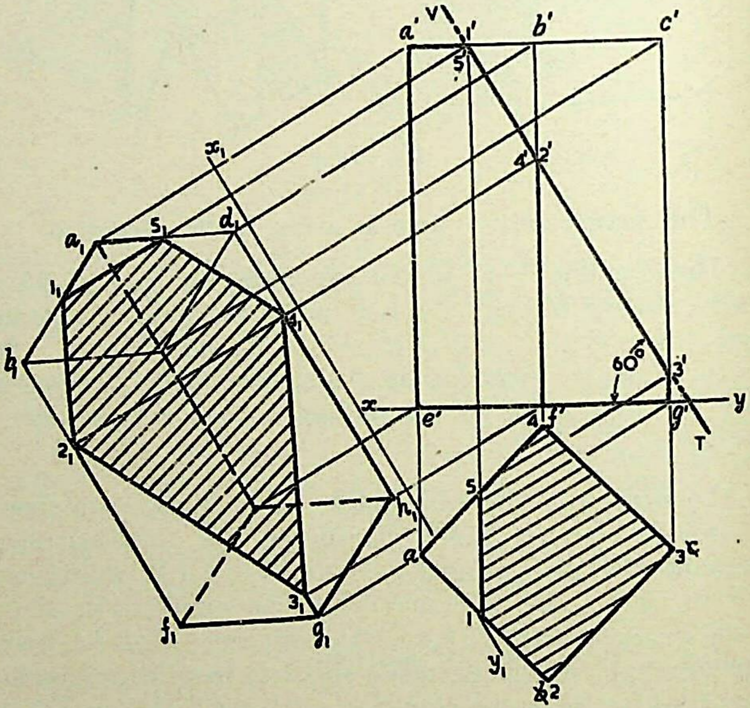


Fig. 16-5

The problem is similar to problem 4 and needs no further explanation. The true shape of the section is seen in the auxiliary plan.

Problem 6:

A hexagonal prism has a face in the H.P. and the axis parallel to the V.P. It is cut by a vertical plane, the H.T. of which makes an angle of 45° with xy, and which cuts the axis at a point 2 cm ($\frac{3}{4}''$) from one of its ends. Draw its sectional elevation and the true shape of the section. Side of base 2·5 cm (1″) long, height 6·5 cm ($2\frac{1}{2}''$).

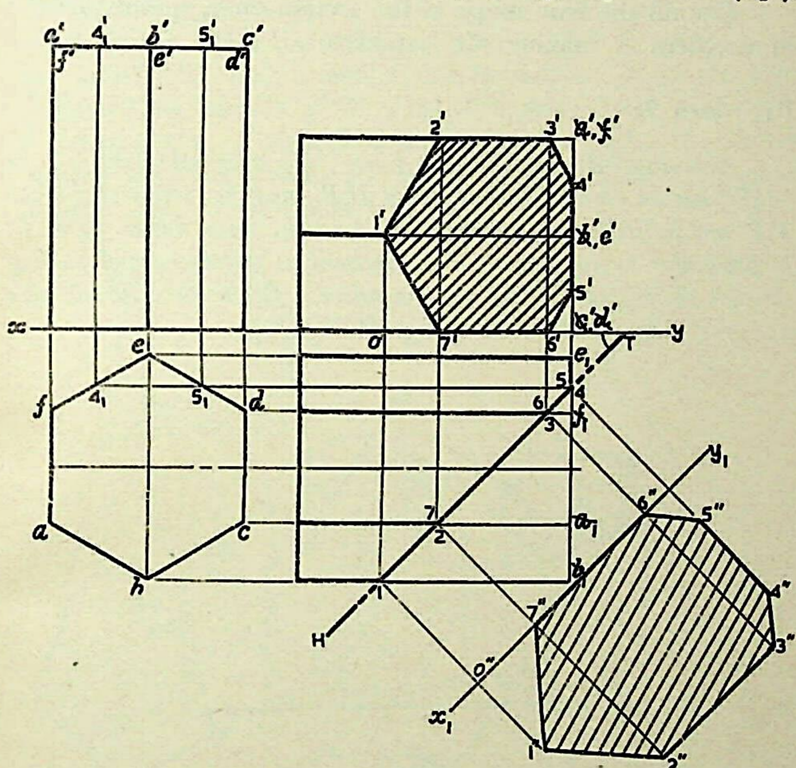


FIG. 16-6

Draw the elevation and plan of the prism and show the H.T. of the section plane in the plan (fig. 16-6).

Name in proper sequence, the points at which the lines are cut. Project them on the corresponding lines in the elevation. The positions of points 4 and 5 cannot be located directly. Hence, project them on the first plan to 4_1 on *ef* and 5_1 on *ed*. From this plan, obtain their positions $4_1'$ and

$5_1'$ on the corresponding lines in the first elevation. As the two elevations are identical, these points can now be transferred to the second elevation by making $b'4'$ equal to $b'4_1'$ and $b'5'$ equal to $b'5_1'$. $4'$ and $5'$ are the projections of points 4 and 5 respectively. Complete the sectional elevation as shown.

Obtain the true shape of the section on x_1y_1 as explained in problem 3, making $o''1''$ equal to $o'1'$, etc.

Problem 7:

A pentagonal prism, base 2·8 cm ($1\frac{1}{8}''$) side and height 6·5 cm ($2\frac{1}{2}''$), has an edge of its base in the H.P., and the axis parallel to the V.P. and inclined at 60° to the H.P. A section plane, having its H.T. perpendicular to xy, and the V.T. inclined at 60° to xy and passing through the highest corner, cuts the prism. Draw the sectional plan and true shape of the section (fig. 16-7).

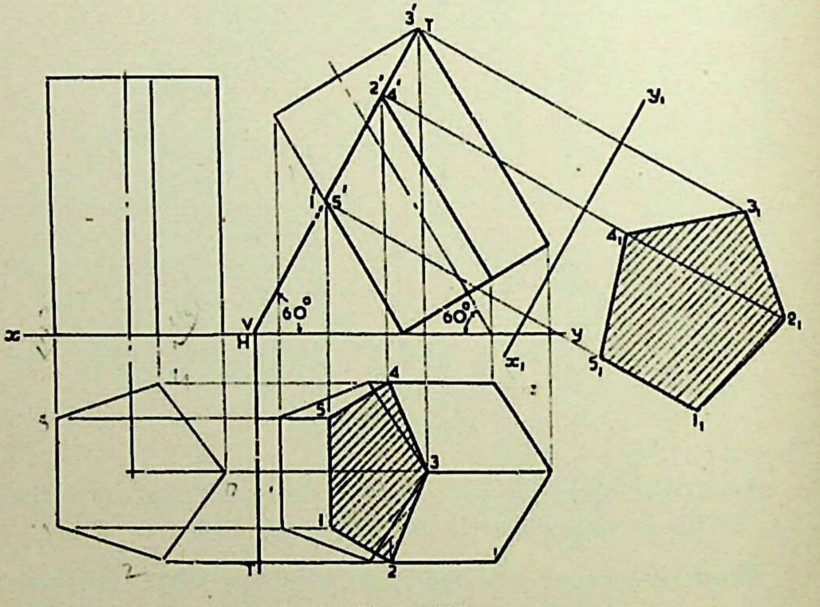


FIG. 16-7

Draw the projections of the prism in the required position. Draw the line V.T. passing through the highest

corner 3′ and inclined at 60° to *xy*. A perpendicular to *xy* through *V*, will be the H.T. of the section plane.

Project the sectional plan and the true shape of the section, as shown in the figure.

Problem 8:

A hollow square prism, base 4 cm ($1\frac{5}{8}''$) side (outside), height 6·5 cm ($2\frac{1}{2}''$) and thickness 0·8 cm ($\frac{5}{16}''$) is resting on its base in the H.P. with a vertical face inclined at 30° to the V.P. A section plane, inclined at 30° to the H.P., perpendicular to the V.P. and passing through the axis at a point 1·2 cm ($\frac{1}{2}''$) from its top end, cuts the prism. Draw its sectional plan, sectional end elevation and true shape of the section (fig. 16-8).

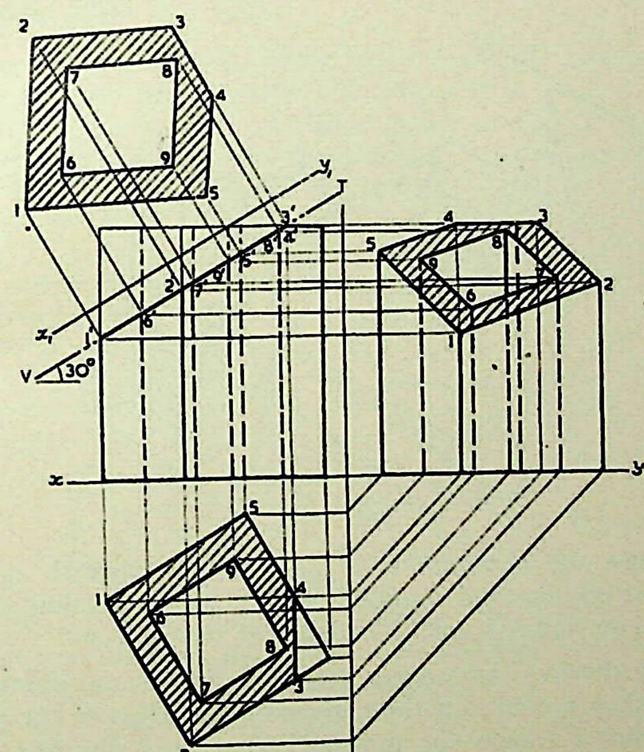


FIG. 16-8

Draw the projections of the prism in the given position, showing the hidden edges by dotted lines. Draw a line

V.T. for the cutting plane and mark points at which the inside and outside edges are cut. Project the sectional plan, true shape of the section and the sectional end view as shown.

SECTIONS OF PYRAMIDS:

(a) Section plane parallel to the base of the pyramid.

Problem 9:

A pentagonal pyramid, base 2·8 cm ($1\frac{1}{8}''$) side, axis 6·5 cm ($2\frac{1}{2}''$) long, has its base in the H.P. and an edge of the base parallel to the V.P. A horizontal section plane cuts it at a distance of 2·5 cm (1") above the base. Draw its elevation and sectional plan (fig. 16-9).

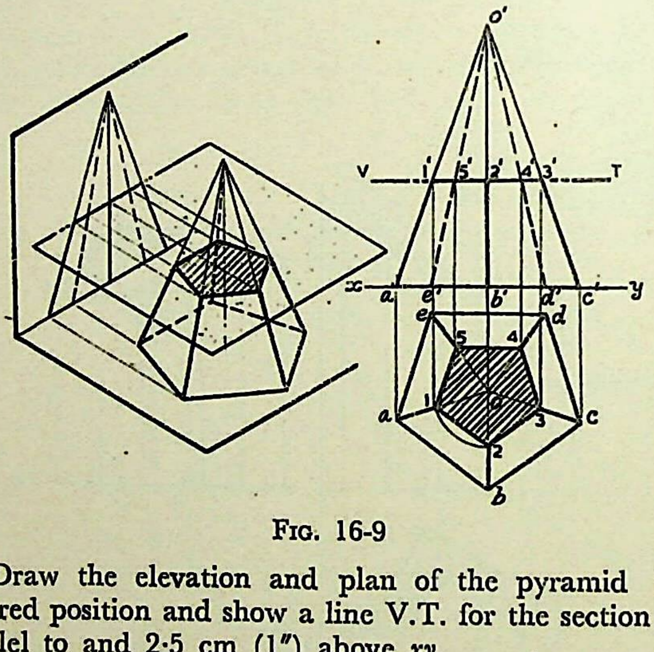


FIG. 16-9

Draw the elevation and plan of the pyramid in the required position and show a line V.T. for the section plane, parallel to and 2·5 cm (1") above *xy*.

All the five slant edges are cut. Project the points at which they are cut, on the corresponding edges in the plan. The point 2′ cannot be projected directly as the line *ob* is perpendicular to *xy*. But it is quite evident from the projections of other points that the lines of the section in plan, viz. 3-4, 4-5 and 5-1 are parallel to the edges of the base in their respective faces and that the points 1, 3, 4 and 5 are

equidistant from *o*. Hence, line 1-2 also will be parallel to *ab* and *o*2 will be equal to *o*1, *o*3 etc. Therefore, with *o* as centre and radius *o*1, draw an arc cutting *ob* at a point 2 which will be the projection of 2′. Complete the sectional plan in which the true shape of the section, viz. the pentagon 1 2 3 4 5 is also seen.

Hence, when pyramid is cut by a plane parallel to its base, the true shape of the section will be a figure, similar to the base; the sides of the section will be parallel to the edges of the base in the respective faces and the corners of the section will be equidistant from the axis.

(b) Section plane parallel to the V.P.

Problem 10:

A triangular pyramid, having base 2·5 cm (1″) side and axis 5 cm (2″) long, is lying in the H.P. on one of its faces, with the axis parallel to the V.P. A section plane, parallel to the V.P. cuts the pyramid at a distance of 0·6 cm (¼″) from the axis. Draw its sectional elevation and plan (fig. 16-10).

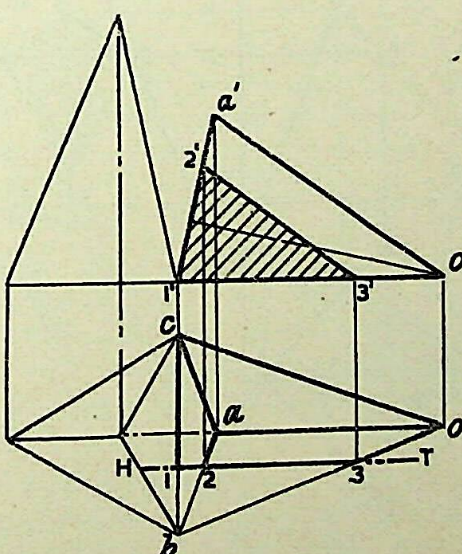


FIG. 16-10

Draw the projections of the pyramid in the required position and show a line H.T. (for the cutting plane) in the

plan, parallel to xy and 0·6 cm ($\frac{1}{4}''$) from the axis.

Project points 1, 2 and 3 (at which the edges are cut) on corresponding edges in the elevation and join them. Figure 1′2′3′ shows the true shape of the section.

(c) Section plane inclined to the H.P. and perpendicular to the V.P.

Problem 11:

A square pyramid, base 4 cm ($1\frac{1}{2}''$) side and axis 6·5 cm ($2\frac{1}{2}''$) long, has its base in the H.P. and all the edges of the base equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and which bisects the axis. Draw its sectional plan, sectional end view and true shape of the section (fig. 16-11).

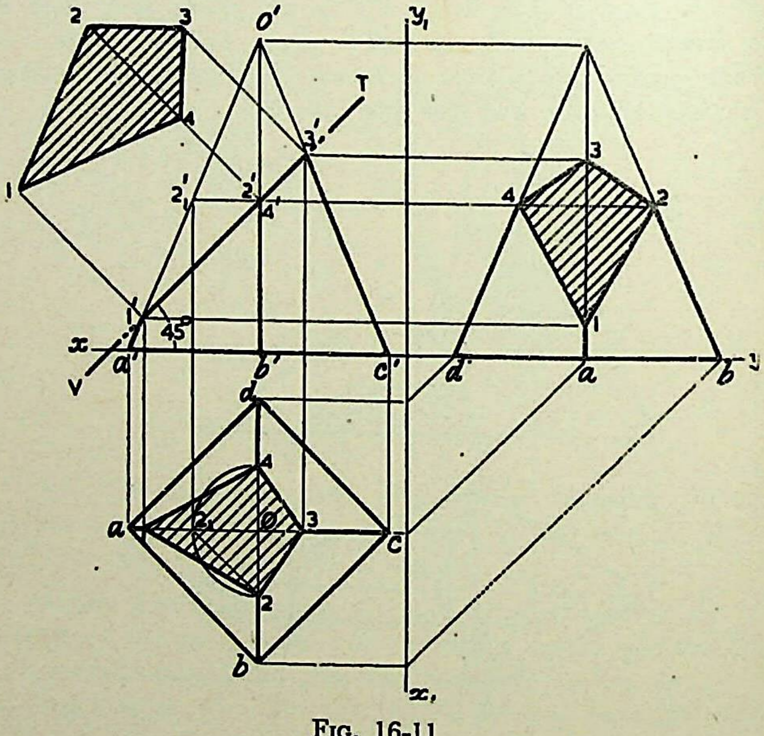


FIG. 16-11

Draw the plan and elevation of the pyramid in the required position. The section plane will be seen as a line in

the elevation. Hence, draw a line V.T. through the mid-point of the axis and inclined at 45° to xy. Name in correct sequence, the points at which the four edges are cut and project them in the plan. Here also, points 2′ and 4′ cannot be projected directly. Hence, assume a horizontal section through 2′ and draw a line parallel to the base, cutting $o'a'$ at $2_1'$. Project $2_1'$ to 2_1 on oa in the plan. From 2_1, draw a line parallel to ab and cutting ob at a point 2; or, with o as centre and radius $o2_1$, draw an arc cutting ob at 2 and od at 4. Complete the section 1 2 3 4 by joining the points and draw section lines in it.

Assuming the V.T. to be the new ground line, draw the true shape of the section. Project the end elevation from the two views. The removed portion of the pyramid may be shown by thin and fainter lines.

(d) Section plane inclined to the V.P. and perpendicular to the H.P.

Problem 12:

A pentagonal pyramid has its base in the H.P. and the edge of the base nearer the V.P., parallel to it. A vertical section plane, inclined at 45° to the V.P., cuts the pyramid at a distance of 0·6 cm ($\frac{1}{4}''$) from the axis. Draw the plan, sectional elevation and the auxiliary elevation on an A.V.P. parallel to the section plane. Base of pyramid 2·8 cm ($1\frac{1}{8}''$) side; axis 5 cm (2″) long (fig. 16-12).

The section plane will be seen as a line in the plan. It is to be at a distance of 0·6 cm ($\frac{1}{4}''$) from the axis. Hence, draw a circle with o as centre and radius equal to 0·6 cm ($\frac{1}{4}''$). Draw a line H.T., tangent to this circle and inclined at 45° to xy. It can be drawn in four different positions, of which any one may be selected.

Project points 1, 2 etc. from the plan to the corresponding edges in the elevation. Here again, point 2 cannot be projected directly. The process shown in problem 11 must be reversed. With centre o and radius $o2$, draw an arc cutting any one of the slant edges, say oc, at 2_1. Project 2_1 to $2_1'$ on $o'c'$. Through $2_1'$, draw a line parallel to the base,

cutting $o'b'$ at $2'$. Then $2'$ is the required point. Complete the view; it will show the apparent section.

Draw a ground line x_1y_1 parallel to the H.T. and project an auxiliary elevation which will show the true shape of the section.

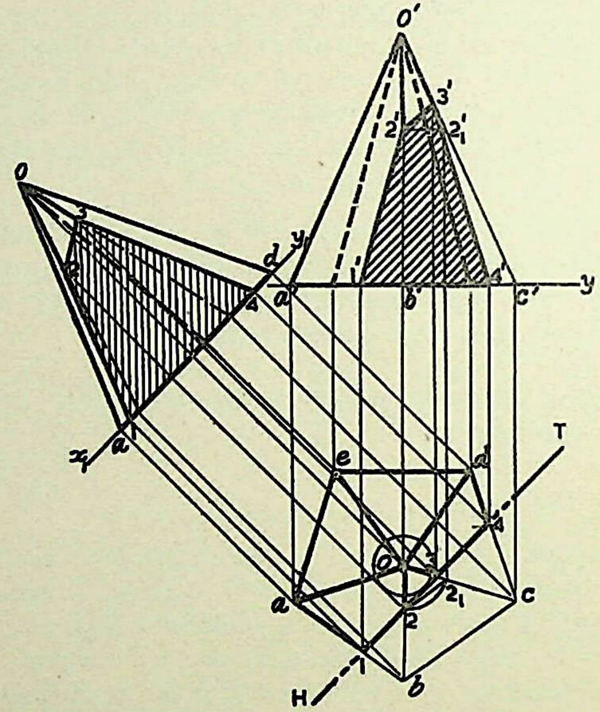


FIG. 16-12

ADDITIONAL PROBLEMS ON SECTIONS OF PYRAMIDS:

Problem 13:

A hexagonal pyramid, base 2·8 cm ($1\frac{1}{8}''$) side and axis 6·5 cm ($2\frac{1}{2}''$) long, is resting on its base in the H.P. with two edges parallel to the V.P. It is cut by a section plane, perpendicular to the V.P. inclined at 45° to the H.P. and intersecting the axis at a point 2·2 cm ($\frac{7}{8}''$) above the base. Draw elevation, sectional plan and true shape of the section (fig. 16-13).

This problem is similar to problem 11. In this case, the base is also cut and hence, the section is a heptagon. Care must be taken to name the points in proper sequence.

The true shape may be drawn on the V.T. as a new ground line or around the centre line a_1d_1, drawn parallel to the V.T. as shown. The distances of the points 1_1, 2_1 etc.

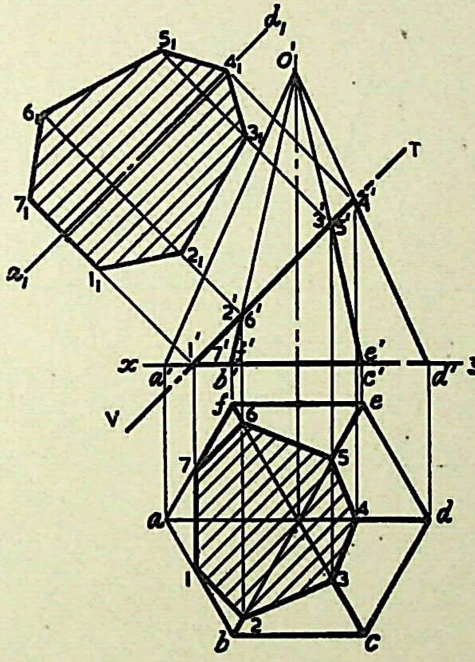


FIG. 16-13

from a_1d_1 are taken equal to the distances of points 1, 2 etc. from the line *ad* (which is parallel to *xy*).

Problem 14:

A pentagonal pyramid, side of base equal to 2·5 cm (1") and axis 5 cm (2") long, is lying on one of its triangular faces in the H.P. with the axis parallel to the V.P. A vertical section plane, whose H.T. bisects the plan of the axis and makes an angle of 30° with the ground line, cuts the pyramid, removing its top part. Draw the plan, sectional elevation, true shape of the section and development of the surface of the remaining portion of the pyramid (fig. 16-14).

Draw the H.T. of the section plane and name the points at which the edges are cut, in correct sequence, i.e. mark the visible edges first and then the hidden edges.

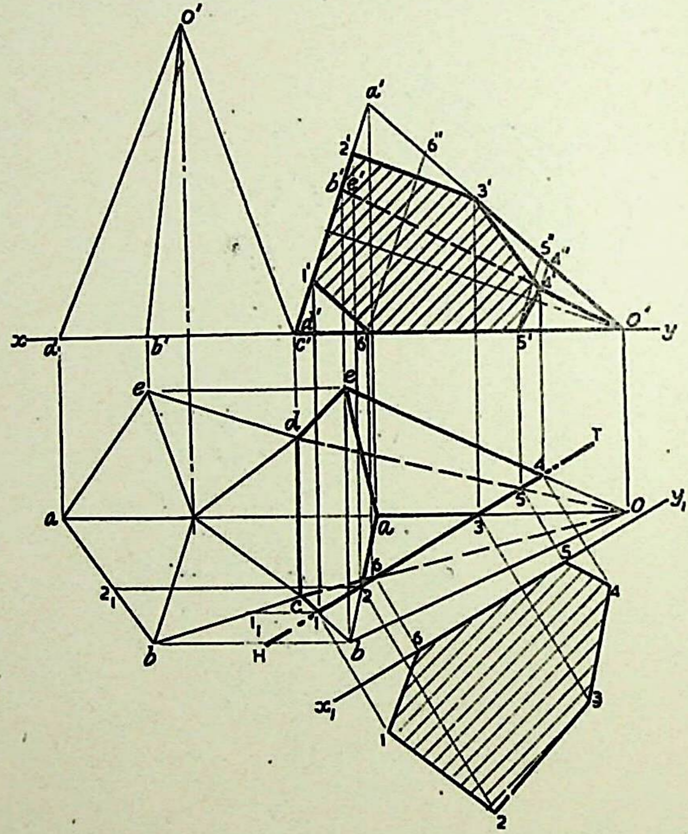


FIG. 16-14

Project the sectional elevation which will show the apparent section.

Obtain the true shape of the section on a ground line x_1y_1, drawn parallel to the H.T.

Development (fig. 16-15):

The line $o'a'$ shows the true length of the slant edge. With any point o as centre and radius $o'a'$, draw an arc and construct the development of the whole pyramid. Mark

points on it, taking the positions of 1 and 2 from the first plan and those of other points by projecting them on the

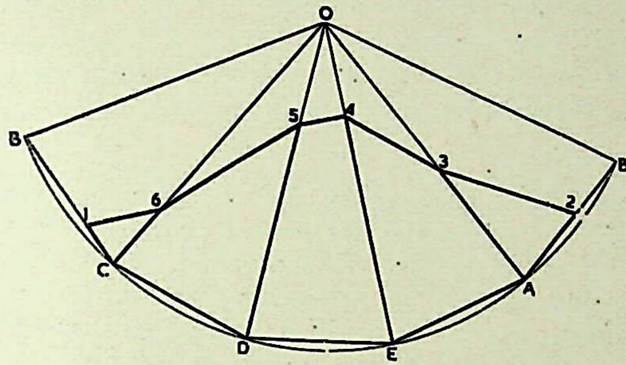


FIG. 16-15

true-length-line *o'a'*. Draw lines joining these points and complete the development as shown in the figure.

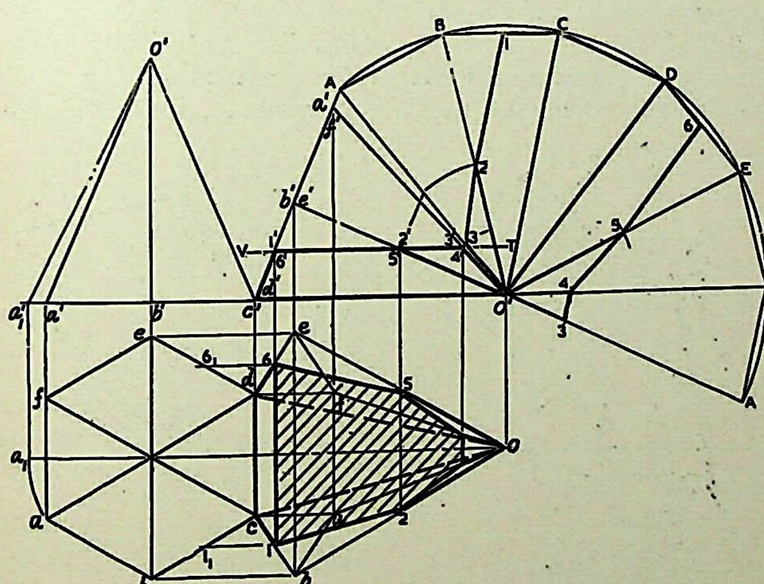


FIG. 16-16

Problem 15:

A hexagonal pyramid, base 2·5 cm (1") side and axis 5 cm (2")

long, has a triangular face in the H.P. and the axis parallel to the V.P. It is cut by a horizontal section plane which bisects the axis. Draw the elevation and sectional plan and develop the surface of the cut-pyramid (fig. 16-16).

The V.T. cuts six edges. The sectional plan shows the true shape of the section also.

Development:

None of the edges shows the true length of the slant edge. Hence, determine the true length $o'a_1'$ and draw the development of the whole pyramid. Locate positions of the points 1 and 6 by projecting them on the first plan and positions of other points by drawing lines through them, parallel to the base and upto the true length line $o'A$. Mark these points on the development and complete it as shown.

Problem 16:

A hexagonal pyramid, base 2·8 cm ($1\frac{1}{8}''$) side and axis 7·5 cm ($3''$) long, resting on its base in the H.P. with two of its edges parallel to the V.P. is cut by two section planes, both perpendicular to the V.P. The horizontal section plane cuts the axis at a point 4 cm ($1\frac{1}{2}''$) from the apex. The other plane which makes an angle of 45° with

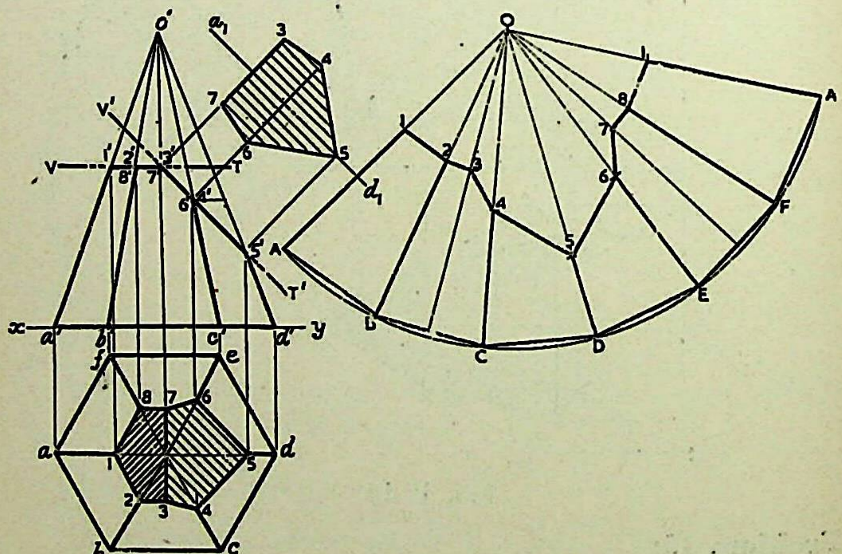


FIG. 16-17

the H.P., also intersects the axis at the same point. Draw the elevation, sectional plan and development of the surface of the remaining part of the pyramid (fig. 16-17).

Draw lines *V.T.* and *V'T'* for the two section planes. The plan will show the true shape of the horizontal section, the sides of which are parallel to the respective sides of the base.

The true shape of the other section may be obtained on *V'T'* as the ground line or around a_1d_1.

Draw the development with *o'a'* or *o'd'* as radius and locate the points on it, as shown in the figure.

SECTIONS OF CYLINDERS:

(a) Section plane parallel to the base.

When a cylinder is cut by a section plane parallel to the base, the true shape of the section is a circle of the same diameter.

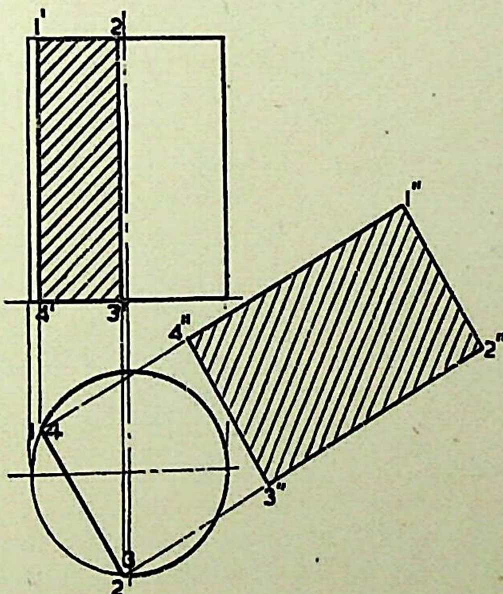


FIG. 16-18

(b) Section plane parallel to the axis.

When a cylinder is cut by a section plane parallel to the axis, the true shape of the section is a rectangle, the sides

of which are respectively equal to the length of the axis and the length of the section plane within the cylinder (fig. 16-18). When the section plane contains the axis, the rectangle will be of the maximum size.

(c) Section plane inclined to the base.

Problem 17:

A cylinder of 5 cm (2") diameter, 7 cm ($2\frac{3}{4}$") height and having its base in the H.P., is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 4 cm ($1\frac{1}{2}$") above the H.P. Draw its elevation, sectional plan, sectional end view and true shape of the section (fig. 16-19).

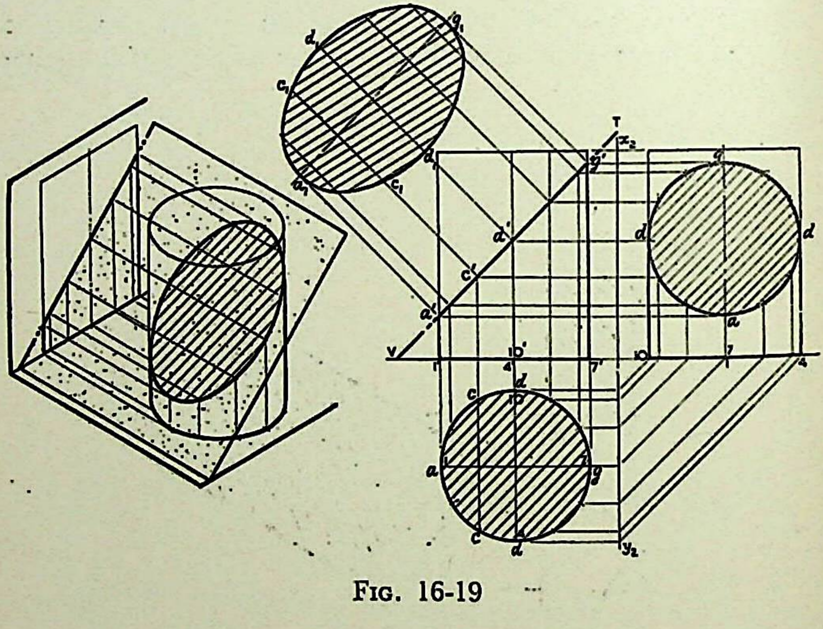


Fig. 16-19

As the cylinder has no edges, a number of lines representing the generators may be assumed on its curved surface by dividing the base-circle into, say 12 equal parts. Name the points at which these lines are cut by the V.T. In plan, these points lie on the circle and hence, the same circle is the plan of the section. The width of section at any point, say *c′*, will be equal to the length of the chord *cc* in the plan.

The true shape of the section may be drawn around the centre line a_1g_1 drawn parallel to V.T. as shown. It is an ellipse, the major axis of which is equal to the length of the section plane, viz. $a'g'$, and the minor axis equal to the diameter of the cylinder.

Project the end elevation as shown. The section will be seen as a circle because the section plane makes 45° angle with x_2y_2.

ADDITIONAL PROBLEMS ON SECTIONS OF CYLINDERS:

Problem 18:

A cylinder, 5 cm (2") diameter and 6·5 cm (2½") long, is resting on its base in the H.P. It is cut by a section plane perpendicular to the V.P., the V.T. of which cuts the axis at a point 4 cm (1½") from the base and makes an angle of 45° with the H.P. Draw its elevation, sectional plan and another plan on an A.I.P., parallel to the section plane (fig. 16-20).

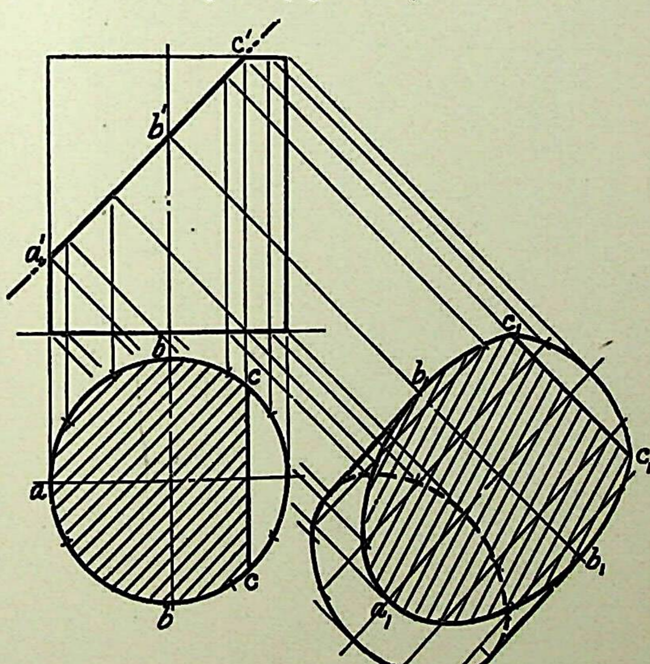


FIG. 16-20

In this case, the top end of the cylinder is also cut. Hence, the true shape of the section is a part of an ellipse, as shown in the auxiliary plan.

Problem 19:

A cylinder, 5 cm (2") diameter and 6·5 cm ($2\frac{1}{2}$") long, is lying in the H.P. with its axis parallel to both the H.P. and the V.P. It is cut by a vertical section plane inclined at 30° to the V.P., so that the axis is cut at a point 2·5 cm (1") from one of its ends and both the bases of the cylinder are partly cut. Draw its sectional elevation and true shape of the section (fig. 16-21).

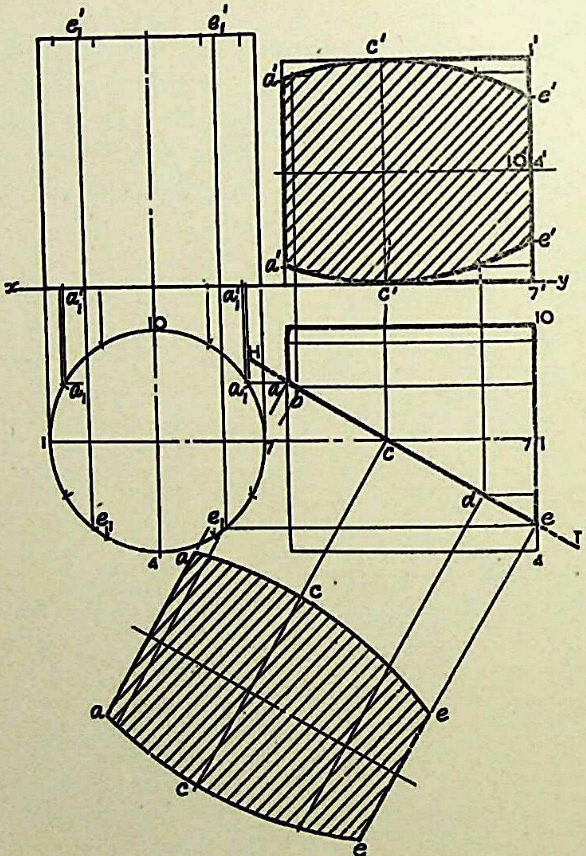


FIG. 16-21

Draw the projections of the cylinder and a line H.T. for the section plane. Project the points at which the bases and

the lines are cut. The points on the bases cannot be projected directly. Therefore, project them (i) to the first plan, i.e. a to a_1 and e to e_1, (ii) then to the first elevation, i.e. a_1 to a_1' and e_1 to e_1' and (iii) finally, transfer them to the second elevation to a' and e' each, at two places as shown.

Draw the true shape of the section either on a new ground line or symmetrically around the centre line and making aa equal to $a'a'$, cc equal to $c'c'$ etc.

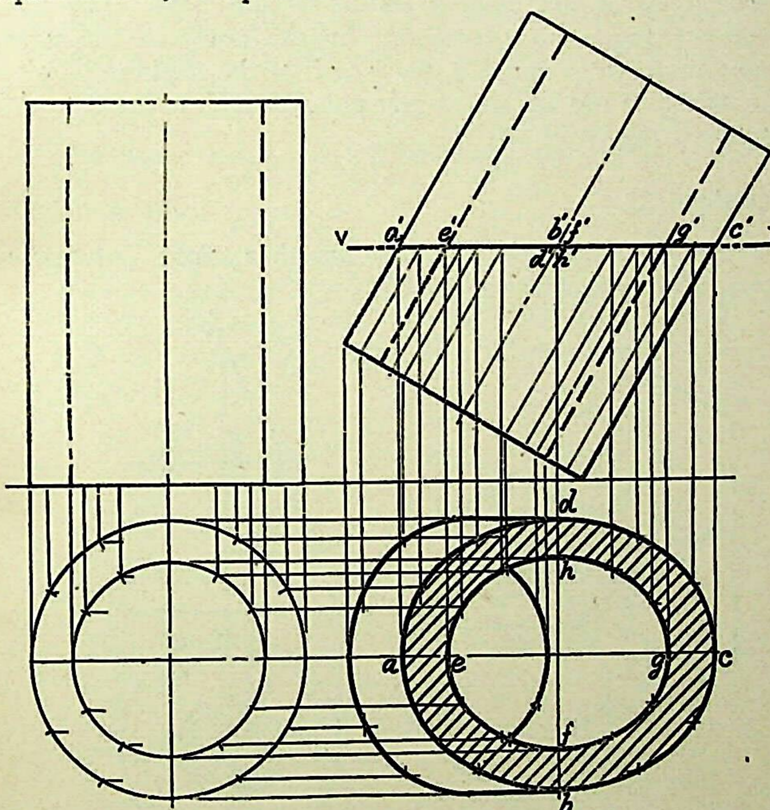


FIG. 16-22

Problem 20:

A hollow cylinder, 6·5 cm (2½") diameter, axis 9 cm (3½") long and thickness 1 cm (⅜"), resting in the H.P. with the axis inclined at 30° to the vertical, is cut in two equal halves by a horizontal section plane. Draw its sectional plan (fig. 16-22).

The figure is self-explanatory.

Note that a part of the ellipse for the inside bottom will also be visible.

SECTIONS OF CONES:

(a) Section plane parallel to the base.

The cone resting in the H.P. on its base (fig. 16-23) is cut by a section plane parallel to the base. The true shape of the section is shown by the circle in the plan, whose diameter is equal to the length of the section viz. *a′a′*. The width of the section at any point, say *b′*, is equal to the length of the chord *bb*.

Problem 21:

To locate the position in plan of any given point c′ in the elevation of the above cone (fig. 16-23).

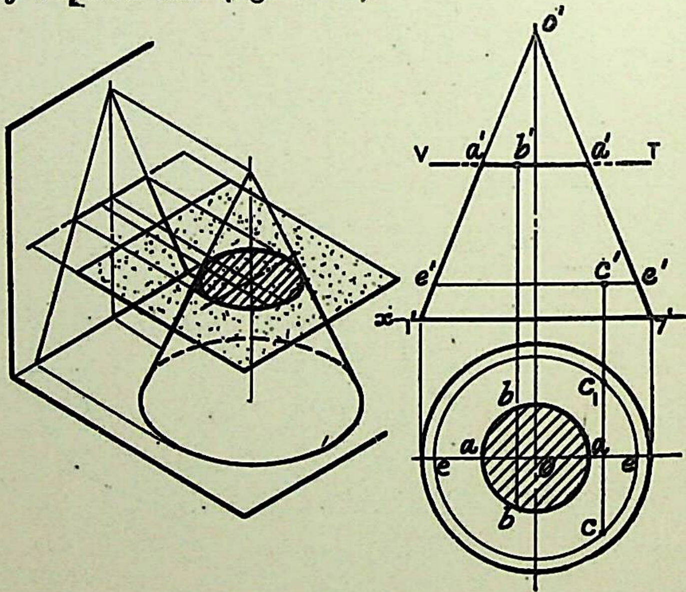


Fig. 16-23

Through *c′*, draw a line *e′e′* parallel to the base. With *o* as centre and diameter equal to *e′e′*, draw a circle in the plan. Project *c′* to points *c* and c_1 on this circle. *c* is the plan

of c'. c_1 is the plan of another point on the back side of the cone and coinciding with c'. The chord cc_1 shows the width of the horizontal section of the cone at the point c'. This method may be called *circle method.*

When the position of a point in the plan say c is given, its elevation c' can be determined by reversing the above process. With centre o and radius oc, draw a circle cutting the horizontal centre line at e. Through e, draw a projector cutting the slant side $o'1'$ or $o'7'$ at e'. Draw the line $e'e'$ parallel to the base, intersecting a projector through c at the required point c'.

(b) Section plane passing through the apex.

Problem 22:

A cone, diameter of base 5 cm (2") and axis 6·5 cm ($2\frac{1}{2}$") long is resting on its base in the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 75° to the H.P. and passing through the apex. Draw its elevation, sectional plan and true shape of the section (fig. 16-24).

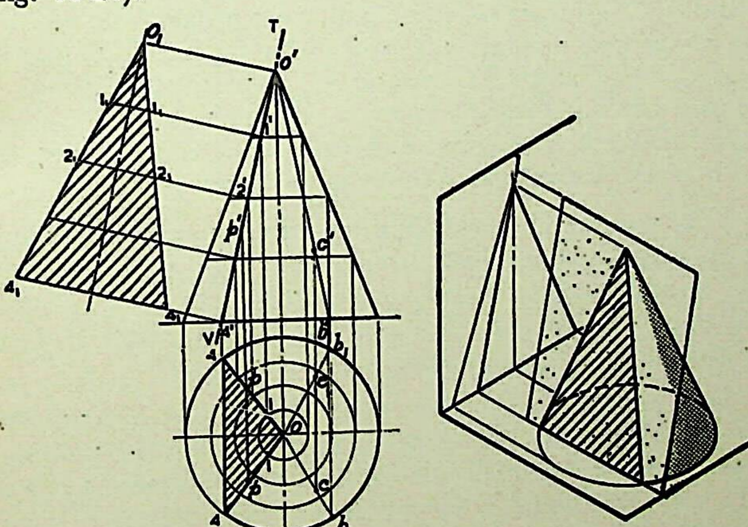


FIG. 16-24

Draw the elevation and plan of the cone and show on it, the line V.T. for the section plane.

Mark a number of points 1′, 2′ etc. on the V.T. and project them to points 1,2 etc. in the plan by the circle method. It will be found that these points lie on a straight line through *o*. Thus, *o*4 is the plan of the line or generator *o*′4′ and triangle *o*44 is the plan of the section. The width of the section at any point *p*′ on the section is the line *pp*, obtained by projecting *p*′ on the triangle. This method is called *generator method*.

Project the true shape of the section. It is an isosceles triangle, the base of which is equal to the length of the chord on the base-circle and the altitude is equal to the length of the section plane within the cone.

Problem 23:

To determine by generator method, the position in the plan, of a given point c′ in the elevation of the above cone (fig. 16-24).

Join *o*′*c*′ and produce it to cut the base at *b*′. Project *b*′ to points *b* and b_1 on the base-circle in the plan. Draw lines *ob* and ob_1. Thus, *ob* is the plan of the generator *o*′*b*′, and ob_1 that of the generator (at the back) which coincides with *o*′*b*′. Project *c*′ to *c* and c_1 on *ob* and ob_1 respectively. Thus, *c* is the plan of *c*′, and c_1 is the plan of another point on the other side of the cone and coinciding with *c*′. The line cc_1 is the width of the horizontal section of the cone at *c*′.

The position in elevation, of any given point in the plan, say *c*, may be determined by reversing the above method. Join *oc* and produce it to cut the base-circle at *b*. Project *b* to *b*′ on the base in elevation. Join *o*′*b*′. Through *c*, draw a projector to cut *o*′*b*′ at the required point *c*′.

Sectional views of cones may be obtained by applying any one of the above two methods for locating the positions of points. The generator method is more suitable particularly when the cone is in inclined positions.

(c) Section plane inclined to the base at an angle smaller than the angle of inclination of the generators with the base.

Problem 24:

A cone, diameter of base 6·5 cm ($2\frac{1}{2}$"), and axis 7·5 cm (3") long is resting on its base in the H.P. It is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and cutting the axis at a point 3·5 cm ($1\frac{3}{8}$") from the apex. Draw its elevation, sectional plan, sectional end view and the true shape of the section.

Draw a line V.T. in the required position, in the elevation of the cone. The positions of points on this line and the width of section at each point can be determined by one of the two methods explained in problems 21 and 23 and as shown below.

(1) *Generator method* (fig. 16-25):

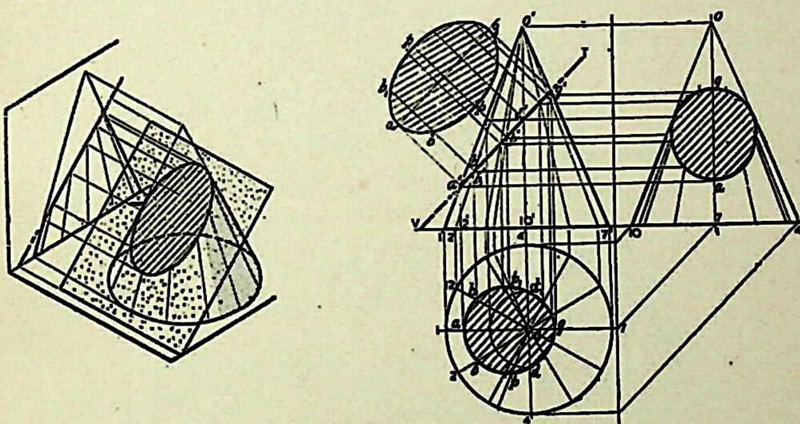


Fig. 16-25

Divide the base-circle into a number of equal parts, say 12. Draw lines (i.e. generators) joining these points with o and project them on the line representing the base in the elevation. Draw lines joining $o'2'$, $o'3'$ etc. cutting the line for section at points b', c' etc. Project these points on the corresponding lines in the plan. For example, point b' on $o'2'$, also represents point b_1' on o'-12$'$ which coincides with $o'2'$. Therefore, project b' to b on $o2$ and to b_1 on o-12. b and b_1 are the points on the section (in the plan). Similarly, obtain other points. Point d' cannot be projected directly. Hence, the same method as in case of pyramids should be employed to determine the positions d and d_1, as shown. In

addition to these, two more points for the maximum width of the section at its centre should also be obtained. Mark p', the mid-point of the section and obtain the points p and p_1. Draw a smooth curve through these points.

The true shape of the section may be obtained on the V.T. as a new ground line or symmetrically around the centre line *ag*, drawn parallel to the V.T. as shown. It is an ellipse whose major axis is equal to the length of the section and minor axis equal to the width of the section at its centre.

Draw the sectional end elevation by projecting the points on corresponding generators, as shown.

(2) *Circle method* (fig. 16-26):

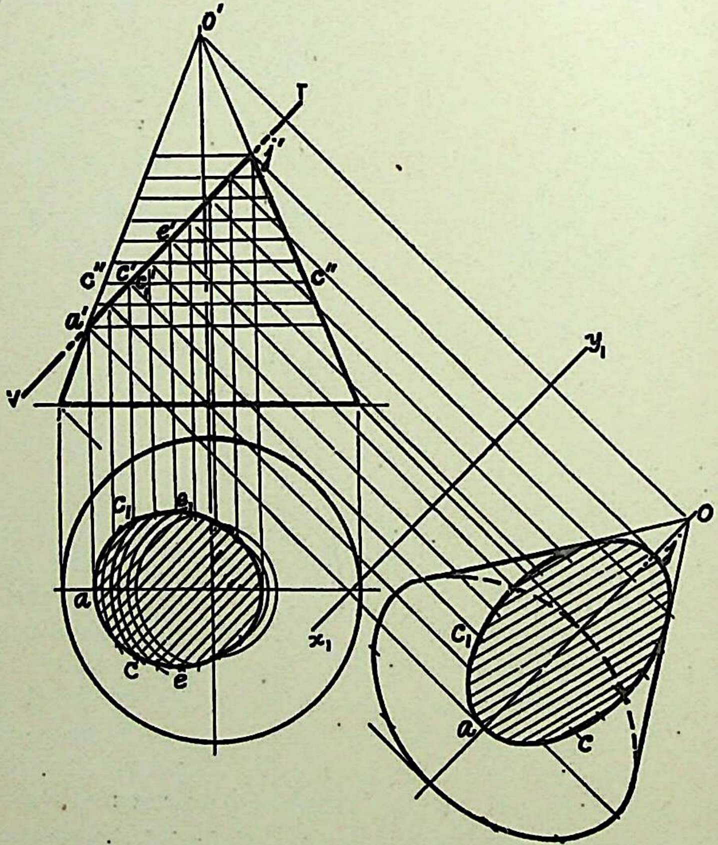


FIG. 16-26

Divide the line of section into a number of equal parts. Determine the width of section at, and the position of, each division-point in the plan by the circle method. For example, through c', draw a line $c''c''$ parallel to the base. With o as centre and radius equal to half of $c''c''$, draw an arc. Project c' to c and c_1 on this arc; then c and c_1 are the required points. The straight line joining c and c_1 will be the width of the section at c'. Similarly, obtain all other points and draw a smooth curve through them. This curve will show the apparent section. The maximum width of the section will be at the mid-point e'. It is shown in the plan by the length of the chord joining e and e_1.

Draw a ground line x_1y_1 parallel to the V.T. and project the true shape of the section. In the figure, the auxiliary plan of the truncated cone is shown. It shows the true shape of the section.

The sectional end elevation may be obtained by projecting all the division-points horizontally and then marking the width of the section at each point, symmetrically around the axis of the cone.

(d) Section plane parallel to a generator.

Problem 25:

The cone in same position as in problem 24, is cut by a section plane perpendicular to the V.P. and parallel to and 1·2 cm (½") away from one of its end generators. Draw its elevation, sectional plan and true shape of the section (fig. 16-27).

Draw a line V.T. (for the section plane) parallel to and 1·2 cm (½") away from the generator $o'1'$. Draw the twelve generators in the plan and project them to the elevation. All the generators except $o'1'$, $o'2'$ and o'-$12'$ are cut by the section plane. Project the points at which they are cut, to the corresponding generators in the plan. The width of the section at the point where the base is cut will be the chord aa_1. Draw a curve through $a..f..a_1$. The figure enclosed between aa_1 and the curve is the apparent section.

Obtain the true shape of the section as explained in the previous problem. It will be a parabola.

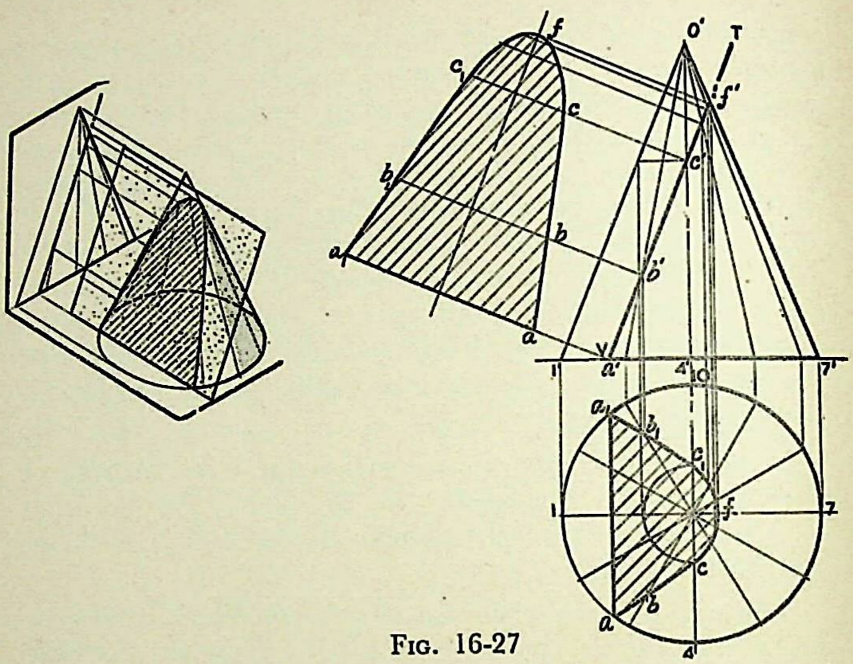


FIG. 16-27

(e) Section plane inclined to the base at an angle greater than the angle of inclination of the generators with the base.

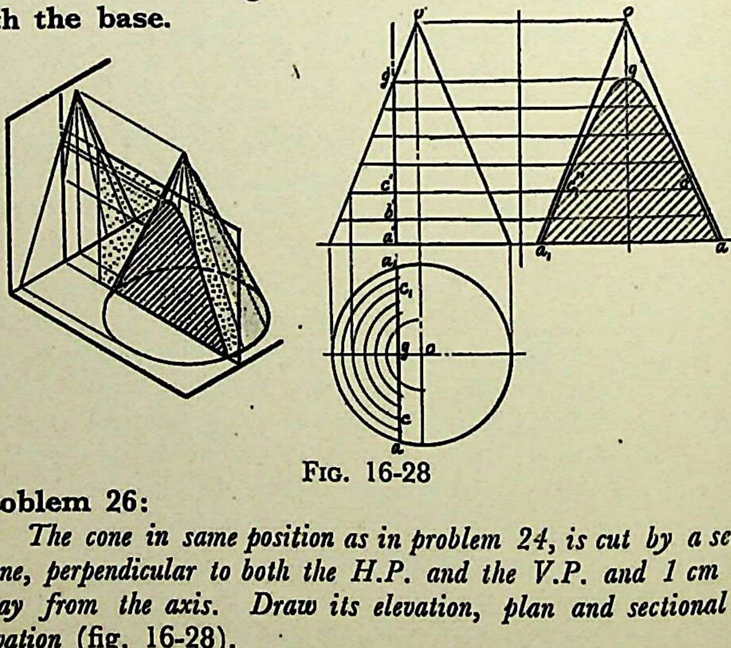


FIG. 16-28

Problem 26:

The cone in same position as in problem 24, is cut by a section plane, perpendicular to both the H.P. and the V.P. and 1 cm ($\frac{3}{8}''$) *away from the axis. Draw its elevation, plan and sectional end elevation* (fig. 16-28).

The section will be seen as a line, perpendicular to xy, in both the elevation and the plan. The end elevation will show the true shape of the section. The width of the section at any point, say c', will be equal to cc_1 obtained by the circle method.

Draw the end elevation of the cone. Project the points (on the section) in the end view taking the widths from the plan. For example, through c' draw a horizontal line. Mark on it points c'' and c_1'' equidistant from and on both sides of the axis so that $c''c_1'' = cc_1$. Draw a curve through the points thus obtained. It will be a hyperbola.

ADDITIONAL PROBLEMS ON SECTIONS OF CONES:

Problem 27:

A cone, diameter of base 5 cm (2") and axis 6·5 cm ($2\frac{1}{2}$") long, is lying in the H.P. on one of its generators with the axis parallel to the V.P. It is cut by a horizontal section plane 1·2 cm ($\frac{1}{2}$") above the H.P. Draw its elevation, sectional plan and development of its surface (fig. 16-29).

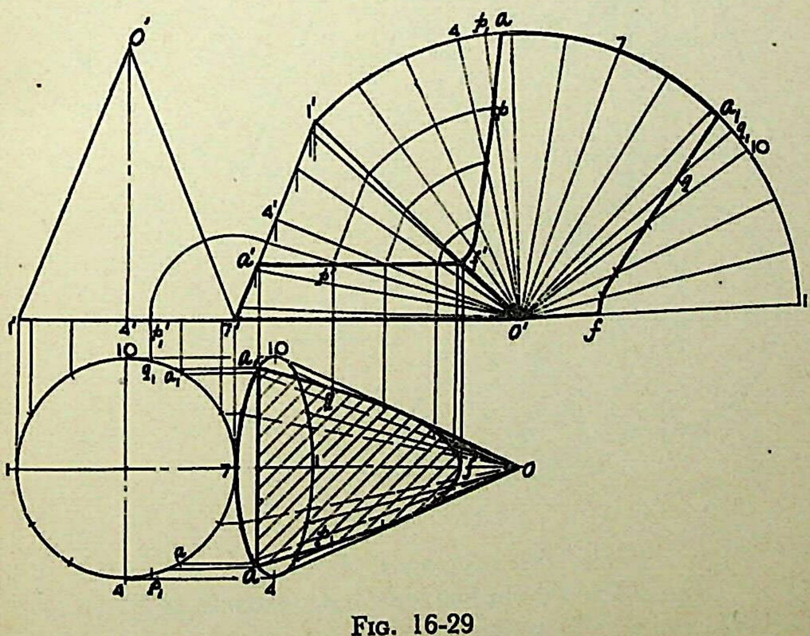


FIG. 16-29

Use generator method and project the points in the plan. The curve will show the true shape of the section (viz. a parabola).

For development, the true lengths of the cut-generators are obtained by drawing lines parallel to the base. Positions of points a and a_1 are determined by projecting them on the base-circle in the first plan.

Problem 28:

A cone, base 6·5 cm (2½") diameter, axis 7·5 cm (3") long and resting on its base in the H.P., is cut by a vertical section plane, the H.T. of which makes an angle of 60° with the ground line and is 1·2 cm (½") from the plan of the axis. Draw the sectional elevation, true shape of the section and development of the surface of the sectioned cone (fig. 16-30).

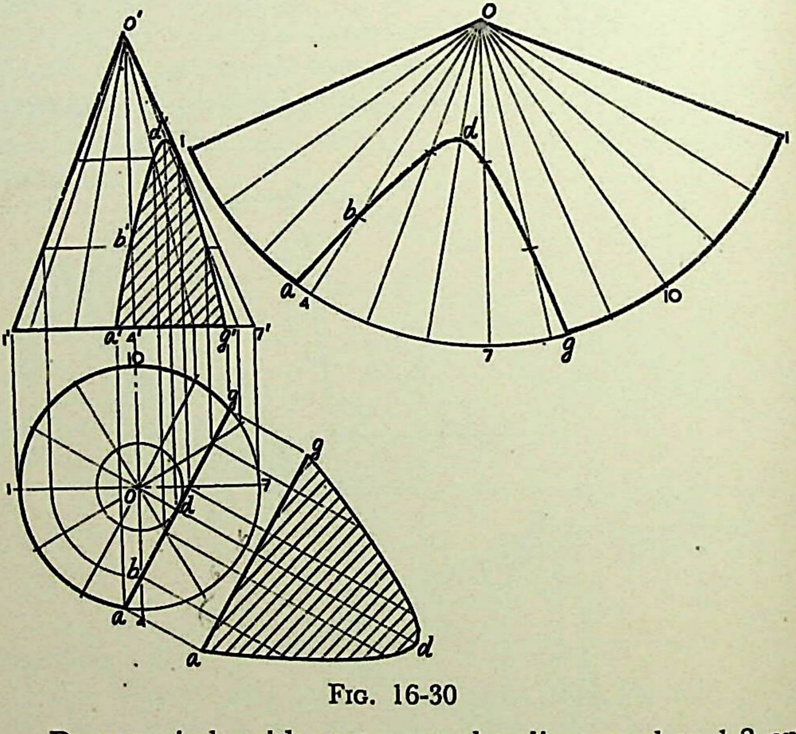


Fig. 16-30

Draw a circle with centre o and radius equal to 1·2 cm (½"). Draw a line for the section plane, tangent to this circle and inclined at 60° to xy.

Project the elevation by the generator method as shown. Note how the point b' is obtained in the elevation. Draw the true shape of the section and the development as already explained.

Problem 29:

A cone, base 6·5 cm ($2\frac{1}{2}$") diameter and axis 7·5 cm (3") long is lying in the H.P. on one of its generators with the axis parallel to the V.P. A vertical section plane parallel to the generator which is tangent to the ellipse (for the base) in the plan, cuts the cone bisecting the axis and removing a portion containing the apex. Draw its sectional elevation and true shape of the section (fig. 16-31).

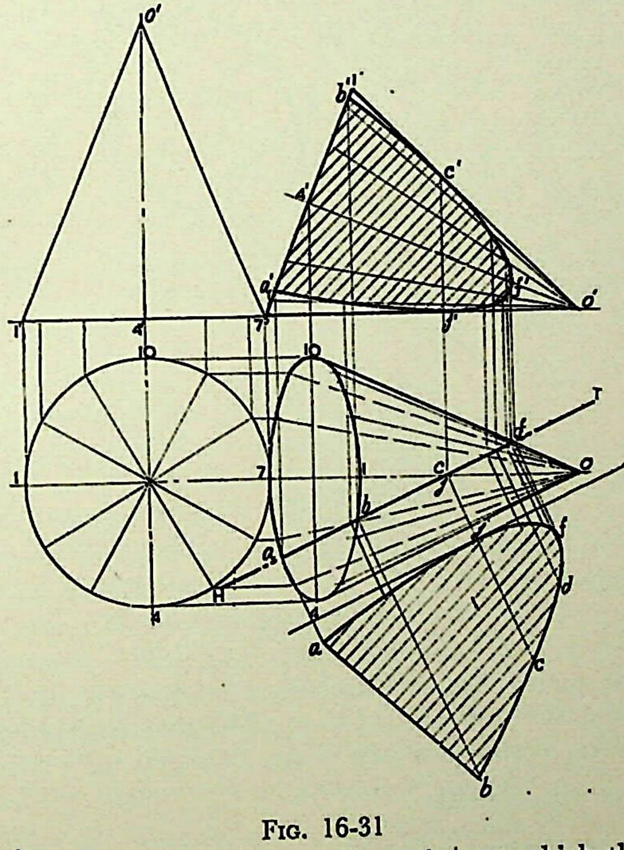


Fig. 16-31

Name in correct sequence the points at which the base and the generators are cut and project them in the elevation.

Project the true shape of the section on a ground line parallel to the H.T.

Problem 30:

A cone, base 7 cm ($2\frac{3}{4}''$) diameter and axis 10 cm (4″) long, is resting in the H.P. on its base. It is cut by a section plane, perpendicular to the V.P., inclined at 45° to the H.P. and intersecting the axis 3·5 cm ($1\frac{3}{8}''$) above the base. Draw its plan when it is lying in the H.P. on its cut-surface, with the axis parallel to the V.P. (fig. 16-32).

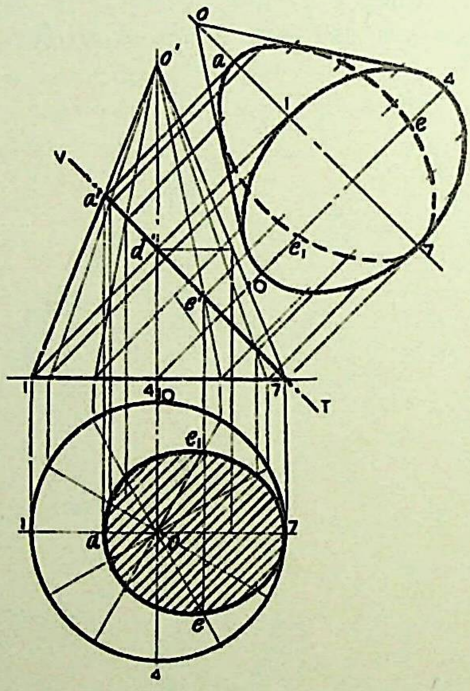


FIG. 16-32

The auxiliary plan is drawn on the line V.T. of the section plane as a new ground line. The cone is thus lying on its cut-face in the auxiliary plane.

SECTIONS OF SPHERES:

When a sphere is cut by a plane, the true shape of the section is always a circle.

The sphere in fig. 16-33 is cut by a horizontal section plane. The true shape of the section (seen in plan) is a circle of diameter $a'a'$. The width of the section at any point, say b', is equal to the length of the chord bb.

When the sphere is cut by a section plane parallel to the V.P. (fig. 16-34), the true shape of the section, seen in elevation, is a circle of diameter cc. The width of section at any point d is equal to the length of the chord $d'd'$.

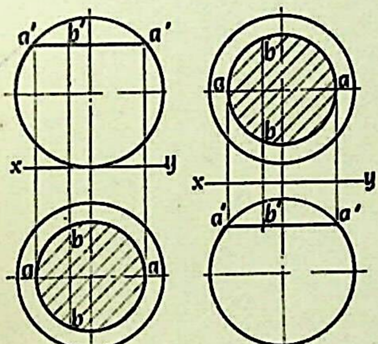


Fig. 16-33

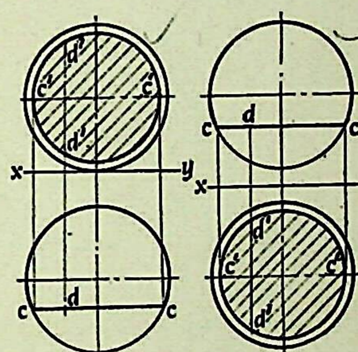


Fig. 16-34

Problem 31:

A sphere of 5 cm (2") diameter, is cut by a section plane perpendicular to the V.P., inclined at 45° to the H.P. and at a distance of 1 cm (0·4") from its centre. Draw the sectional plan and true shape of the section [fig. 16-35(*i*)].

Draw a line (for the section plane) inclined at 45° to xy and tangent to a circle of 1 cm (0·4") radius drawn with o' as centre. Mark a number of points on this line.

Method I: Find the width of section at each point in the plan as shown in fig. 16-33. For example, the chord cc is the width of section at a point c'. Draw a curve through the points thus obtained. It will be an ellipse. The true shape of the section will be a circle of diameter $a'g'$.

Method II: We know that the true shape of the section is a circle of diameter equal to $a'g'$. The width of section at any point say c' is equal to the chord c_1c_1 on this circle. Therefore,

project c' to points c in the plan so that $cc = c_1c_1$. Similarly, obtain other points and draw the ellipse through them.

Fig. 16-35(*ii*) shows in third-angle projection, the sectional elevation and true shape of the section when the section plane is vertical and inclined to the V.P.

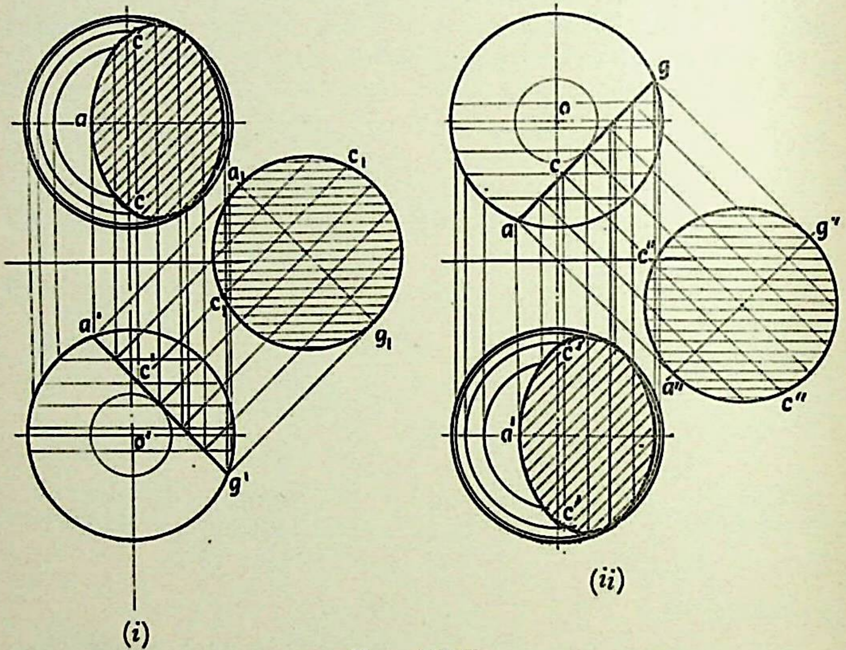


FIG. 16-35

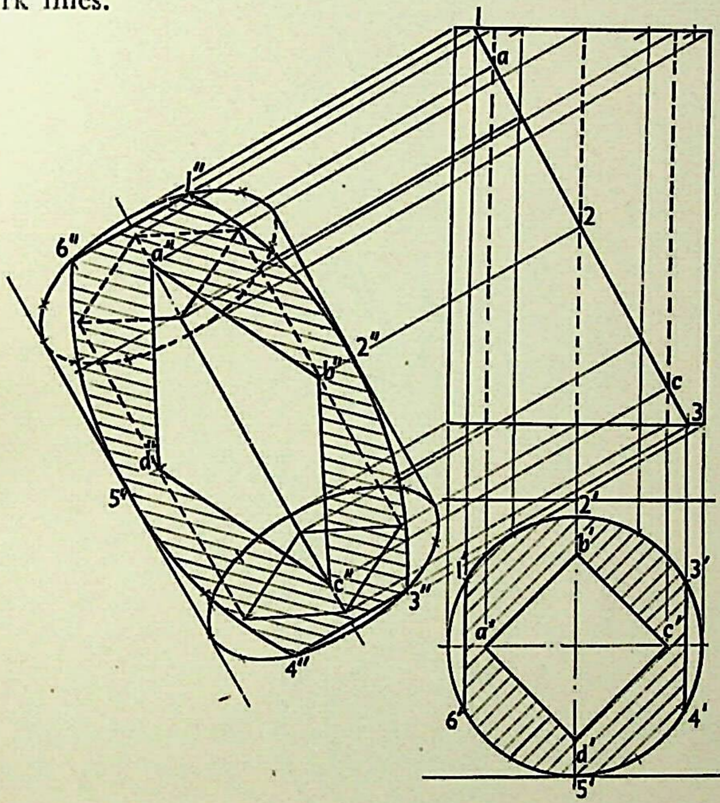
TYPICAL PROBLEMS:

Problem 32:

A cylinder, base 5 cm (2") diameter and axis 7·5 cm (3") long, has a square hole of 2·5 cm (1") side cut through it so that the axis of the hole coincides with that of the cylinder. The cylinder is lying on the ground with the axis perpendicular to the V.P. and the faces of the hole equally inclined to the H.P. A vertical section plane, inclined at 60° to the V.P. cuts the cylinder in two equal halves. Project the elevation of the cylinder on an A.V.P. parallel to the section plane (fig. 16-36).

Assuming the cylinder to be whole, draw its auxiliary elevation. Project the points at which the generators of the cylinder and the edges of the hole are cut. The section of

the cylinder will be a part of an ellipse. Join the points at which the edges of the hole are cut. The lines for the back edges of the hole will be visible within the section and hence, must be shown as full lines. Complete the view by showing the section and the remaining portion of the cylinder with dark lines.



Third-angle projection

FIG. 16-36

Problem 33:

A solid composed of a half-cone and a half-hexagonal pyramid is shown in fig. 16-37. It is cut by a section plane, whose V.T. makes an angle of 30° with xy and whose H.T. is perpendicular to xy and coincides with an edge of the base. Draw its sectional plan, true shape of the section and development of the surface of the remaining portion.

Draw a line H.T. coinciding with the vertical edge of the base in the plan. From the point of its intersection with *xy*, draw a line V.T. inclined at 30° to *xy*.

Project the sectional plan. Note how points b and b_1 are obtained. The true shape of the section will be partly elliptical.

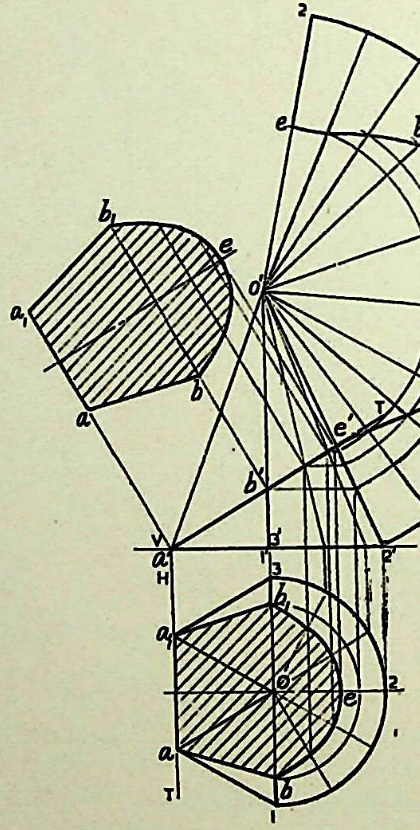


FIG. 16-37

Draw the development of the half-cone and half-pyramid and show the lines for the section in it.

Problem 34:

The projections of a hemisphere 4 cm (1½") diameter, placed centrally on the top of a frustum of a hexagonal pyramid, base 2·5

cm (1″) side, top 1·6 cm ($\frac{5}{8}$″) side and axis 4 cm (1$\frac{1}{2}$″) long are given. Draw the sectional elevation when the vertical section plane H.T. inclined at 45° to the V.P. and 1 cm ($\frac{3}{8}$″) from the axis, cuts them (fig. 16-38).

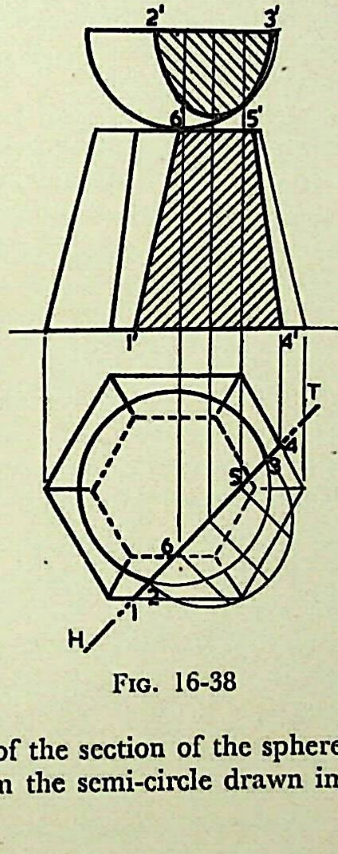


FIG. 16-38

The widths of the section of the sphere at various points are obtained from the semi-circle drawn in the plan.

EXERCISES XVI

(1) A cube of 5 cm (2″) edge is resting in the H.P. with a vertical face inclined at 30° to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 30° to the H.P. and passing through a point on the axis, 4 cm (1$\frac{1}{2}$″) above the H.P. Draw the sectional plan, true shape of the section and the development of the surface of the remaining portion of the cube.

(2) A hexagonal prism, side of base 3·5 cm ($1\frac{3}{8}''$) and height 7·5 cm ($3''$) is resting on one of its corners in the H.P., with a longer edge containing that corner inclined at 60° to the H.P. and a rectangular face parallel to the V.P. A horizontal section plane cuts the prism in two equal halves. (i) Draw the elevation and sectional plan of the cut prism. (ii) Draw another plan on an auxiliary inclined plane which makes an angle of 45° with the H.P.

(3) A pentagonal prism, side of base 5 cm ($2''$) and length 10 cm ($4''$), has a rectangular face in the H.P. and the axis parallel to the V.P. It is cut by a vertical section plane, the H.T. of which makes an angle of 30° with *xy* and bisects the axis. Draw the sectional elevation, plan and true shape of the section. Develop the surface of the remaining half of the prism.

(4) A hollow square prism, base 5 cm ($2''$) side (outside), length 7·5 cm ($3''$) and thickness 1 cm ($\frac{3}{8}''$) is lying in the H.P. on one of its rectangular faces, with the axis inclined at 30° to the V.P. A section plane, parallel to the V.P. cuts the prism, intersecting the axis at a point 2·5 cm ($1''$) from one of its ends. Draw the plan and sectional elevation of the prism.

(5) A cylinder, 6·5 cm ($2\frac{1}{2}''$) diameter and 9 cm ($3\frac{1}{2}''$) long, is lying in the H.P. with the axis parallel to the H.P. and inclined at 30° to the V.P. It is cut by a vertical section plane in such a way that the true shape of the section is an ellipse having the major axis 7·5 cm ($3''$) long. Draw its sectional elevation and true shape of the section.

(6) A cube of 6·5 cm ($2\frac{1}{2}''$) edge is resting in the H.P. with its vertical faces equally inclined to the V.P. It is cut by a section plane, perpendicular to the V.P., so that the true shape of the section is a regular hexagon. Determine the inclination of the cutting plane with the H.P. and draw the sectional plan and true shape of the section.

(7) A vertical hollow cylinder, outside diameter 6·5 cm ($2\frac{1}{2}''$), length 9 cm ($3\frac{1}{2}''$) and thickness 1 cm ($\frac{3}{8}''$), is cut by two section planes which are normal to the V.P. and which intersect each other at the top end of the axis. The planes cut the cylinder on opposite sides of the axis and are inclined at 30° and 45° respectively to it. Draw the elevation, sectional plan

and auxiliary sectional plans on planes parallel to the respective section planes.

(8) A square pyramid, base 5 cm (2″) side and axis 7·5 cm (3″) long, is resting in the H.P. on one of its triangular faces, the plan of the axis making an angle of 30° with the V.P. It is cut by a horizontal section plane, the V.T. of which intersects the axis at a point 0·6 cm (¼″) from the base. Draw the elevation, sectional plan and the development of the sectioned pyramid.

(9) A pentagonal pyramid, base 3 cm (1¼″) side and axis 7·5 cm (3″) long, has its base in the H.P. and an edge of the base parallel to the V.P. It is cut by a section plane, perpendicular to the V.P., inclined at 60° to the H.P. and bisecting the axis. Draw the elevation and plan when the pyramid is tilted so that it lies on its cut-face in the H.P. with the axis parallel to the V.P. Show the shape of the section by dotted lines. Develop the surface of the truncated pyramid.

(10) A tetrahedron of 6·5 cm (2½″) edge is lying in the H.P. on one of its faces, with an edge perpendicular to the V.P. It is cut by a section plane which is perpendicular to the V.P. so that the true shape of the section is an isosceles triangle of base 5 cm (2″) long and altitude 4 cm (1½″). Find the inclination of the section plane with the H.P. and draw the elevation, sectional plan and the true shape of the section.

(11) A hexagonal pyramid, base 5 cm (2″) side and axis 10 cm (4″) long, is lying in the H.P. on one of its triangular faces with the axis parallel to the V.P. A vertical section plane, the H.T. of which makes an angle of 30° with the ground line, passes through the centre of the base, and cuts the pyramid, the apex being retained. Draw the plan, sectional elevation, true shape of the section and the development of the surface of the cut-pyramid.

(12) A cone, base 7·5 cm (3″) diameter and axis 7·5 cm (3″) long, has its axis parallel to the V.P. and inclined at 45° to the H.P. A horizontal section plane cuts the cone through the mid-point of the axis. Draw the elevation, sectional plan and an auxiliary plan on a plane parallel to the axis.

(13) A cone, base 6·5 cm ($2\frac{1}{2}''$) diameter and axis 7·5 cm ($3''$) long, is lying on one of its generators in the H.P. with the axis parallel to the V.P. A section plane which is parallel to the V.P. cuts the cone 0·6 cm ($\frac{1}{4}''$) away from the axis. Draw the sectional elevation and development of the surface of the remaining portion of the cone.

(14) The cone in problem 13 is cut by a horizontal plane passing through the centre of the base. Draw the sectional plan and another plan on an auxiliary plane parallel to the axis of the cone.

(15) A hemisphere of 6·5 cm ($2\frac{1}{2}''$) diameter, lying in the H.P. on its flat face, is cut by a vertical section plane inclined to the V.P. so that the semi-ellipse seen in the elevation has its minor axis 4·5 cm ($1\frac{3}{4}''$) long and half major axis 2·5 cm ($1''$) long. Draw the plan, sectional elevation and true shape of the section.

(16) The plan of a cylinder 7·5 cm ($3''$) diameter, 12·5 cm ($5''$) long, placed on top of the frustum of a cone, base 10 cm ($4''$) diameter, top 5 cm ($2''$) diameter and axis 12·5 cm ($5''$) long is shown in fig. 16-39. Both the solids are cut by a vertical section plane, the H.T. of which is 1·2 cm ($\frac{1}{2}''$) from the axis of the frustum and makes 30° angle with *xy*. Draw the sectional elevation and true shape of the sections.

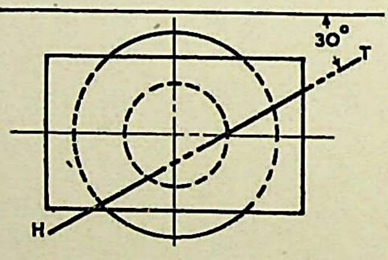


FIG. 16-39

(17) A sphere of 7·5 cm ($3''$) diameter resting in the H.P. is cut by a section plane, perpendicular to the V.P. and inclined at 30° to the H.P. in such a way that the true shape of the section is a circle of 5 cm ($2''$) diameter. Draw its elevation, sectional plan and sectional end elevation.

(18) A frustum of a cone, base 7·5 cm ($3''$) diameter, top 5 cm ($2''$) diameter and axis 7·5 cm ($3''$) long, has a hole of 3 cm

($1\frac{1}{4}''$) diameter drilled centrally through its flat faces. It is resting on its base on the H.P. and is cut by a section plane, the V.T. of which makes an angle of 60° with *xy* and bisects the axis. Draw its sectional plan and an auxiliary plan on a ground line parallel to the V.T., showing clearly the shape of the section.

(19) A hexagonal prism, side of base 2·5 cm (1″) and axis 6·5 cm ($2\frac{1}{2}''$) long is resting on an edge of the base in the H.P., its axis being inclined at 60° to the H.P. and parallel to the V.P. A sectional plane, inclined at 45° to the V.P. and normal to the H.P., cuts the prism and passes through a point on the axis at a distance of 2 cm ($\frac{3}{4}''$) from the top end of the axis. Draw its sectional elevation and true shape of the section.

(20) A pentagonal pyramid, edge of base 2·5 cm (1″) and height 5 cm (2″) is resting on a corner of its base in such a way that the slant edge containing that corner makes an angle of 60° with the H.P. and is parallel to the V.P. It is cut by a section plane making an angle of 30° with the V.P., perpendicular to the H.P. and passing through a point on the axis at a distance of 0·6 cm ($\frac{1}{4}''$) from its base. Draw its sectional elevation and true shape of the section.

(21) The distance between the opposite parallel faces of a 5 cm (2″) thick hexagonal block is 7·5 cm (3″). The block is resting on one of its rectangular faces in the H.P. with its axis making an angle of 30° with the V.P. It is cut by a section plane making an angle of 30° with the H.P., normal to the V.P. and bisecting the axis. Draw its sectional plan and another plan on a plane parallel to the section.

(22) *PQR* is an isosceles triangle having base *PR* horizontal and 5 cm (2″) long, and altitude 5 cm (2″). A point *A* is taken on *PR* at a distance of 1·5 cm ($\frac{5}{8}''$) from *P* and a straight line *AB* is drawn parallel to *PQ* cutting *QR* at *B*. If *AB* is regarded as the V.T. of an inclined plane perpendicular to the V.P., cutting a cone of which *PQR* is the elevation, draw the sectional plan, sectional end view and true shape of the section.

CHAPTER XVII

INTERSECTION OF SURFACES

In this chapter, we shall learn about the intersection of surfaces. These intersecting surfaces may be two plane surfaces or two curved surfaces of solids. (The lateral surface of every solid taken as a whole is a curved surface. This surface may be made of only curved surface as in case of cylinders cones etc. or of plane surfaces as in case of prisms, pyramids etc.). In the former case, the problem is said to be on the intersection of surfaces and in the latter case, it is commonly known as the problem on interpenetration of solids. It may, however, be noted that when two solids meet or join or interpenetrate, it is the curved surfaces of the two that intersect each other. The latter problem also is, therefore, on the intersection of surfaces.

Line of intersection:

In engineering practice, objects constructed may have constituent parts, the surfaces of which intersect one another in lines which are called *lines of intersection.* A dome fitted on a boiler is one such example. The surface of the dome extends upto the line of intersection only. For accurate development of the surface of the dome, this line of intersection must be accurately located and shown in two orthographic views. The shape of the hole to be cut in the boiler-shell is also determined from the shape of the same line of intersection.

Thus, the line of intersection of the two surfaces is a line common to both. It is composed of points at which the lines of one surface intersect those on the other surface. The line of intersection may be straight or curved, depending upon the nature of intersecting surfaces. Two plane surfaces (e.g. faces of prisms and pyramids) intersect in a straight line. The line of intersection between two curved surfaces (e.g. of cylinders and cones) or between a plane surface and a curved surface is a curve.

When a solid completely penetrates another solid, there will be two lines of intersection. These lines are, sometimes, called *the lines or curves of interpenetration.* The portion of the penetrating solid which lies hidden within the other solid is shown by dotted lines.

METHODS OF DETERMINING THE LINE OF INTERSECTION BETWEEN SURFACES OF TWO INTERPENETRATING SOLIDS:

(1) *Line method:* A number of lines are drawn on the lateral surface of one of the solids and in the region of the line of intersection. Points of intersection of these lines with the surface of the other solid are then located. These points will obviously lie on the required line of intersection. They are more easily located from the view in which the lateral surface of the second solid appears edge-wise (i.e. as a line). The curve drawn through these points will be the line of intersection.

(2) *Cutting-plane method:* The two solids are assumed to be cut by a series of cutting planes. The cutting planes may be vertical (i.e. perpendicular to the H.P.), edge-wise (i.e. perpendicular to the V.P.) or oblique. The cutting planes are so selected as to cut the surface of one of the solids in straight lines and that of the other in straight lines or circles.

Each method is explained in detail while solving illustrative problems. Sound knowledge of projections of solids in various positions is quite essential while dealing with these problems.

Intersection of two prisms:

Prisms have plane surfaces as their faces. The line of intersection between two plane surfaces is obtained by locating the positions of points at which the edges of one surface intersect the other surface and then joining the points by a straight line. These points are called *vertices* (plural of vertex). The line of intersection between two prisms is therefore a closed figure composed of a number of such lines meeting at the vertices. It is determined by locating the points at which

edges of one prism intersect edges or faces of the other prism and then joining them in correct sequence.

Problem 1 (fig. 17-1):

A prism, base 5 cm (2") side, resting vertically in the H.P. is completely penetrated by a horizontal square prism, base 4 cm ($1\frac{1}{2}$") side, so that their axes intersect. The faces of the two prisms are equally inclined to the V.P. Draw the projections of the solids, showing lines of intersection.

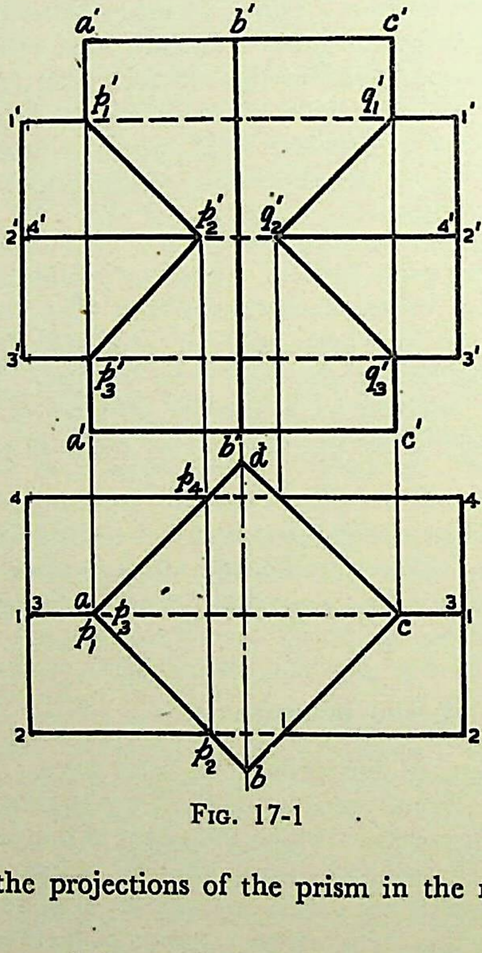


FIG. 17-1

Draw the projections of the prism in the required position.

The faces of the vertical prism are seen as lines in the plan. Hence, let us first locate the points of intersection in

that view. Lines 1-1 and 3-3 intersect the edge of the vertical prism in points p_1 and p_3 (coinciding with a). Lines 2-2 and 4-4 intersect the faces at p_2 and p_4 respectively. The exact positions of these points along the length of the prism may now be determined by projecting them on corresponding lines in the elevation. For example, p_2 is projected to p_2' on the line $2'2'$. Note that p_4' coincides with p_2'. Draw lines $p_1'p_2'$ and $p_2'p_3'$. Lines $p_1'p_4'$ and $p_3'p_4'$ coincide with the front lines. These lines show the line of intersection. Lines $q_1'q_2'$ and $q_2'q_3'$ on the other side are obtained in a similar manner. Note that the lines for the hidden portion of the edges are shown dotted. The portions $p_1'p_3'$ and $q_1'q_3'$ of vertical edges $a'a'$ and $c'c'$ do not exist and hence, must be removed or kept fainter.

Problem 2 (fig. 17-2):

A square prism, base 5 cm (2″) side and height 9 cm ($3\frac{1}{2}$″) is standing vertically in the H.P. with a face inclined at 30° to the V.P. It is completely penetrated by another square prism, base 4 cm ($1\frac{1}{2}$″) side and axis 10 cm (4″) long, faces of which are equally inclined to the V.P. The axes of the two prisms are parallel to the V.P. and bisect each other at right angles. Draw the elevation and plan showing curves of intersection.

Points $p_1' \ldots p_4'$ at which edges of the horizontal prism intersect faces of the vertical prism may be located from the plan. In addition to these points, it will be necessary to find points at which edges of the vertical prism are cut. They will be the points at which these edges intersect the faces of the horizontal prism. For this purpose, draw the end elevation. In this view, all faces of the horizontal prism are seen as lines. Mark points e and f at which the line $a''a''$ intersects the faces. Project these two points to e' and f' on the line $a'a'$ in the elevation. Join all the points of intersection in correct sequence. Care must be taken to determine visible and hidden lines. Only two lines viz. $p_1'p_2'$ and $p_2'p_3'$ are visible. Locate points (on the other side) at which the edges come out and also the two points g' and h' at which the edge $c'c'$ is cut. Draw lines joining these points

in correct order. In this case, lines $q_2'g'$ and $q_2'h'$ only are visible.

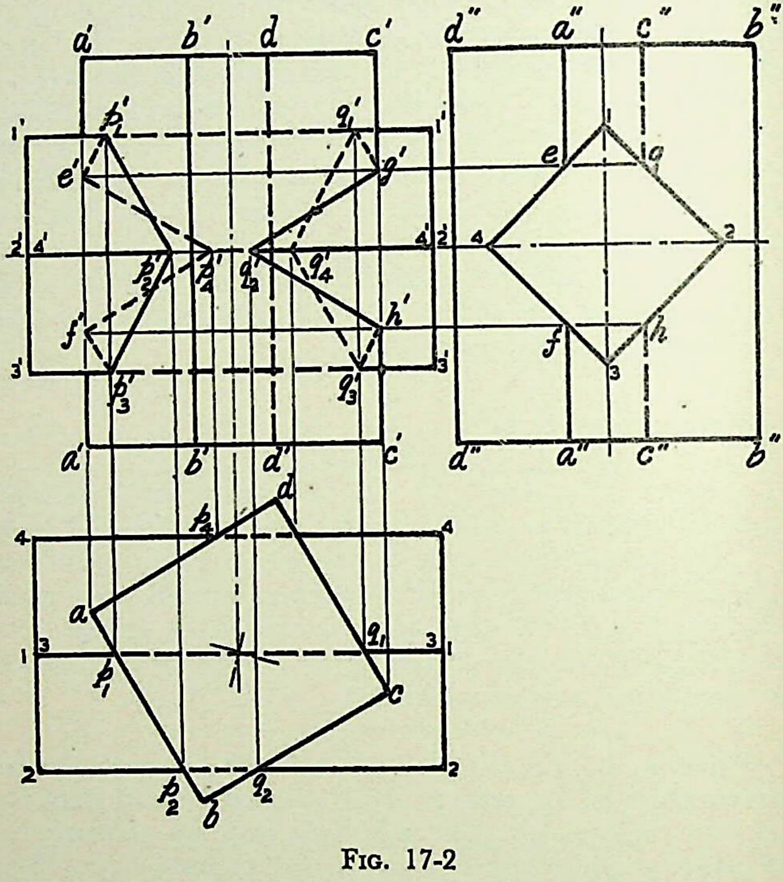


FIG. 17-2

A square pipe having a similar branch of smaller size is shown in fig. 17-3(*i*). The axis of the main pipe is vertical and is intersected by that of the branch at an angle of 45°. All the fàces of both the pipes are equally inclined to the V.P. The line of intersection between the two pipes is obtained in the same manner as shown in problem 1. As the axes are intersecting, the edge $a'a'$ is cut by the two edges of the branch at points p_1' and p_3'. The other two edges of the branch enter the faces of the main pipe at points p_2' and p_4'.

Developments of the surface of the two pipes are shown in fig. 17-3(*ii*). Heights of all the points above *C-C* and 3-3 are obtained from the elevation, e.g. $P_1A = p_1'a'$, $P_11 = p_1'1'$ etc. The exact position of point P_2 is located from the plan by making $AE = ap_2$ and then erecting a perpendicular at *E*. The point P_4 is similarly located.

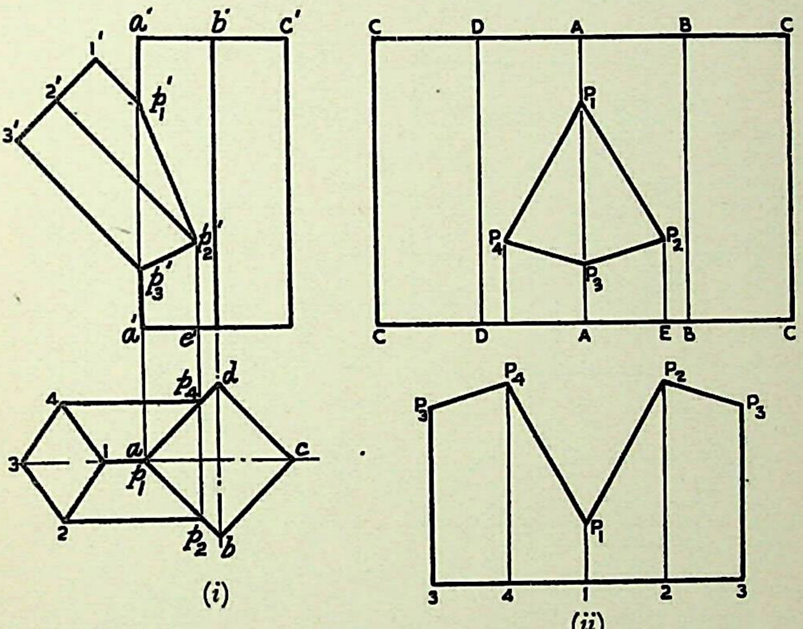


FIG. 17-3

Problem 3 (fig. 17-4):

A vertical square prism, base 5 cm (2") side, is intersected by another square prism, base 4 cm (1½") side, the axis of which is parallel to the V.P. and inclined at 30° to the H.P. The axes of the two prisms are 0·6 cm (¼") apart and their faces are equally inclined to the V.P. Draw the projections showing the line of intersection.

Obtain points of intersection of the edges of the inclined prism from the plan. For the points at which the edge *a'a'* of the vertical prism is cut, it will be necessary to project

a view in which faces of the inclined prism will be seen as lines. Therefore, project an auxiliary plan on a ground line x_1y_1 drawn perpendicular to the axis of the inclined prism. Mark points e and f at which $a''a''$ is pierced by the faces and project them to points e' and f' on the corresponding line $a'a'$ in the elevation. Draw straight lines joining the six points in correct sequence.

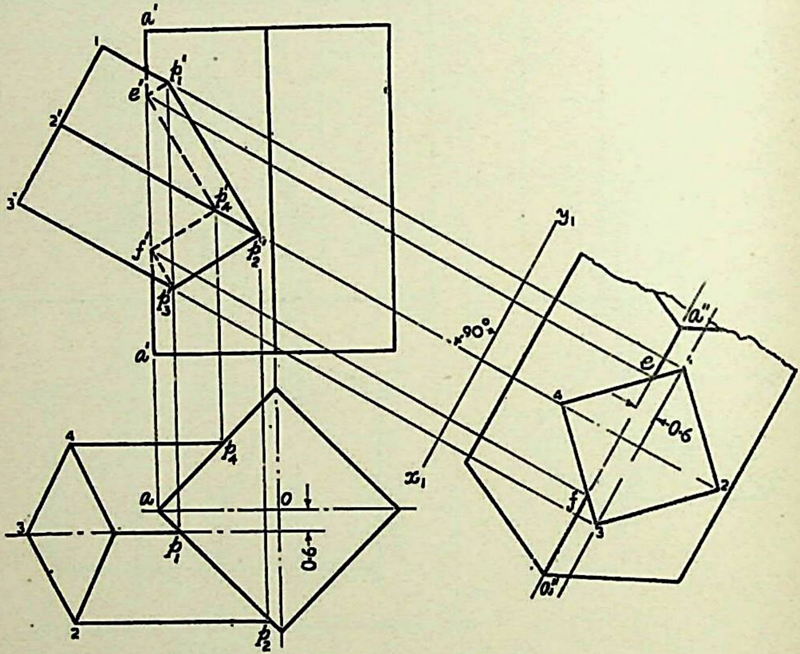


FIG. 17-4

Intersectoin of two cylinders:

As cylinders have their lateral surfaces curved, the line of intersection between them will also be curved. Points on this line may be located by any one of the two methods. For plotting an accurate curve, certain *critical or key points*, at which the curve changes direction, must also be located. These are the points at which outermost or extreme lines of each cylinder pierce the surface of the other cylinder. In prisms, vertices are the key points.

Problem 4 (fig. 17-5):

A cylinder of 7·5 cm (3") diameter, standing on its base in the H.P., is completely penetrated by another cylinder of 5·6 cm ($2\frac{1}{4}$") diameter, their axes bisecting each other at right angles. Draw their projections showing curves of penetration, assuming the axis of the penetrating cylinder to be parallel to the V.P.

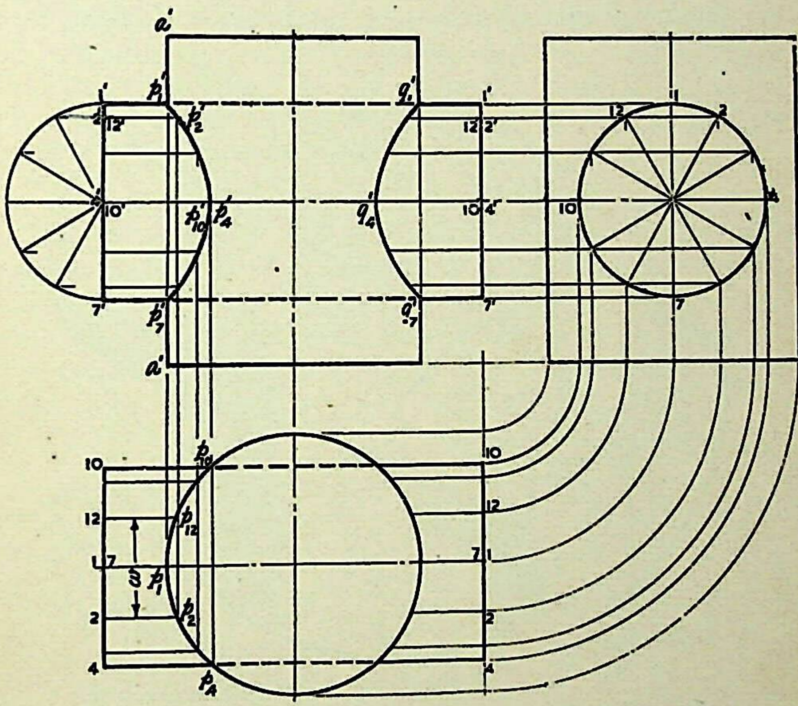


FIG. 17-5

Draw the plan and elevation of the cylinders. To mark lines, spaced equally apart, on the surface of the horizontal cylinder, project an end elevation. Divide the circle into twelve equal parts. Project the division-points in the elevation and draw lines through them. Project these lines in the plan and name them. Mistakes are often committed in projecting and naming the lines from one view to the other; hence, these lines are shown projected in every detail. Positions of these lines may also be obtained from the semi-circle drawn at an end of the cylinder.

(a) *Line method:* Mark points p_1, p_2 etc. at which lines 1-1, 2-2 etc. intersect the circle (showing the surface of the vertical cylinder) in the plan and project them to p_1', p_2' etc. on corresponding lines 1′1′, 2′2′ etc. in the elevation. Draw the required curves on both sides of the axis through points thus located. Hidden portions of the curves coincide with the visible portions. Points p_1', p_4', p_7' and p_{10}' are the key points where the curve changes direction.

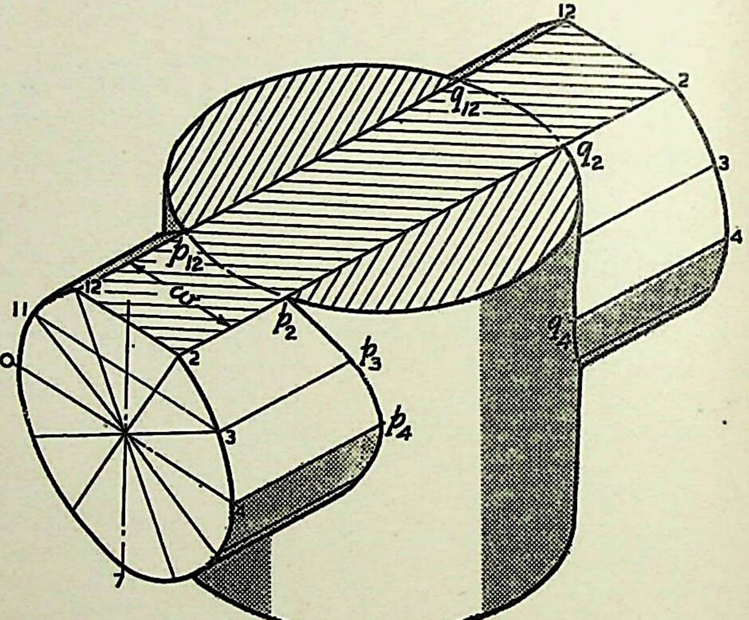


FIG. 17-6

(b) *Cutting-plane method:* It will be seen that in this problem, there is practically no difference between the line method and the cutting-plane method. But the latter method proves more useful in solving problems in which none of the projections shows a line-view of the surface of a solid.

Assume a series of horizontal cutting planes passing through the lines on the horizontal cylinder and cutting both cylinders. Sections of the horizontal cylinder will be rectangles, while those of the vertical cylinder will always be circles of the same diameter as its own. Points at which

sides of the rectangles intersect the circle will lie on the curve of intersection. For example, let a horizontal section plane pass through points 2 and 12 (fig. 17-6). In elevation, it will be seen as a line coinciding with the line 2′2′. The section of the horizontal cylinder will be a rectangle of width w (i.e. the line 2-12). The section of the vertical cylinder will be a circle. Points p_2 and p_{12} at which the sides (2-2 and 12-12) of the rectangle cut the circle, lie on the curve. These points are first marked in the plan (fig. 17-5) and then projected to points p_2' and p_{12}' on lines 2′2′ and 12′-12′ in the elevation. Points on the other side of the vertical axis are located in the same manner.

The problem may also be solved by assuming cutting planes to be vertical and parallel to both axes. They will be seen as lines in plan and end elevation. Sections of both cylinders will be rectangles and will be seen in their true sizes in the elevation. Points at which sides of sections of one cylinder intersect sides of corresponding sections of the other, will lie on the curve of intersection.

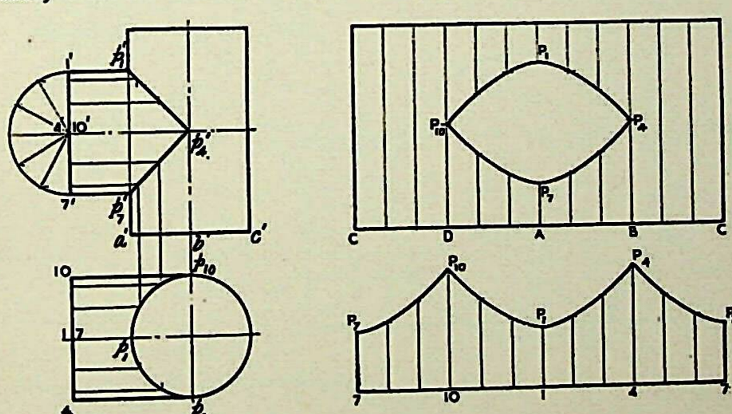


FIG. 17-7

When two cylinders are of equal diameters and when their axes intersect, curves of intersection are seen in the elevation as straight lines intersecting at right angles. Fig. 17-7 shows a pipe joining another pipe of the same size at right angles. Developments of the surfaces of the pipes are shown in the same figure.

Problem 5 (fig. 17-8):

A cylinder of 7·5 cm (3") diameter, resting on its base in the H.P., is penetrated by another cylinder of 5 cm (2") diameter, the axis of which is parallel to both the H.P. and the V.P. The two axes are 1 cm (⅜") apart. Draw the plan and elevation showing curves of intersection.

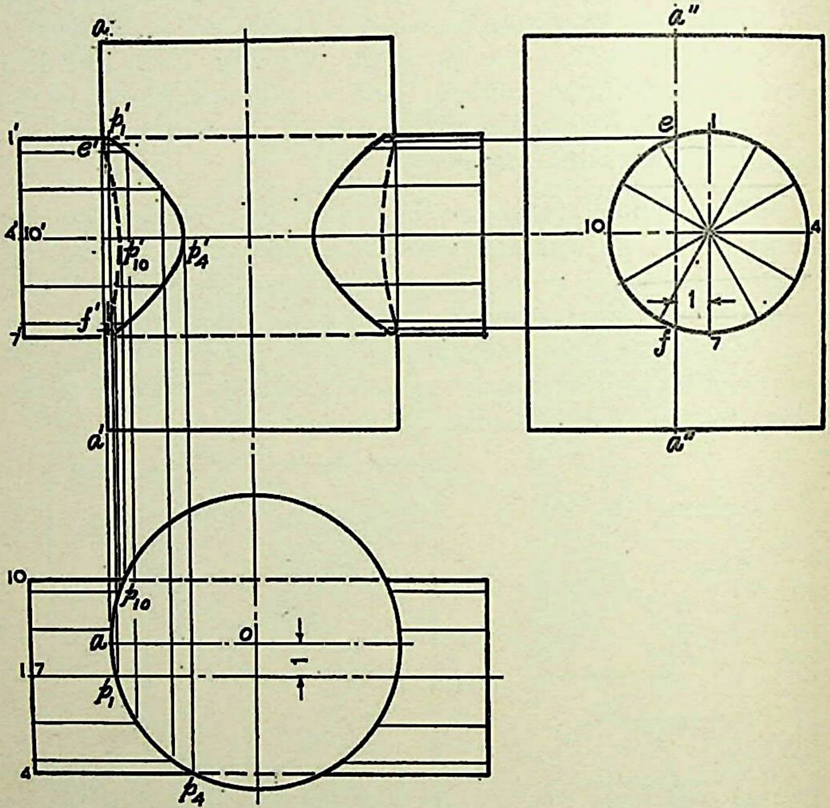


FIG. 17-8

Obtain the twelve points as in problem 4. Two more key points at which the extreme line *a'a'* of the vertical cylinder is cut, must be located. These are found from the end elevation. Mark points *e* and *f* at which the line *a"a"* intersects the circle. Project these points to *e'* and *f'* on the line *a'a'*. Draw the curve passing through all the points in correct sequence, showing the hidden portion by dotted lines. Plot a similar curve on the other side of the axis.

Problem 6 (fig. 17-9):

A cylinder of 7·5 cm (3") diameter, resting on its base in the H.P., is penetrated by another cylinder of the same size. The axis of the penetrating cylinder is parallel to both the H.P. and the V.P. and is 1 cm ($\frac{3}{8}$") away from the axis of the vertical cylinder. Draw the projections showing curves of intersection.

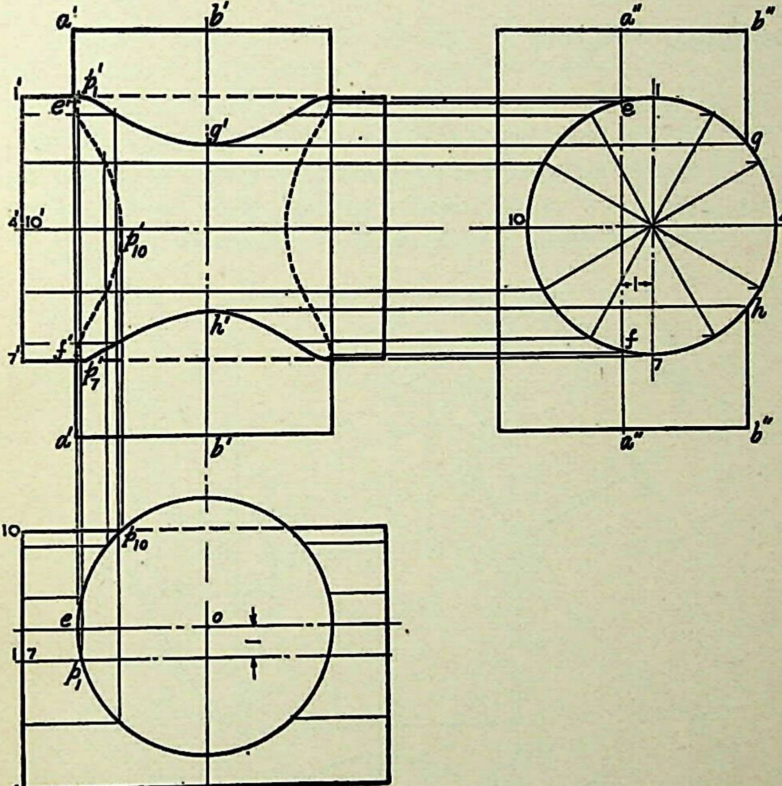


FIG. 17-9

As the cylinders are of the same size and their axes are apart, a portion of the surface of the penetrating cylinder will be outside the vertical cylinder.

Draw the three views and project points of intersection of lines which lie within the vertical cylinder. Locate key points e' and f' as shown in problem 5. In addition to these,

mark two more key points g and h (in the end elevation) where the circle cuts extreme line $b''b''$ of the vertical cylinder. Project these points to g' and h' on the line $b'b'$ in the elevation. Draw a curve through all the points, showing the hidden portion by dotted lines. Instead of two separate curves of intersection we have one continuous curve. Note that there are twelve key points in this curve.

Problem 7 (fig. 17-10):

A cylinder of 7·5 cm (3") diameter, having its axis vertical is penetrated by another cylinder of 5 cm (2") diameter. The axis of the penetrating cylinder is parallel to the V.P. and bisects the axis of the vertical cylinder, making an angle of 60° with it. Draw the plan and elevation, showing curves of intersection.

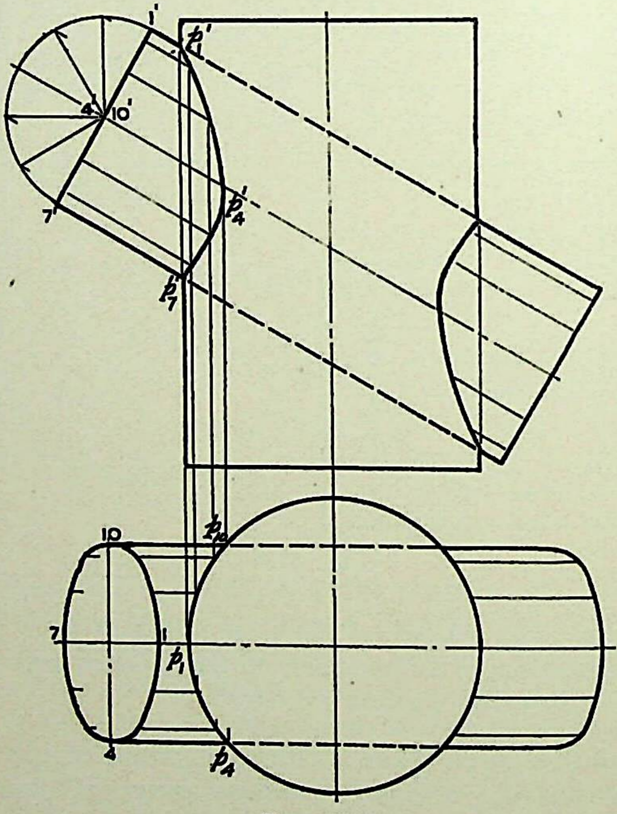


FIG. 17-10

Draw the projections of the cylinders in the required position and proceed to locate the points on curves of intersection as shown in problem 4. The back and front curves will coincide with each other.

Problem 8 (fig. 17-11):

A vertical pipe of 7·6 cm (3″) diameter has a branch of 3·8 cm (1½″) diameter. The axis of the branch is inclined at 45° to the ground and is 1·9 cm (¾″) away from the axis of the main pipe. Draw the elevation and plan of the pipes showing the curve of intersection. Also, develop the surfaces of the two pipes, assuming any lengths.

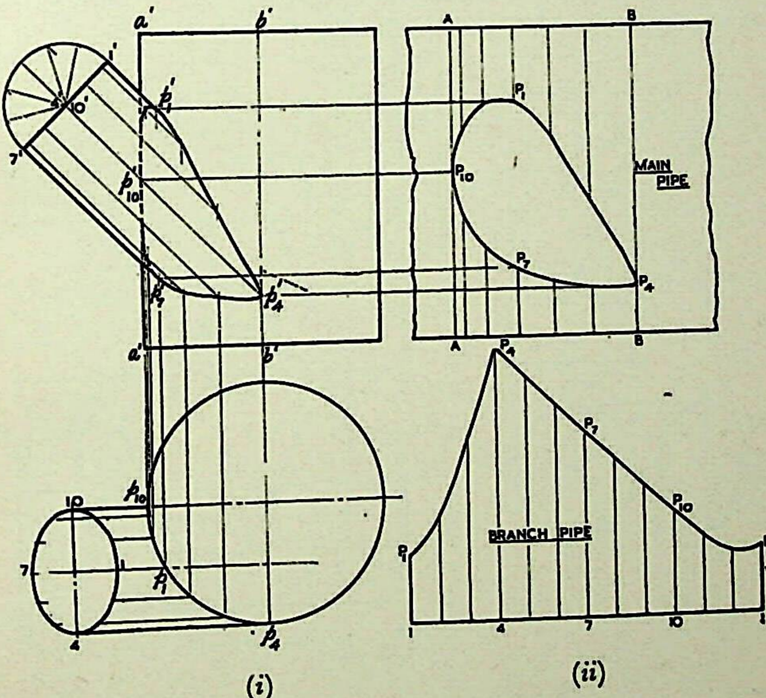


Fig. 17-11

Apply the same method as in problem 4. As the diameter of the branch and the distance between the two axes are respectively one-half and one-fourth of the diameter of

the main pipe, one extreme line of the branch (in the plan) will be tangent to the circle. In the elevation, the visible part of the curve will extend upto the centre line of the main pipe, while the hidden part will just touch the line *a'a'*.

Only a part of the development of the main pipe, just sufficient to show the shape of the hole in it, is shown in fig. 17-11(*ii*). Distances along the length of the stretch-out line are taken from the plan and positions of points are projected from the elevation.

In the development of the branch, the stretch-out line is divided into twelve equal parts and heights of points are taken from the elevation, e.g. $4P_4 = 4'p_4'$ etc.

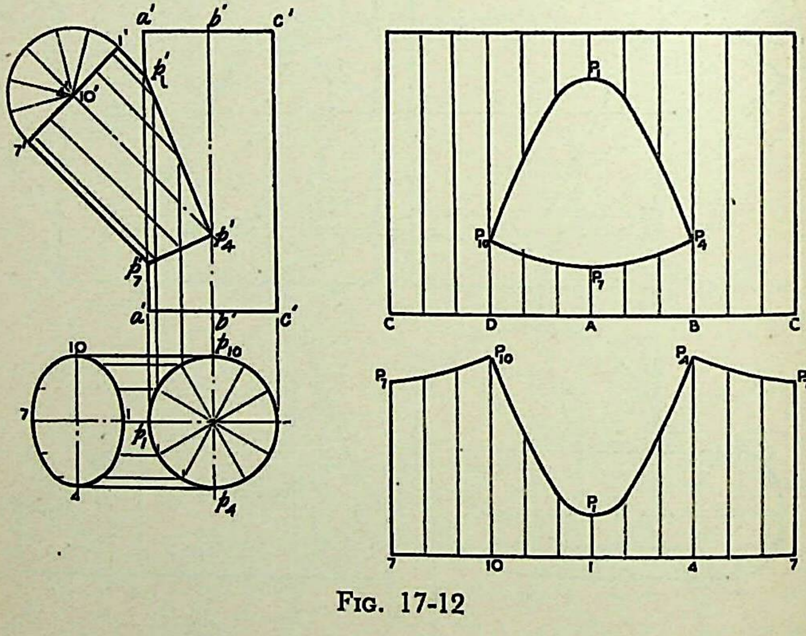


FIG. 17-12

Fig. 17-12 shows two equal pipes meeting at an angle of 45°. The curve of intersection in the elevation is made up of two straight lines meeting at right angles. Developments of the surfaces of the pipes are also shown in the same figure.

Intersection of cylinder and prism:

Problem 9 (fig 17-13):

A cylinder of 7·5 cm (3") diameter, resting on its base in the H.P., is penetrated by a horizontal square prism, base 5 cm (2") side, the axis of which is parallel to the V.P. and 1·2 cm (½") away from the axis of the cylinder. A face of the prism makes an angle of 30° with the H.P. Draw their projections, showing curves of intersection.

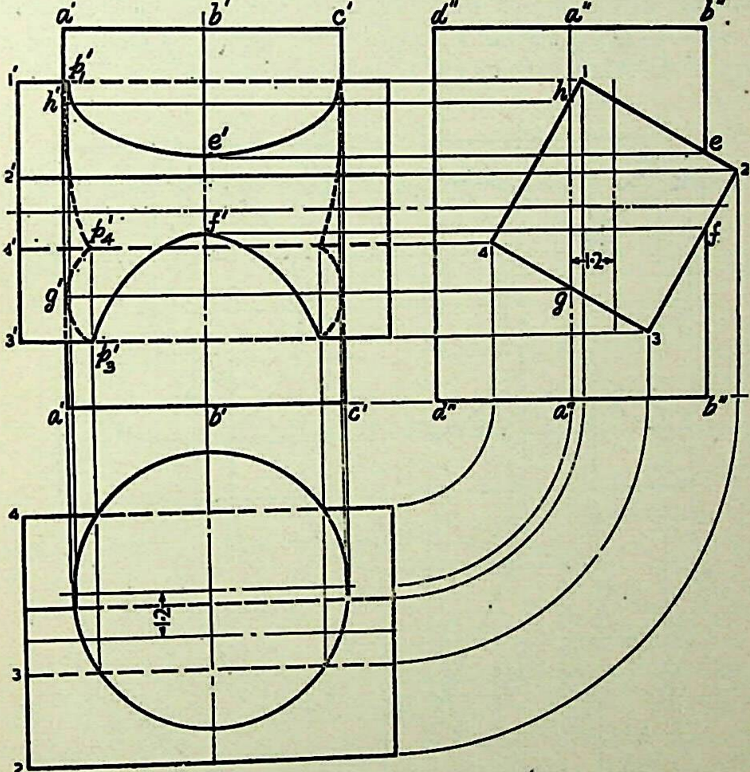


FIG. 17-13

Draw the views in the required position. One longer edge of the prism will remain outside the cylinder. Project all the key points viz. points of intersection of the edges of the prism with the cylinder and those of the extreme lines of the cylinder with the surface of the prism, as shown in the figure.

To obtain accurate shape of the curves project few more intermediate points also. These are omitted from the figure. Draw the required curve of intersection through these points, taking precaution to show the hidden part by dotted lines.

Problem 10 (fig. 17-14):

A square prism, base 5 cm (2″) side, has its base in the H.P. and a face inclined at 30° to the V.P. It has a hole of 6·5 cm ($2\frac{1}{2}$″) diameter drilled through it. The centre line of the hole is parallel to both the H.P. and the V.P. and is 0·5 cm ($\frac{3}{16}$″) away from the axis of the prism. Draw the projections of the prism.

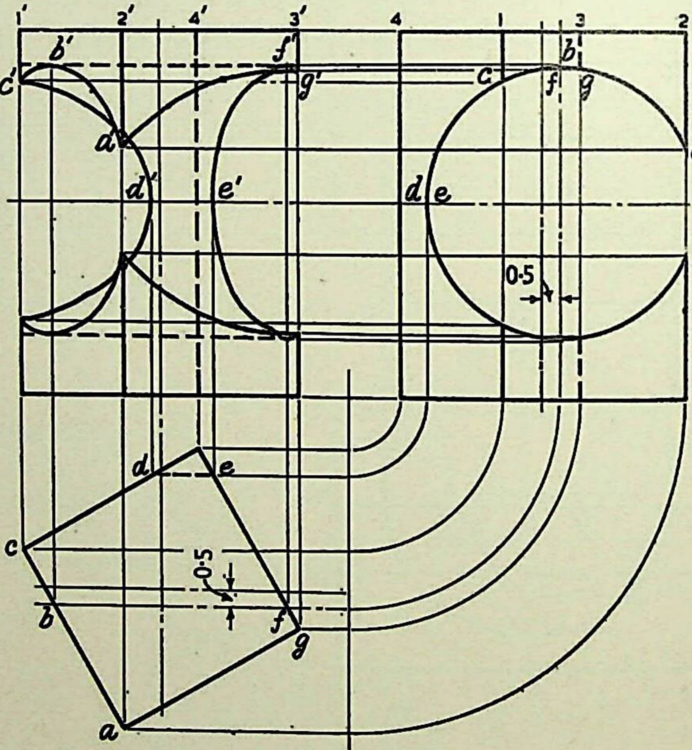


Fig. 17-14

Draw the views and project all the key points as shown. Plot few intermediate points (between the key points) also.

Draw a curve joining the points in correct sequence. As it is a hole cut in the prism, the curve at the back will also be visible. Only a small portion of the curve will not be seen.

Problem 1(fig. 1 17-15):

A connecting rod, 5 cm (2") diameter, has a rectangular block 6·5 cm (2½") wide and 2·5 cm (1") thick, forged at its end. The rod joins the block with a turned radius of 2 cm (¾"). Draw the elevation and plan of the rod showing curve of intersection.

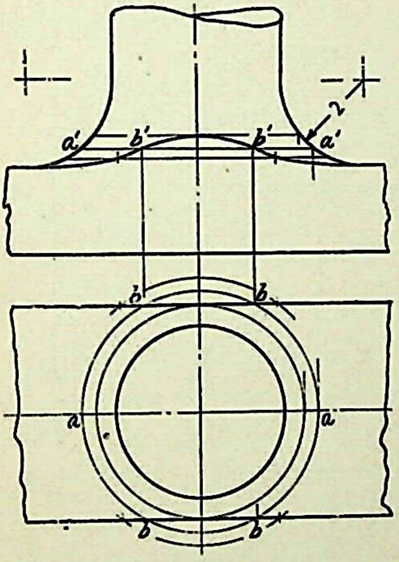


FIG. 17-15

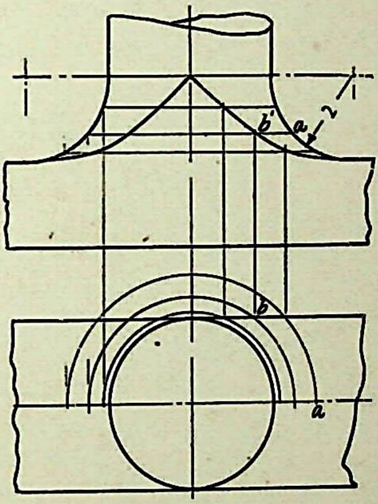


FIG. 17-16

The rod increases in diameter as it approaches the block. This forms what is called a fillet of 2 cm (¾") radius. As the width of the block is not as large as the biggest diameter of the rod, a curve of intersection is formed. Points on the curve are found as shown below.

Assume a horizontal cutting plane passing through a line, say *a'a'*. The section of the rod is a circle (see plan) which cuts the sides of the block at points *b*. Project these points to points *b'* on the line *a'a'*. Then the points *b'* lie

on the curve. Obtain more points by assuming additional sections. The highest point on the curve will be on a line where the diameter of the section of the rod is equal to the width of the block.

Fig. 17-16 shows the shape of the curve when the diameter of the rod is equal to the width of the block. In this case, the highest point on the curve is on a horizontal line through the centre of the arc for the fillet.

Problem 12:

Draw two views of the forked end of the connecting rod shown in fig. 17-17, showing the curve of intersection.

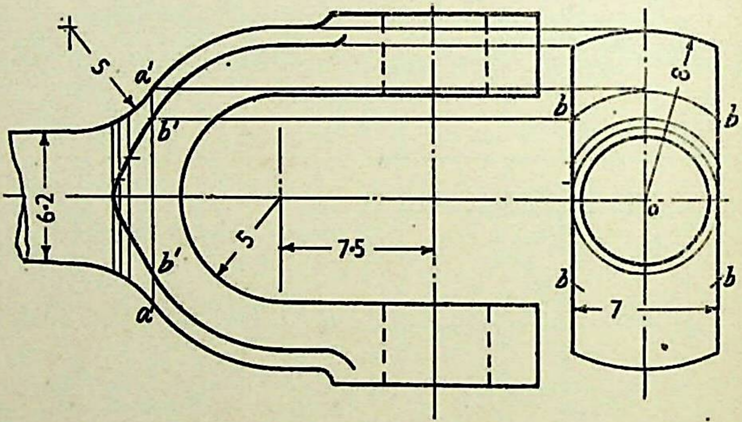


FIG. 17-17

In this case, a cylindrical rod meets a forked piece, the sides of which are flat surfaces, while the top and bottom surfaces are cylindrical.

Assume a series of section planes perpendicular to the axis of the rod. Points at which circles for the sections intersect the flat faces of the fork, lie on the curve. Take a section plane say at *a'a'*. With centre *o* and radius equal to one-half of *a'a'*, draw arcs cutting the sides of the fork at points *b* (in the end view). Project these points to points *b'* on *a'a'*. Obtain other points in the same manner and draw the required curve through them.

Problem 13:

Projections of a steel chimney of 1·2 m (4′) diameter erected on a roof, are given in fig. 17-18(i). The axis of the chimney is at a distance of 0·3 m (1′) from the ridge. Project an end elevation showing the curve of intersection. Determine the real shape of the hole in the roof.

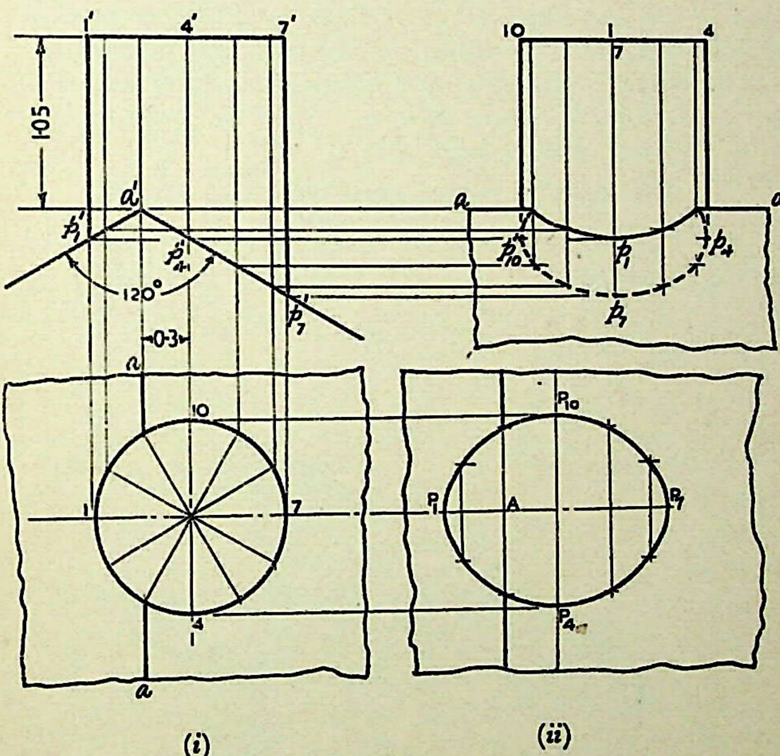


FIG. 17-18

Draw the given views and project the end elevation. To obtain points on the curve, draw lines on the surface of the chimney by dividing the circle (in plan) into twelve equal parts. Project points at which these lines intersect the two lines for the roof in the elevation, to corresponding lines in the end view. Draw a curve through the points thus obtained. The lower part of the curve will be hidden.

The real shape of the hole in the roof is shown below the end view. Horizontal distances are taken from elevation and widths are projected from the plan.

Intersection of cone and cylinder:

Problem 14:

A cone, diameter of base 7·5 cm (3″) and axis 10 cm (4″) long, resting on its base in the H.P. is completely penetrated by a cylinder of 4·5 cm ($1\frac{3}{4}$″) diameter. The axis of the cylinder is parallel to the H.P. and the V.P. and intersects the axis of the cone at a point 2·8 cm ($1\frac{1}{8}$″) above the base. Draw the projections of the solids showing curves of intersection.

(a) *Cutting-plane method* (figs. 17-19 and 17-20):

Draw lines dividing the surface of the cylinder into twelve equal parts.

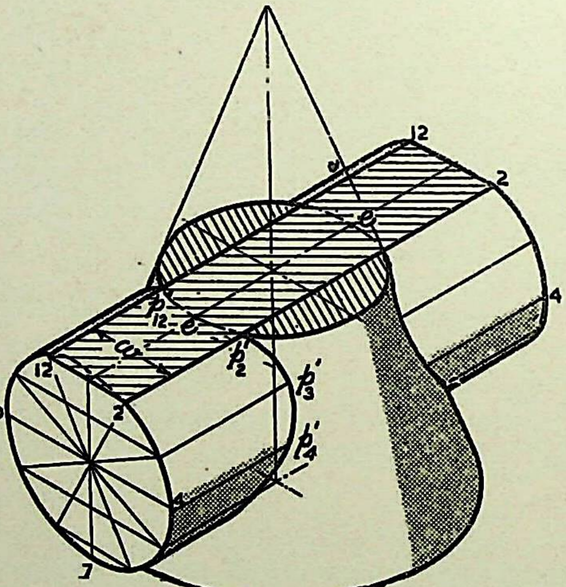


FIG. 17-19

Assume a horizontal cutting plane passing through say, point 2 (fig. 17-19). The section of the cylinder will be a rectangle of width *w* (i.e. the line 2-12), while that of the cone will be a circle of diameter *ee*. These two sections

intersect at points p_2 and p_{12}. These sections are clearly indicated in the plan by the rectangle 2-2-12-12 and the circle of diameter *ee* (fig. 17-20). In the elevation, the cutting plane is seen as a line coinciding with 2′2′. Points p_2 and p_{12} when projected on the line 2′2′ (with which the line 12′-12′ coincides) will give a point p_2' (with which p_{12}' will coincide). Then p_2' and p_{12}' are the points on the curve of intersection.

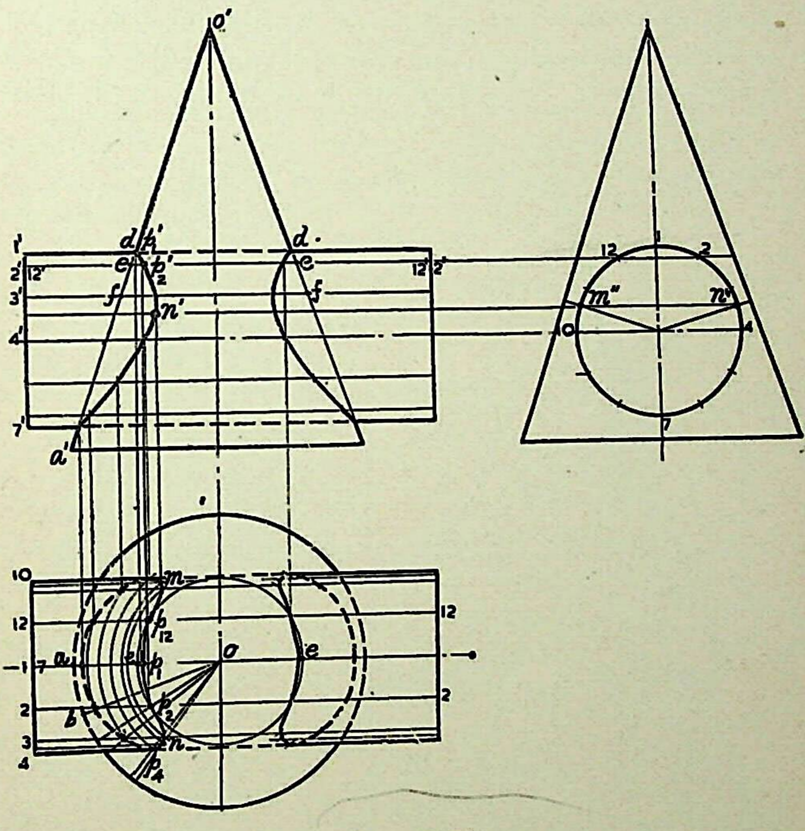


FIG. 17-20

To obtain the points systematically, draw circles with centre *o* and diameters *dd*, *ee*, *ff* etc. cutting lines through 1, 2 and 12, 3 and 11 etc. at points p_1, p_2 and p_{12}, p_3 and p_{11} etc. Project these points to the corresponding lines in the elevation. Two more key points at which the curve changes

direction must also be located. Their positions are determined from the end view. They are the points of nearest approach viz. m'' and n'' at which, lines drawn from the centre of the circle (i.e. the axis of the cylinder) and perpendicular to the extreme generators of the cone, cut the circle. Project these points to m' and n' in the elevation and to m and n in the plan on the corresponding lines. Draw curves through these points in both the views. The back curve in the elevation will coincide with the front curve. In the plan, a part of the curve will lie hidden and hence, it will be dotted. Draw similar curves on the right-hand side of the axis of the cone.

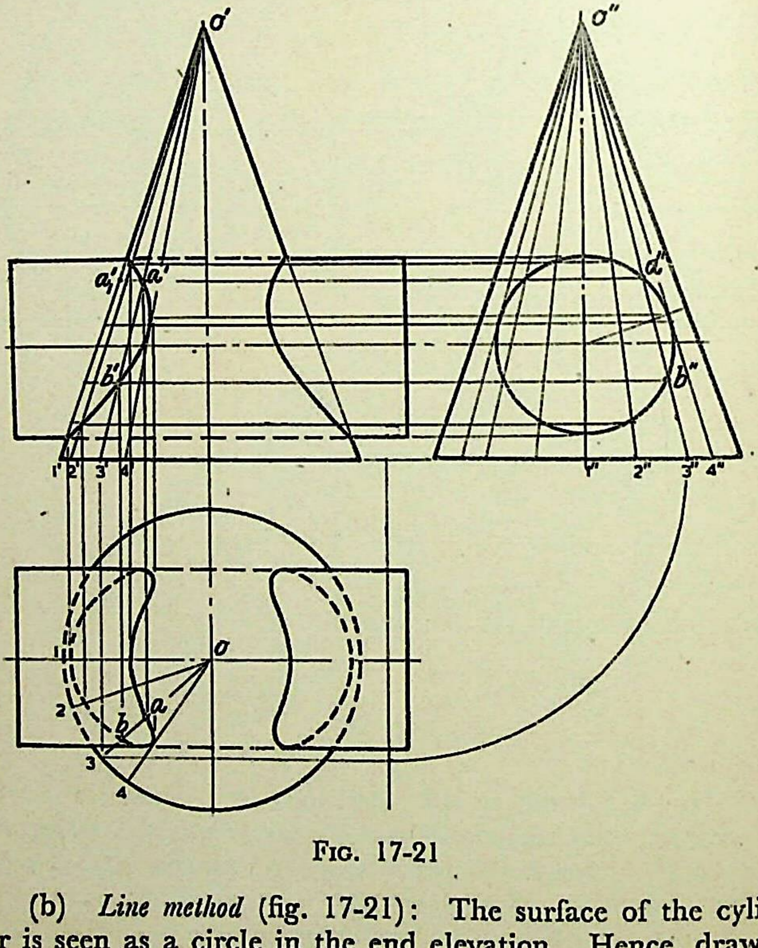


Fig. 17-21

(b) *Line method* (fig. 17-21): The surface of the cylinder is seen as a circle in the end elevation. Hence, draw a

number of lines (representing generators of the cone) $o''1''$, $o''2''$ etc. in the region of the circle and symmetrical on both sides of the axis. Points where these lines intersect the circle, lie on the curve of intersection. To project them in the elevation and plan, first project the lines in both the views and then, locate the positions of these points on them. Let us take the line $o''3''$ in the end view. Locate its position $o3$ in the plan as shown and project its elevation $o'3'$. Project points a'' and b'' to points a' and b' on $o'3'$ and from there, to a and b on $o3$.

Project all points in a similar manner and draw the required curves through them. This method is a type of cutting-plane method in which cutting planes pass through the apex and are parallel to the axis of the cylinder.

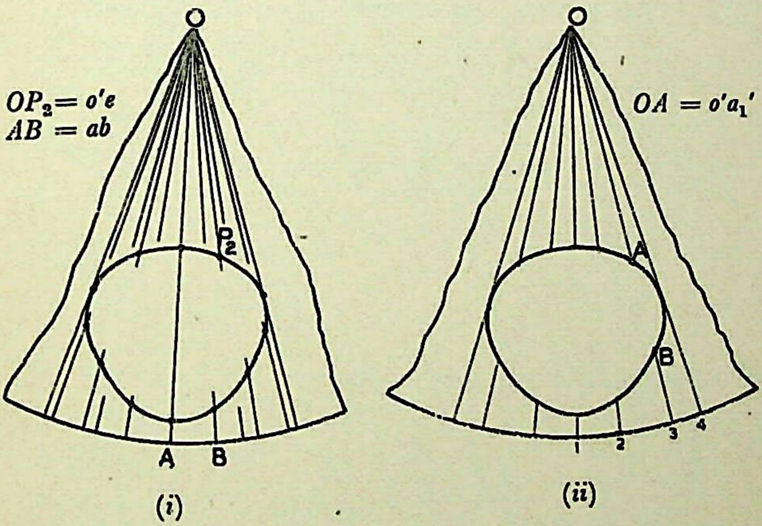


FIG. 17-22

Fig. 17-22 shows the development of a portion of the surface of the cone showing the hole in it. In fig. 17-22(*i*), the positions of points are taken from fig. 17-20, while in fig. 17-22(*ii*), they are obtained from fig. 17-21. In each case, distances along the arc are measured from the plan; while the distances of points (on the generators) from o are taken from the elevation, after projecting them on the true length line.

Problem 15 (fig. 17-23):

A cone, base 7·5 cm (3") diameter, axis 10 cm (4") long, resting on its base in the H.P., is penetrated by a horizontal cylinder of 4 cm ($1\frac{1}{2}$") diameter, the axis of which is 2·5 cm (1") above the H.P., parallel to the V.P. and 0·6 cm ($\frac{1}{4}$") away from the axis of the cone. Draw the projections, showing curves of intersection.

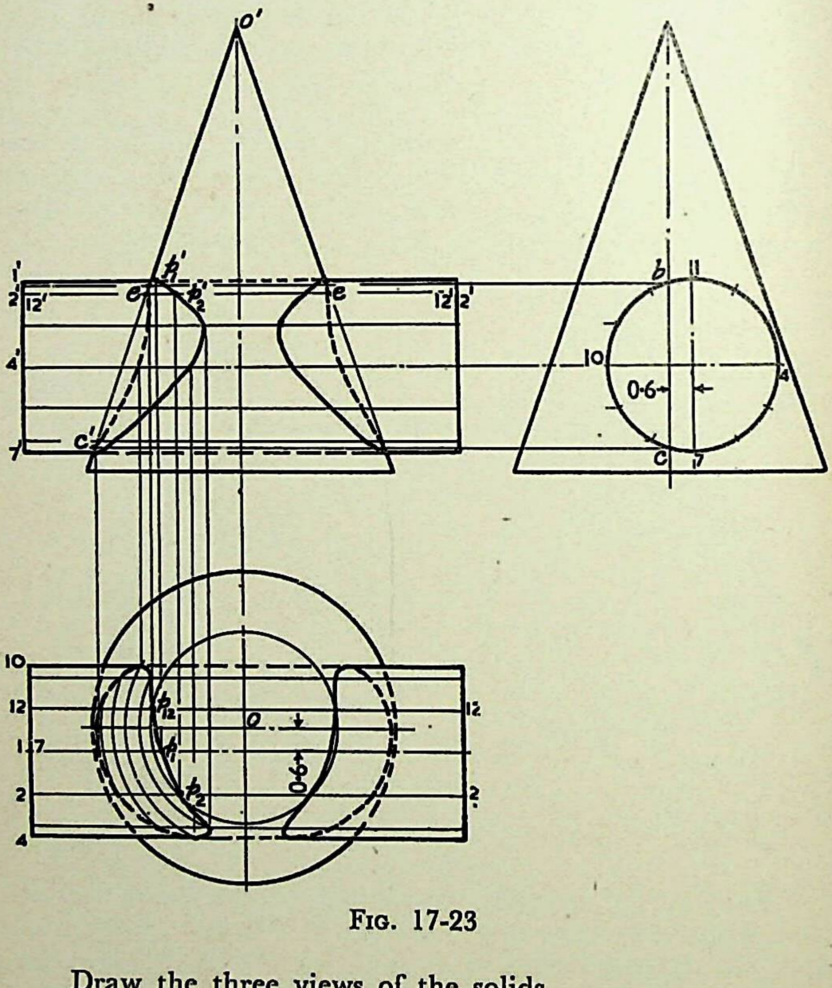


FIG. 17-23

Draw the three views of the solids.

Assuming a horizontal cutting plane through the line 2'2', draw a circle in the plan with centre *o* and diameter *ee*, cutting lines 2-2 and 12-12 at points p_2 and p_{12}. Project these

points to p_2' and p_{12}' on the line 2′2′ (with which 12′-12′ coincides). Obtain other points and the key points b' and c' in the same manner and draw the curves as shown.

As the axes do not intersect, the back curves in the elevation are different from the front curves.

Fig. 17-24 shows the axis of the penetrating cylinder so displaced that a portion of the cylinder remains outside the cone. The curve of intersection is obtained in the same

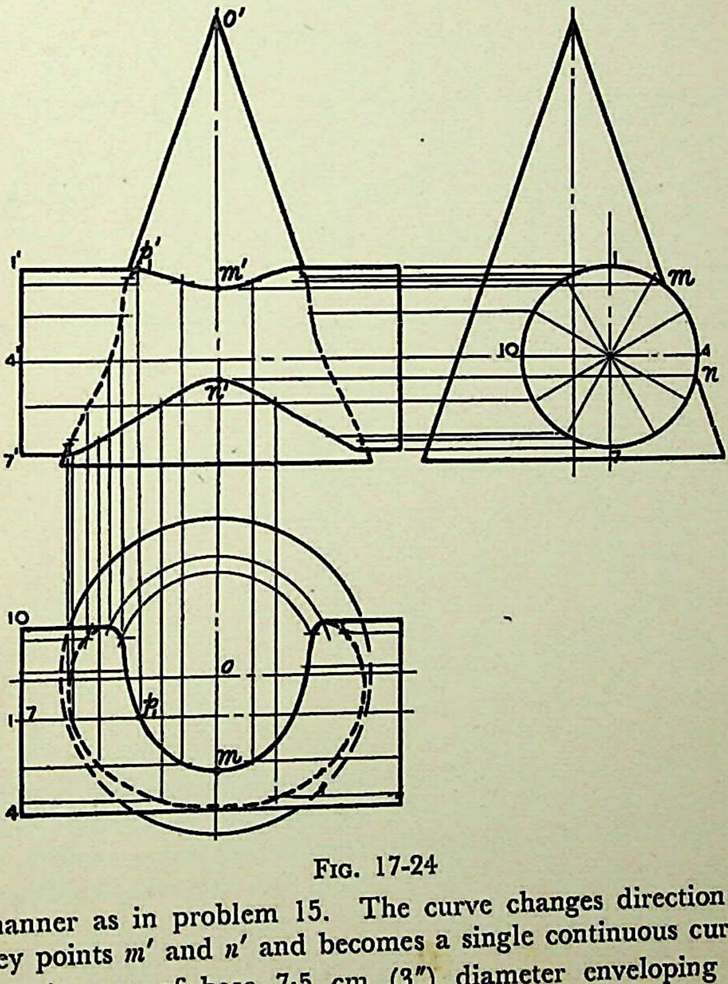


FIG. 17-24

manner as in problem 15. The curve changes direction at key points m' and n' and becomes a single continuous curve.

A cone of base 7·5 cm (3″) diameter enveloping an imaginary sphere of 5 cm (2″) diameter and having its

centre 3 cm ($1\frac{1}{4}''$) above the base of the cone, is shown in fig. 17-25. It is penetrated by a cylinder which also envelopes the same sphere. The axes of the cone and the cylinder intersect each other at right angles and lie in a plane parallel to the V.P.

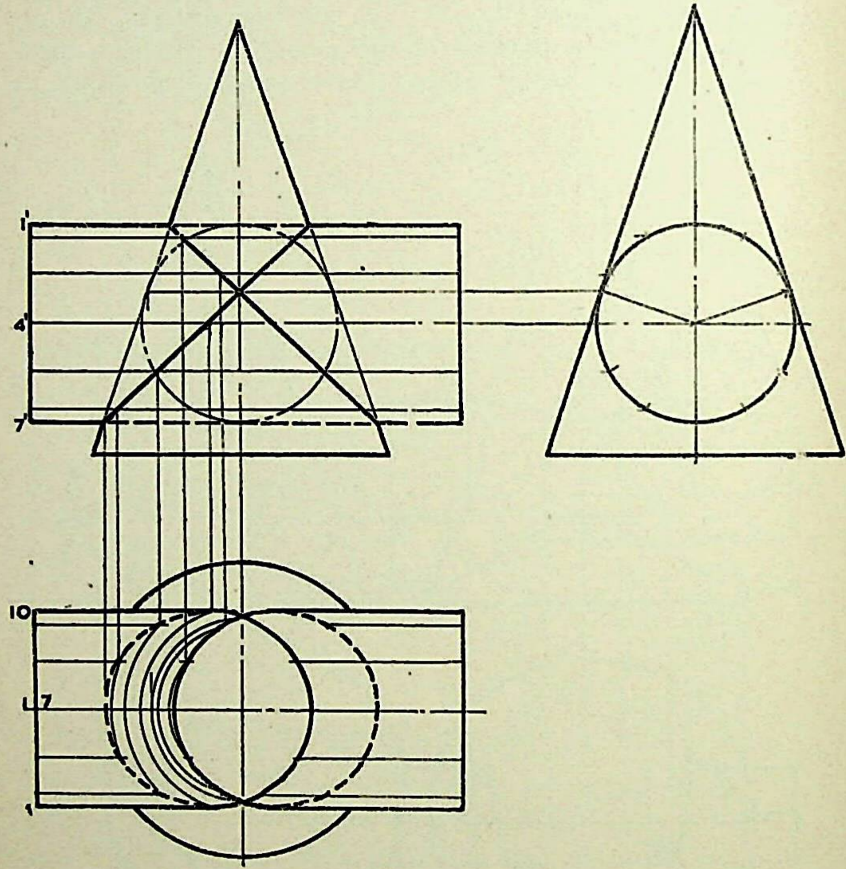


FIG. 17-25

The lines of intersection are determined by finding the points in the same manner as described in problem 14. In the elevation, they appear as straight lines intersecting each other at right angles. In the plan, they are seen as ellipses.

Problem 16 (fig. 17-26):

A funnel is made of two constituent parts (i) a cylindrical pipe and (ii) a conical part, both enveloping a common sphere of 4 cm

(1$\frac{1}{2}$") diameter, with their axes intersecting at right angles. The diameter at the mouth of the funnel is 7·5 cm (3") and the total height of the funnel is 5·6 cm (2$\frac{1}{4}$"). Draw projections showing the curve of intersection when it is placed with its mouth in the H.P. and the two axes parallel to the V.P.

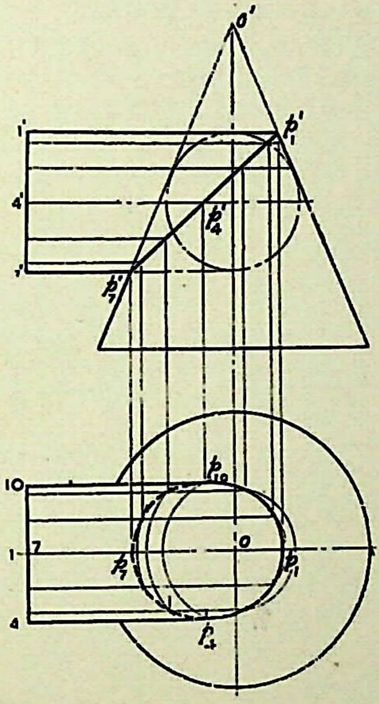


Fig. 17-26

Draw the elevation and plan of the two parts. To form the funnel, the pipe must be extended beyond the centre line of the conical part. The points of intersection of the upper part of the pipe with the cone will therefore be on the right-hand side of the centre line. The curve of intersection will be seen as a straight line in elevation and will be elliptical in plan.

Problem 17:

A cone, diameter of base 7·5 cm (3") and axis 9 cm (3$\frac{1}{2}$") long, resting on its base in the H.P., is penetrated by a cylinder of

5 cm (2″) diameter, the axis of which is parallel to and 1 cm ($\frac{3}{8}$″) away from that of the cone. Draw the projections showing curves of intersection, when (i) the plane containing the two axes is parallel to the V.P.; (ii) the plane containing the two axes is inclined at 45° to the V.P.

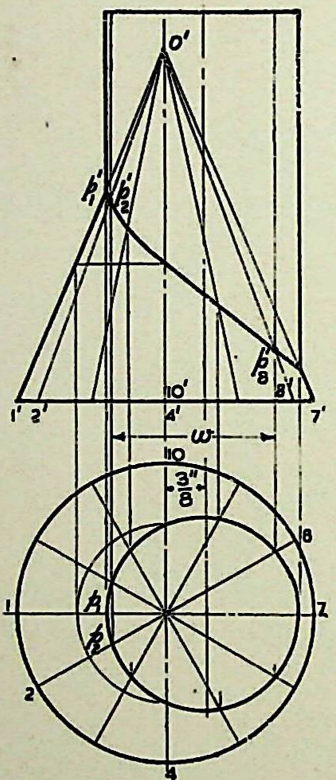


Fig. 17-27

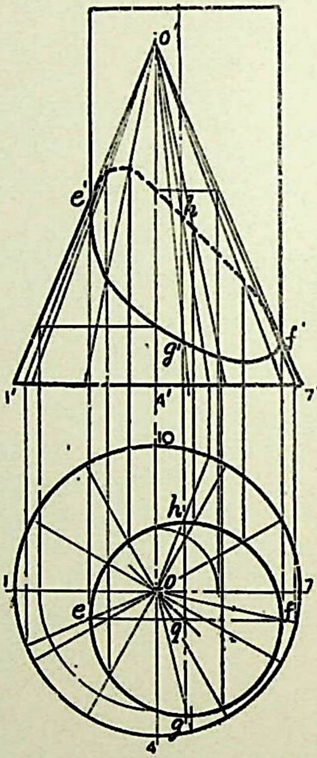


Fig. 17-28

(i) Draw the plan and elevation (fig. 17-27). The centre of the circle for the cylinder (in the plan) will lie on the horizontal centre line and 1 cm ($\frac{3}{8}$″) away from the centre of the circle for the cone.

Divide the base-circle of the cone into twelve equal parts and draw twelve generators in both the views. The surface of the cylinder is seen as a circle in plan. It cuts the lines on the surface of the cone at points p_1, p_2 etc. Project these points to p_1', p_2' etc. on the corresponding lines

in elevation. Draw the required curve of intersection through these points. The curve at the back will coincide with the front curve.

The cutting-plane method is exactly similar. Assume a series of vertical cutting planes passing through the apex. The sections of the cone will be triangles and those of the cylinder will be rectangles. Points of intersection between these sections will lie on the curve. For example, take a cutting plane coinciding with the line 2-8. In elevation, the section of the cone will be shown by triangle $o'2'8'$, while that of the cylinder will be a rectangle of width w. Points p_2' and p_8' at which lines $o'2'$ and $o'8'$ cut the sides of the rectangle, lie on the curve of intersection.

(ii) As the plane containing the two axes makes an angle of 45° with the V.P., the centre q of the circle for the cylinder will lie on a line drawn through o, inclined at 45° to xy and 1 cm ($\frac{3}{8}''$) from o (fig. 17-28).

Draw the plan and elevation and adopt the same method as in case (i). In addition to the twelve points, four key points e, f, g and h (where the extreme lines of the cylinder intersect the cone) must be located. Plot the curve which will be partly hidden. If a hole is drilled through the cone instead of a cylinder penetrating it, the curve will be fully visible.

Problem 18 (fig. 17-29):

A cylinder of 7·5 cm (3") diameter, standing on its base in the H.P., is penetrated by a cone, base 7·5 cm (3") diameter and axis 11 cm ($4\frac{3}{8}''$) long, the two axes bisecting each other at right angles. Draw the elevation showing lines of intersection.

Divide the base circle of the cone into twelve equal parts and draw lines for the generators in both views.

Assume a series of cutting planes perpendicular to the V.P. and passing through the apex. They will be seen as lines in elevation. The sections of the cone will be triangles and those of the cylinder will be ellipses.

Take a cutting plane coinciding with a line $3'o'$. In plan, the section of the cone is shown by a triangle o-3-11 and that of the cylinder by the circle (plan of the cylinder). The sides of the triangle intersect the circle at points p_3 and

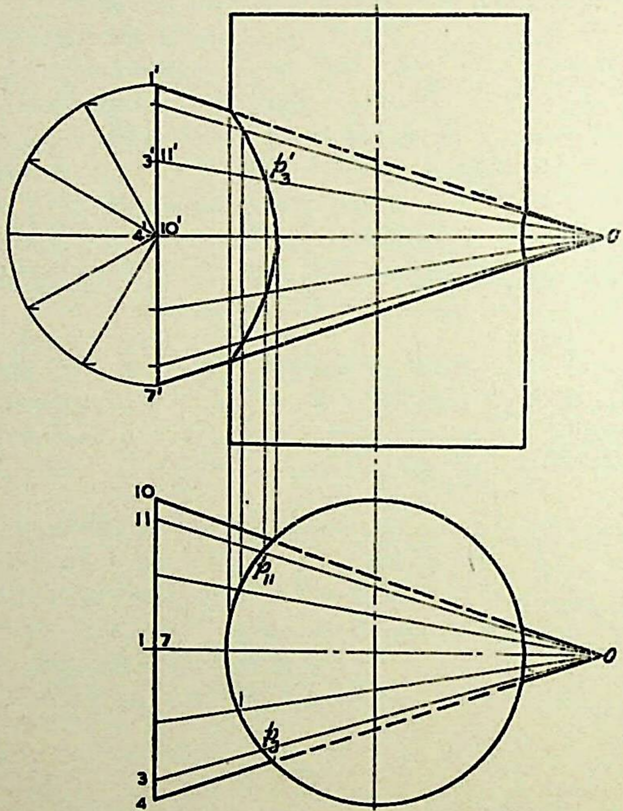


FIG. 17-29

p_{11}. Project these points in elevation to p_3' on the line $3'o'$ with which $11'$-o' coincides. Obtain other points in the same manner and draw a curve through them. Points on the curve on the right-hand side of the axis of the cylinder are obtained simultaneously and in the same manner.

Intersection of cone and prism:

Problem 19 (fig. 17-30):

Draw an equilateral triangle of 10 cm (4") side with one side horizontal. Draw a square of 4 cm ($1\frac{1}{2}$") side in its centre, with

its sides inclined at 45° to the base of the triangle. The figure shows the elevation of a cone standing on its base on the ground and having a square hole cut through it. Draw three views of the cone, using third-angle projection method.

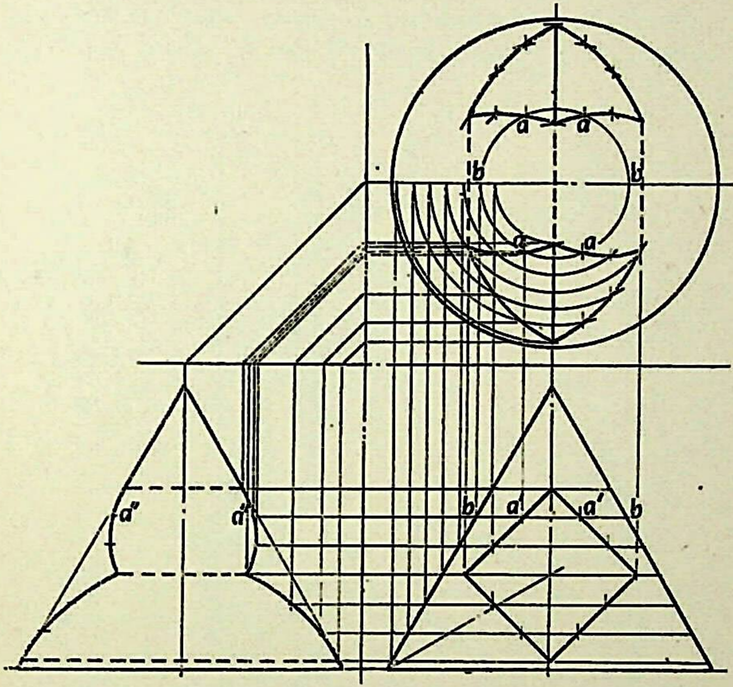


FIG. 17-30

Project the plan and end elevation from the given view. Assume the cone as cut by a horizontal cutting plane passing through points *a′*. The section of the cone will be a circle of diameter *bb*. The hole will be cut in two straight lines through points *a′* and perpendicular to the V.P. Therefore, with centre *o* (the apex in plan) and diameter *bb*, draw a circle cutting the projectors through *a′* at points *a*. Assume additional cutting planes, particularly those which pass through corners of the square and find other points. Draw curves through these points.

The method of locating points *a″* in end elevation is clearly indicated by construction lines. Obtain all points in the same manner.

Problem 20 (fig. 17-31):

A cone, base 7·5 cm (3″) diameter and axis 9 cm ($3\frac{1}{2}$″) long, is penetrated by a square prism of base 3 cm ($1\frac{3}{16}$″) side. The axis of the prism is parallel to and 1·2 cm ($\frac{1}{2}$″) away from that of the cone. Draw the projections when a plane containing the two axes is parallel to the V.P. and the faces of the prism are equally inclined to the V.P.

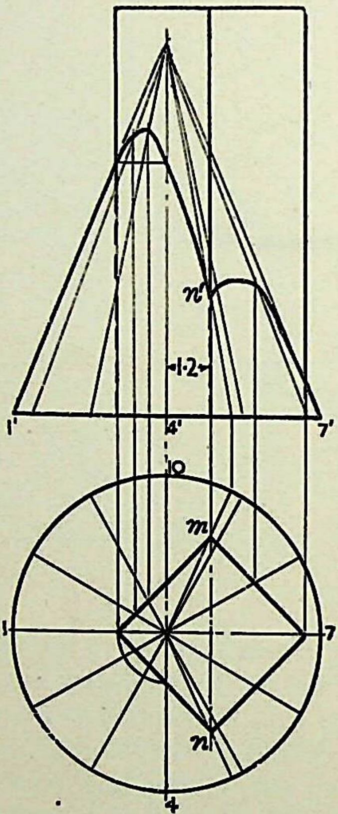


FIG. 17-31

Adopt the same method as shown in problem 17.

Figs. 17-31(*a*) and (*b*) show in third-angle projection, two views of a cone having a square hole cut through it. The axes of the cone and the hole are vertical and 1·2 cm (1″) apart. The faces of the hole are equally inclined to the V.P.

In fig. 17-31(*a*), the vertical plane containing the two axes is perpendicular to the V.P.; while in fig. 17-31(*b*) it is inclined at 45° to the V.P. Third-angle projection is used in both cases.

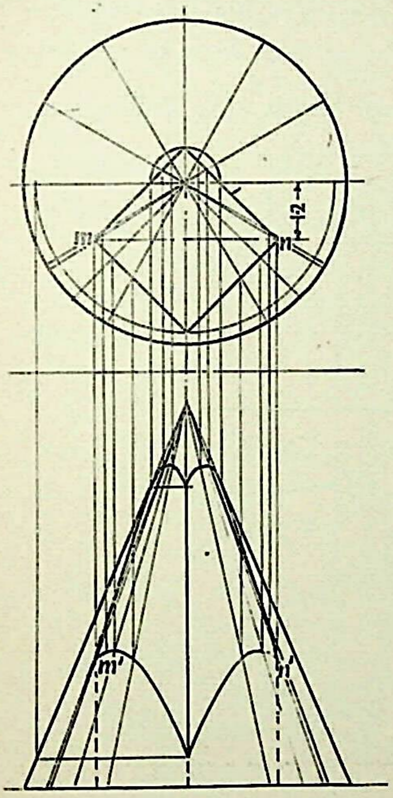


FIG. 17-31(*a*)

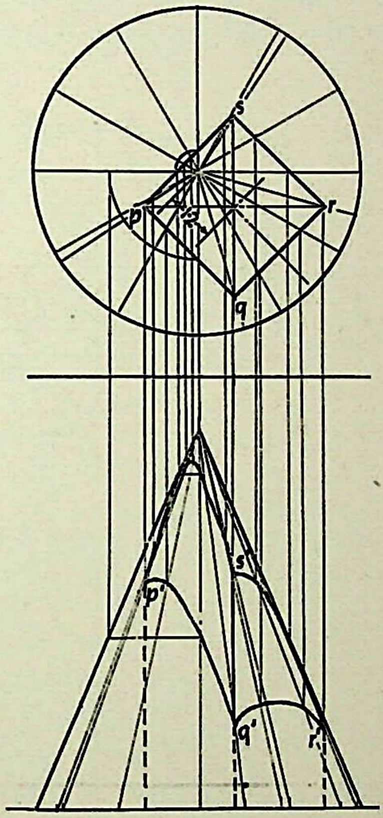


FIG. 17-31(*b*)

Intersection of two cones:

Problem 21 (fig. 17-32):

A cone, base 10 cm (4″) diameter, axis 10 cm (4″) long, and resting vertically in the H.P. is penetrated by another cone of base 7·5 cm (3″) diameter and height 11 cm ($4\frac{3}{8}$″). The axes of the two cones bisect each other at right angles. Draw elevation showing curves of intersection, when the axis of the penetrating cone is parallel to the V.P.

Assume a horizontal cutting plane coinciding with a line *ad*. The section of the vertical cone will be a circle of diameter *ee*. The section of the horizontal cone will be a hyperbola. The width of the hyperbola at the point *a* will be equal to twice the length of the line $a1'$; that at the point *b* will be twice $b2'$, etc. In plan, mark points on the

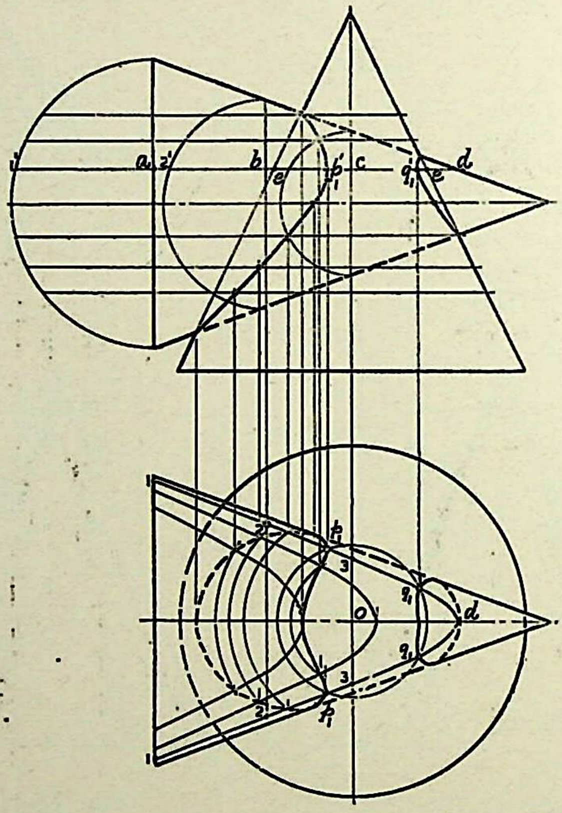


Fig. 17-32

projector through *a* and on both sides of the horizontal axis, so that 1-1 = twice $a1'$. Similarly, mark points on the projector through *b* so that 2-2 = twice $b2'$, etc. Draw the hyperbola through the points thus obtained. Draw a circle with centre *o* and radius *ce*, cutting the hyperbola at points p_1 and q_1. Project p_1 to p_1' and q_1 to q_1' on the cutting-plane line *ad*. Then p_1, q_1, p_1' and q_1' are the points on the curves in the two views. Similarly, assume additional sections, prefer-

ably at equal distances on both sides of the axis (so that the sections will be the same). Draw the hyperbolas and circles in the plan and determine the points of intersection. Draw curves in both the views and on both sides of the vertical axis.

The curves can be determined in the same manner even when the axes do not intersect.

Intersection of sphere and cylinder or prism:

Problem 22 (fig. 17-33):

A hole of 5 cm (2") diameter is drilled through a sphere of 7·5 cm (3") diameter. The axis of the hole is 1 cm ($\frac{3}{8}$") from the centre of the sphere. Draw three views of the sphere when a plane, containing the centre of the sphere and the axis of the hole, is inclined at 60° to the V.P.

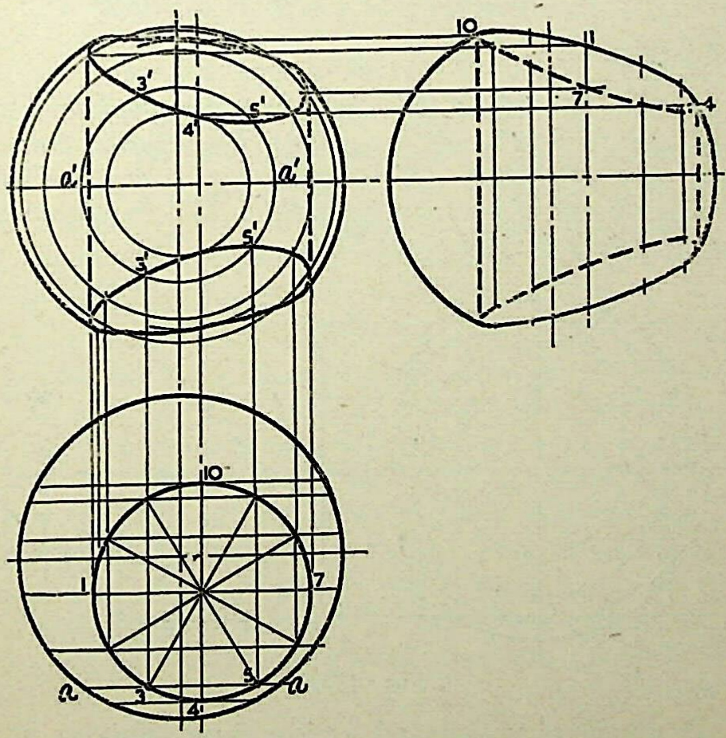


FIG. 17-33

Draw the plan of the sphere and show the circle for the hole in the required position. Project the elevation and end view. Divide the circle for the hole into twelve equal parts.

Assume a number of cutting planes parallel to the V.P. The sections of the sphere will be circles, while the hole will be cut in straight lines. Intersection of the circles with corresponding lines will give points on the curves. Let us take a section plane passing through points 3 and 5. The section of the sphere will be a circle of diameter *aa*. The lines through 3 and 5 intersect this circle at points 3′ and 5′ which lie on the curve. Obtain the twelve points in the same manner. Also plot the key points at which the section through the centre of the sphere is cut by the lines of the hole. Draw curves through all the points. Project the points on the end view, horizontally from the elevation on corresponding lines of the hole and draw curves through them.

Problem 23 (fig. 17-34):

A sphere of 7·5 cm (3″) diameter is penetrated by a square prism, base 5 cm (2″) side, the axis of which passes through the centre. Draw the elevation of the solids showing curves of intersection when the axis of the prism is parallel to both the planes and the faces are equally inclined to the V.P.

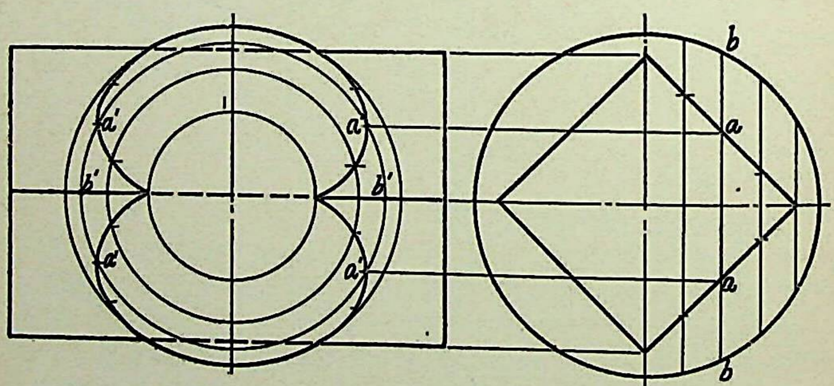


Fig. 17-34

Draw the elevation and project the end view. Assume a vertical cutting plane passing through, say points *a*. The

section of the sphere will be a circle of diameter *bb*. It is cut by the section of the prism (a rectangle of width *aa*) at points *a′* as shown in the elevation. Then points *a′* are on the curve of intersection. Obtain more points together with key points, in the same manner and draw curves through them.

EXERCISES XVII

Note: For conversion of inches into centimetres refer to the table given on page 508.

(1) A square prism, base 5 cm (2″) side resting on its base in the H.P. with its faces equally inclined to the V.P. is completely penetrated by another square prism of base 3 cm ($1\frac{3}{16}$″) side, the axis of which is parallel to both the planes and is 0·6 cm ($\frac{1}{4}$″) away from the axis of the vertical prism. The faces of the horizontal prism also are equally inclined to the V.P. Draw the projections of the solids showing lines of intersection.

(2) Two prisms whose ends are equilateral triangles of 4 cm ($1\frac{1}{2}$″) side and axes 10 cm (4″) long, intersect at right angles. One face of each prism is on the ground. The axis of one of the prisms makes 30° with the V.P. Draw plan, elevation and side elevation, using third-angle projection method.

(3) A square prism of base 5 cm (2″) side and height 12·5 cm (5″) stands on the ground with a side of base inclined at 30° to the V.P. It is penetrated by a cylinder, 5 cm (2″) diameter and 12·5 cm (5″) long, whose axis is parallel to both the H.P. and the V.P. and bisects the axis of the prism. Draw the plan and elevation showing fully the curves of intersection.

(4) A cylindrical boiler is 6′ in diameter and has a cylindrical dome 2′-6″ diameter and 2′ high. The axis of the dome intersects the axis of the boiler. Draw three views of the arrangement. Develop the surface of the dome. Scale 1″ = 1′. (B. U.)

(5) A vertical pipe, 7·5 cm (3″) diameter and 15 cm (6″) long, has two branches, one on each side. The horizontal branch is

of 6 cm ($2\frac{3}{8}''$) diameter while the other is of 5 cm ($2''$) diameter and inclined at 45° to the vertical. Assume the axis of 5 cm ($2''$) branch and the main pipe to be in the same plane, and that of 6 cm ($2\frac{3}{8}''$) branch at 0·6 cm ($\frac{1}{4}''$) from the axis of the main pipe and parallel to the V.P. Draw the views showing curves of intersection. Draw the developments of the three pipes, assuming suitable lengths.

(6) A cylinder of 7·5 cm ($3''$) diameter and 12·5 cm ($5''$) height, stands on its base in the H.P. It is penetrated centrally by a cylinder 5 cm ($2''$) diameter and 12·5 cm ($5''$) long, whose axis is parallel to the H.P. but inclined at 30° to the V.P. Draw plan and elevation, showing curves of intersection. Draw also the development of the penetrated cylinder.

(7) A right circular cylinder of 7·5 cm ($3''$) diameter penetrates another of 10 cm ($4''$) diameter, their axes being at right angles to each other but 1 cm ($\frac{3}{8}''$) apart. Draw the projection of the curves of intersection on a plane parallel to the axes of the cylinders. Use third-angle projection method.

(8) Two circular pipes of 7·5 cm ($3''$) and 5 cm ($2''$) diameters (inside) meet at 30°. The axes of both the pipes are in one plane and the 7·5 cm ($3''$) pipe is vertical. The thickness of the pipes is 0·6 cm ($\frac{1}{4}''$) in both cases. Draw plan and elevation showing curves of intersection.

(9) A square hole of $1\frac{1}{2}''$ side is cut in a cylindrical shaft $3''$ diameter and $5''$ long. The axis of the hole intersects that of the shaft at right angles. All faces of the hole are inclined at 45° to the H.P. Draw three views of the shaft when the plane of the two axes is parallel to the V.P. (B. U.)

(10) A cylinder of 5 cm ($2''$) diameter, branches off another cylinder of 7·5 cm ($3''$) diameter. The axis of the smaller cylinder is vertical and that of the other is horizontal. If the distance between the axes is 1 cm ($\frac{3}{8}''$), draw plan, elevation and end view of the cylinders. Use third-angle projection method.

(11) A cone frustum is $5''$ high, $3\frac{3}{8}''$ diameter at the bottom and $4\frac{5}{8}''$ diameter at top. It is vertically placed and is completely penetrated by a horizontal cylinder $3''$ diameter and $5''$ long, the axis of which bisects the axis of the frustum. Draw plan and elevation of the solids showing curves of intersection. (B. U.)

(12) A conical funnel, 6″ diameter at the mouth and 5″ high has a pipe of $1\frac{1}{2}''$ diameter attached at its apex. The length of the pipe is 4″ and its axis is at right angles to the axis of the funnel and in the same plane, with the outer side of the pipe in line with the apex of the funnel. Draw full size, the arrangement with sufficient number of views to show the joint. (B. U.).

(13) A cone, $3\frac{1}{2}''$ diameter of base, axis $4\frac{1}{2}''$ long, stands on the H.P. and is completely penetrated by a cylinder, 2″ diameter and $4\frac{1}{2}''$ long. The axis of the cylinder is horizontal, parallel to the V.P. and passes through the axis of the cone, 3″ from the apex. Draw the plan and elevation of both curves of intersection. Develop the surface of the cone. (B. U.)

(14) A hole of 1″ diameter is drilled in a cone having 3″ diameter of base and $2\frac{1}{2}''$ height. The axis of the hole is parallel to that of the cone and $\frac{1}{4}''$ from it. Draw three views of the cone when a vertical plane containing the two axes is perpendicular to the V.P. (B. U.)

(15) A conical hopper is fitted in vertical position on a horizontal pipe, 3″ in diameter and 6″ in length. The diameter of the hopper at the top is $4\frac{1}{2}''$ and changes by 1″ in every 1″ of vertical distance. Draw plan and elevation, showing the curves of intersection if the top of the hopper is $2\frac{3}{4}''$ above the axis of the pipe. Develop the surfaces of the hopper and the pipe. (B. U.)

(16) A cylinder, 5 cm (2″) diameter of base and 10 cm (4″) height is centrally penetrated by a cone, 5 cm (2″) diameter of base and 7·5 cm (3″) height. The axis of the cylinder which is vertical, cuts the axis of the cone which is horizontal at 3 cm ($1\frac{3}{16}''$) from the base of the cone. Draw the elevation and end view, showing curves of penetration. Use third-angle projection method.

(17) A cone, base 7·5 cm (3″) diameter and axis 10 cm (4″) long, has an equilateral triangular hole of 4 cm ($1\frac{1}{2}''$) side cut through it. The axis of the hole coincides with the axis of the cone. Draw three views of the cone when it is resting on its base in the H.P. with a face of the hole parallel to the V.P. Develop the surface of the cone.

(18) A sphere of 10 cm (4″) diameter is penetrated vertically by a triangular prism of base 4 cm ($1\frac{1}{2}''$) side. Their axes coincide with each other. Draw the curves of penetration when two faces of the prism are equally seen in the front view.

(19) A cone, base 10 cm (4") diameter and height 12·5 cm (5"), resting in the H.P. on its base, is penetrated by another cone, 5 cm (2") diameter and axis 10 cm (4") long. The axis of the penetrating cone is parallel to the H.P. and the V.P., 4 cm ($1\frac{1}{2}$") above the H.P. and 1 cm ($\frac{3}{8}$") from the axis of the vertical cone. It comes out equally on both sides of the cone. Draw elevation and plan showing curves of intersection.

(20) A cylinder, 5 cm (2") diameter and axis vertical, penetrates a pentagonal pyramid having its base on the ground and one side of the base parallel to the V.P. The diameter of the circumscribing circle of the base of the pyramid is 7·5 cm (3") and its height is 10 cm (4"). The two axes are 1 cm ($\frac{3}{8}$") apart. Draw the projections showing curves of intersection, when the plane containing the two axes is parallel to the V.P. Develop the surface of the pyramid. Use third-angle projection method.

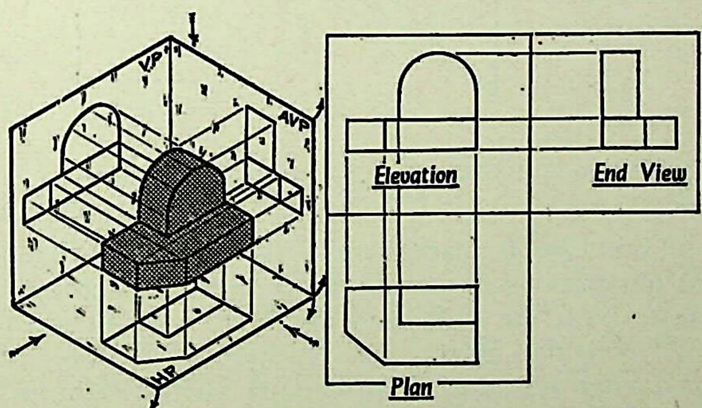
CHAPTER XVIII

CONVERSION OF PICTORIAL VIEWS INTO ORTHOGRAPHIC VIEWS

Having grasped the principles of orthographic and isometric projections, we proceed to deal with the application of the same to conversion of pictorial views into orthographic views.

Orthographic views can be drawn by two methods: (i) first-angle projection method and (ii) third-angle projection method.

In the preceding chapters we have assumed the objects to be situated in front of the V.P. and above the H.P., i.e. in the first quadrant, and then projected them on these planes. This system of projection is known as first-angle projection system. The object lies between the observer and the plane [fig. 18-1(*i*)]. In this system, when the views are drawn in their relative positions [fig. 18-1(*ii*)], the plan is placed below



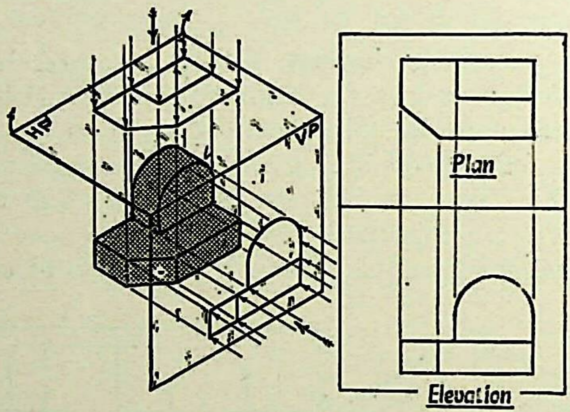
(*i*) (*ii*)

FIG. 18-1

the elevation; the view of the object as seen from the left side is placed to the right side of the elevation and vice-versa. In brief, each projection shows the view of that surface (of

the object) which is remote from the plane on which it is projected, and which is nearest to the observer.

In the third-angle projection system, the object is assumed to be situated in the third quadrant, i.e. behind the V.P. and below the H.P. [fig. 18-2(*i*)]. The planes of projection are assumed to be transparent. They lie between the object and the observer. When the observer views the object from the front, the rays of sight intersect the V.P. The figure formed by joining the points of intersection in correct sequence is the front view or the elevation. The top view or the plan is obtained in a similar manner by looking from above. When the two planes are brought in line with each other, the views will be as shown in fig. 18-2(*ii*). The

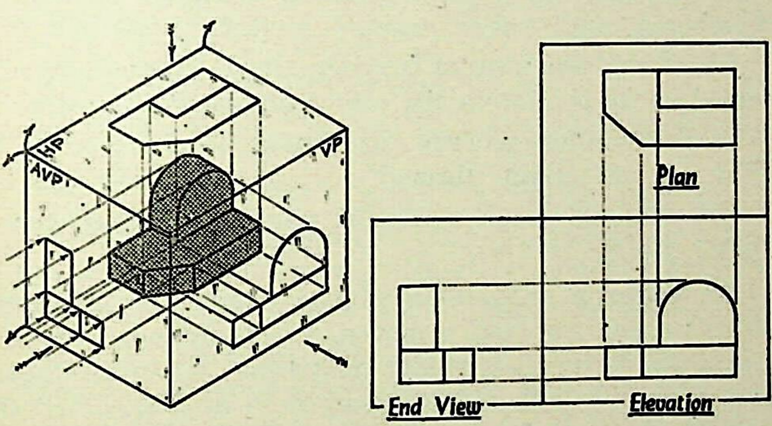


(*i*) (*ii*)

FIG. 18-2

plan in this case is placed above the elevation. The end view is obtained by projecting on an A.V.P. placed perpendicular to both the H.P. and the V.P., and between the observer and the object [fig. 18-3(*i*)]. The three views obtained after the planes are rotated and brought in line with the V.P. are shown in fig. 18-3(*ii*). The left side view of the object is placed on the left side of the elevation. Thus, each projection shows the view of that surface (of the object) which is nearest to the plane on which it is projected. In other words, the view seen from any side of the object is placed on the same side of the elevation.

The system of first-angle projection is the British standard practice. The third-angle projection is the standard practice followed in America and in the continent of Europe. In our country, till recently, the first-angle projection system was in common use. The Indian Standards Institution has recommended the adoption of the system of third-angle projection as a standard practice in our country. Hence, it is fast coming in general use. Persons in engineering profession may come across drawings from countries following any one system. It is, therefore, necessary for them to be conversant with both the systems.



(*i*) (*ii*)

FIG. 18-3

Conversion of a pictorial view into orthographic views requires a sound knowledge of the principles of pictorial projection and some imagination. A pictorial view may have been drawn according to the principles of isometric or oblique projection. In either case, it shows the object as it appears to the eye from one direction only. It does not show the real shapes of its surfaces or the contour. Hidden parts and constructional details are also not clearly shown. All these have to be imagined.

For converting a pictorial view of an object into orthographic views, the direction from which the object is to be viewed for its front elevation is generally indicated by means of an arrow. When this is not done, the arrow may be

assumed to be parallel to a sloping axis. Other views are obtained by looking in directions parallel to each of the other two axes and placed in correct relationship with the elevation.

When looking at the object in the direction of any one of the three axes, only two of the three overall dimensions (viz. length, height and depth or thickness) will be visible. Dimensions which are parallel to the direction of vision will not be seen. Lines which are parallel to the direction of vision will be seen as points, while surfaces which are parallel to it will be seen as lines.

While studying a pictorial view, it should always be remembered that, *unless otherwise specified*,

(i) A hidden part of a symmetrical object should be assumed to be similar to the corresponding visible part.

(ii) All holes, grooves etc. should be assumed to be drilled or cut right through.

(iii) Suitable radii should be assumed for small curve of fillets etc.

An object in its pictorial view may sometimes be shown with a portion cut and removed, to clarify some internal constructional details. While preparing its orthographic views, such object should be assumed to be whole, and the views should then be drawn as required.

It is comparatively easier to prepare a drawing from an actual object. The object is carefully examined and then placed in a suitable position for the front view. All the necessary views are then sketched freehand in a sketch book or on a pad. Measurements of all its details and overall sizes are taken and inserted in the views, along with important notes and instructions. Finally, a scale-drawing is prepared from these sketches.

Procedure for preparing a scale drawing:

A scale-drawing must always be prepared from freehand sketches initially prepared from a pictorial view or a real object. In the initial stages of a drawing, always use a soft pencil viz. HB or F, and work with a light hand, so that lines are thin, faint and easy to erase, if necessary.

1. Determine overall dimensions of the required views. Select a suitable scale so that the views are conveniently accommodated in the drawing sheet.

2. Draw rectangles for the views, keeping sufficient space between them and from the borders of the sheet.

3. Draw centre lines in all the views. When a cylindrical part or a hole is seen as a rectangle, draw only one centre line for its axis. When it is seen as a circle, draw two centre lines intersecting each other at right angles at its centre.

4. Draw details simultaneously in all the views in the following order:

(i) Circles and arcs of circles.

(ii) Straight lines for the general shape of the object.

(iii) Straight lines, small curves etc. for minor details.

5. After the views have been completed in all the details, erase all unnecessary lines completely. Make the outlines so faint that only their impressions exist.

6. Fair the views with 2H or 3H pencil, making the outlines uniform and intensely black (but not too thick). For doing this, adopt the same working order as stated in step 4 above.

7. Dimension the views completely. Keep all centre lines.

8. Draw section lines in the view or views which are shown in section.

9. Print the title, the scale and other particulars and draw the border lines.

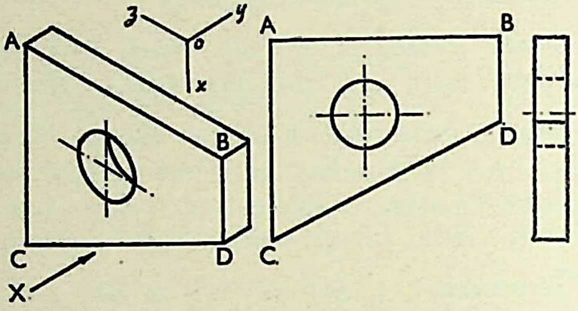
10. Check the drawing carefully and see that it is complete in all respects.

ILLUSTRATIVE PROBLEMS:

Problem 1:

Draw according to third-angle projection method the elevation

and an end view of the wedge-shaped piece shown in pictorial view in fig. 18-4(i).



(i) (ii)

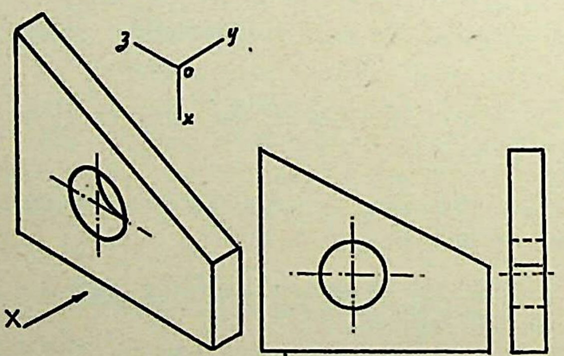
FIG. 18-4

The edge *AB* is parallel to the isometric axis *oz* and hence, it will be seen as a horizontal line in the elevation [fig. 18-4(*ii*)]. The edge *CD* will be seen as an inclined line.

Problem 2:

Draw according to third-angle projection method, the elevation and an end view of the wedge-shaped piece shown pictorially in fig. 18-5(i).

See fig. 18-5(*ii*). Note that the bottom edge is horizontal.



(i) (ii)

FIG. 18-5

Note: The three axes are shown along with the pictorial views, only for the purpose of explanation of the above two problems.

Problem 3:

Draw according to the first-angle projection method, the following views of the block shown pictorially in fig. 18-6(i).

(*i*) *Elevation,* (*ii*) *plan and* (*iii*) *both end views.*

See fig. 18-6(*ii*).

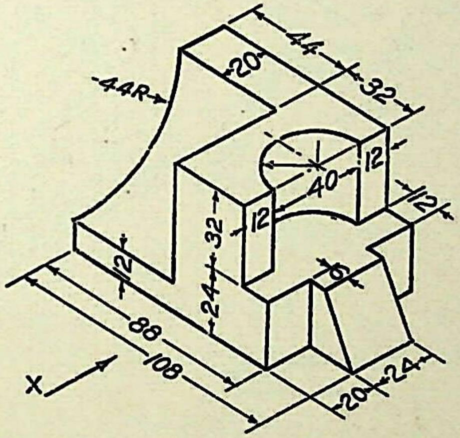


FIG. 18-6(*i*)

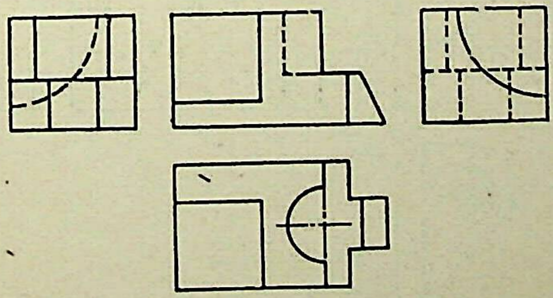


FIG. 18-6(*ii*)

Problem 4:

Draw all the six views of the object shown pictorially in fig. 18-7(i). Use third-angle projection method.

See fig. 18-7(*ii*).

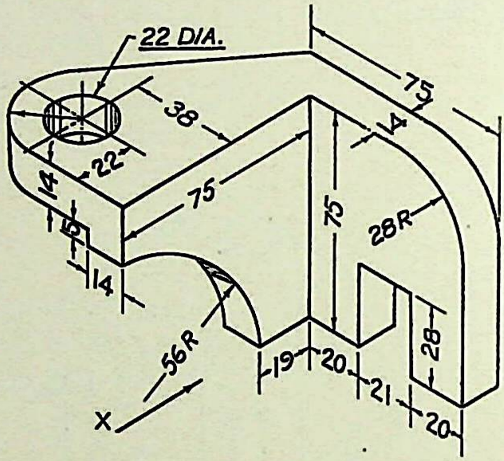


FIG. 18-7(*i*)

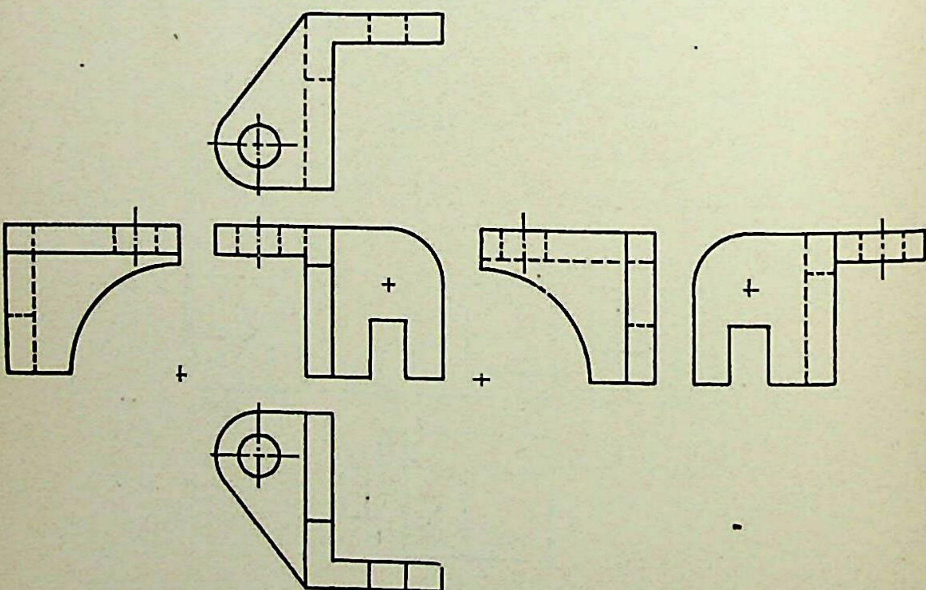


FIG. 18-7(*ii*)

EXERCISES XVIII

Pictorial views of objects are shown in figs. 18-8 to 18-26. Draw, scale full size, views of each object as stated below. The elevation in each case, should be drawn as seen in the direction of the arrow X. Use third-angle projection method, unless otherwise specified. Insert all dimensions in the views.

(1) Fig. 18-8: (i) Elevation. (ii) End view from the left. (iii) Plan. Use first-angle projection method.

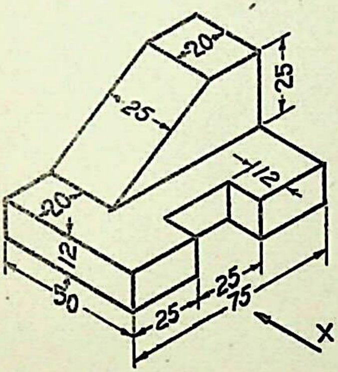


FIG. 18-8

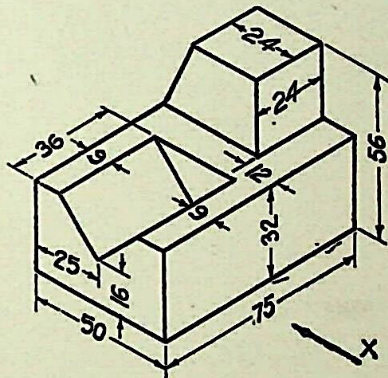


FIG. 18-9

(2) Fig. 18-9: (i) Elevation. (ii) Both end views. (iii) Plan.

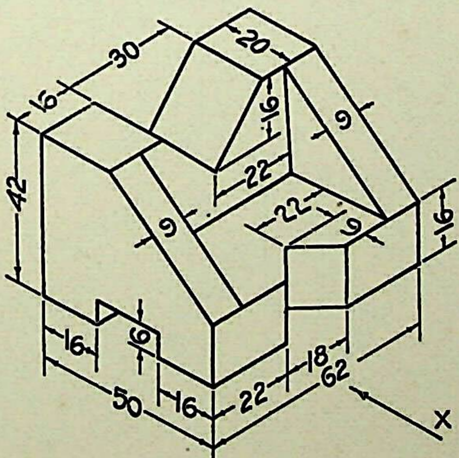


FIG. 18-10

(3) Fig. 18-10: (i) Elevation. (ii) End view. (iii) Plan. Use first-angle projection method.

(4) Fig. 18-11: (i) Elevation. (ii) End view from the left. (iii) Plan.

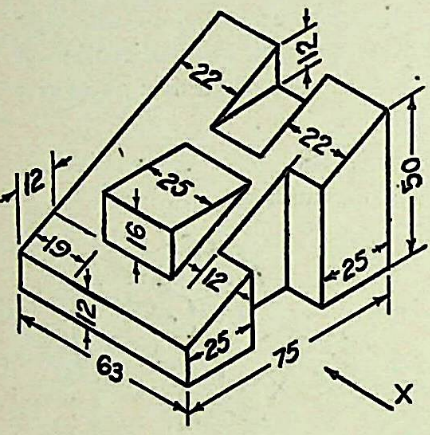


FIG. 18-11

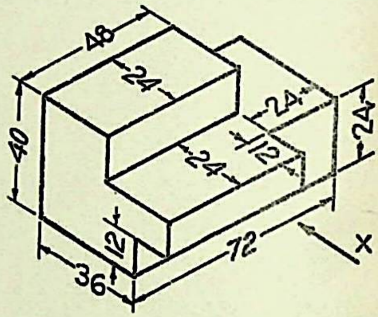


FIG. 18-12

(5) Fig. 18-12: (i) Elevation. (ii) Both end views. (iii) Plan.

(6) Fig. 18-13: (i) Elevation. (ii) Both end views. (iii) Plan.

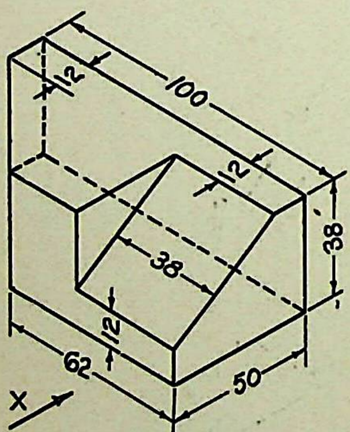


FIG. 18-13

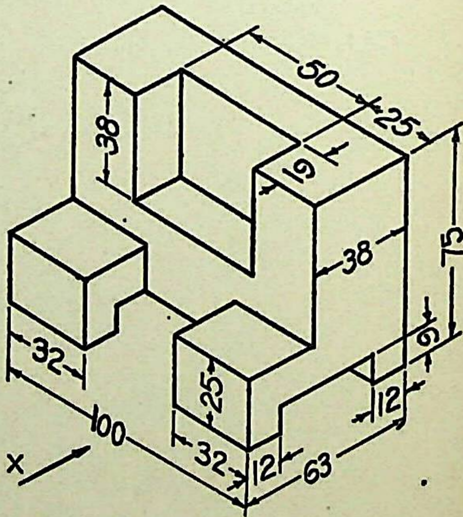


FIG. 18-14

(7) Fig. 18-14: (i) Elevation. (ii) End view. (iii) Plan. Use first-angle projection method.

(8) Fig. 18-15: (i) Elevation. (ii) End view from the right. (iii) Plan.

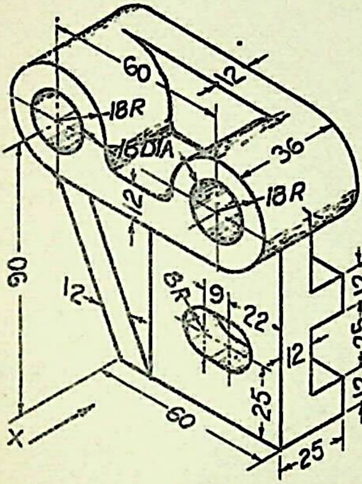


FIG. 18-15

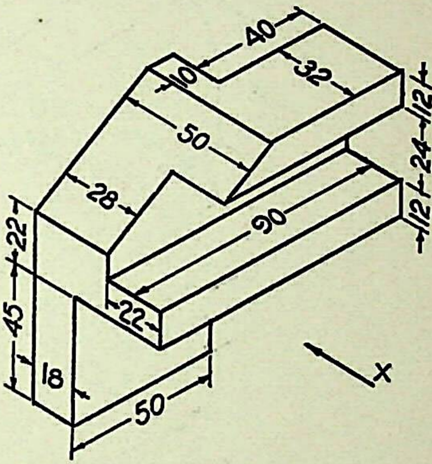


FIG. 18-16

(9) Fig. 18-16: All the six views.

(10) Fig. 18-17: (i) Elevation. (ii) End view from the left. (iii) Plan.

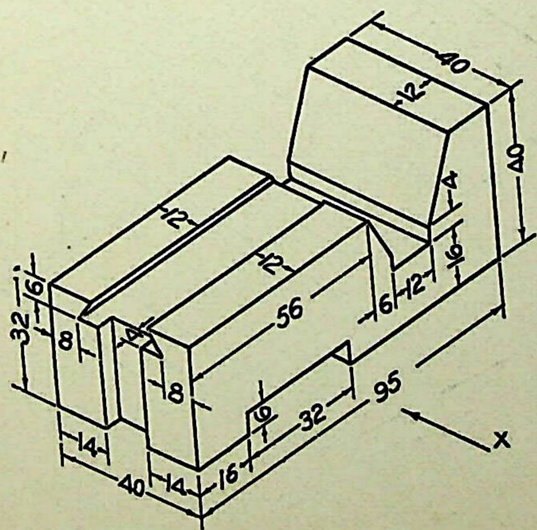


FIG. 18-17

(11) Fig. 18-18: (i) Elevation. (ii) Both end views. (iii) Plan.

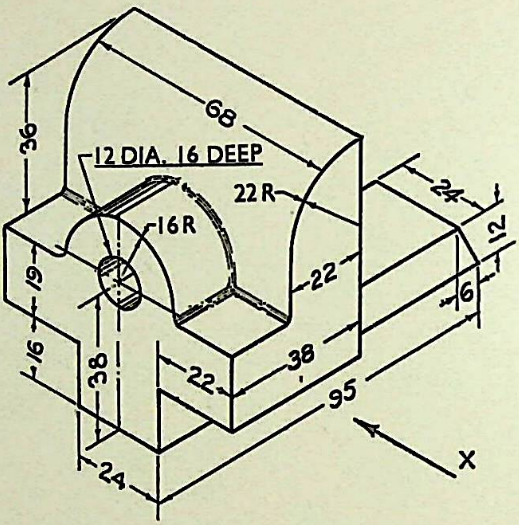


FIG. 18-18

(12) Fig. 18-19: All the six views, according to first-angle projection method.

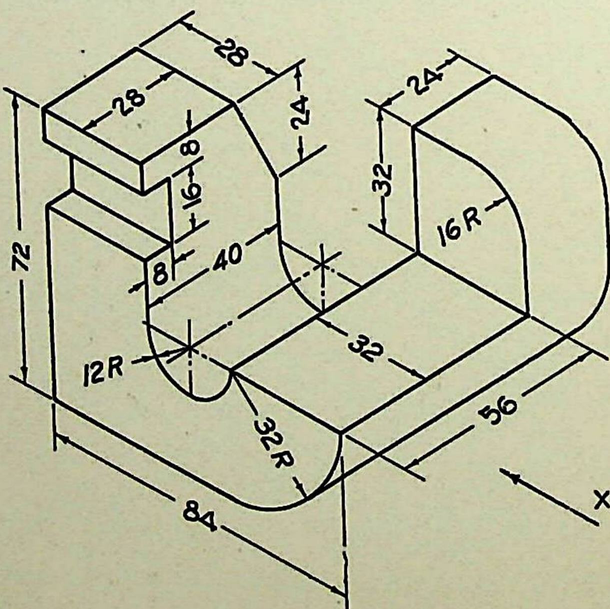


FIG. 18-19

(13) Fig. 18-20: (i) Elevation. (ii) End view from the right. (iii) Plan.

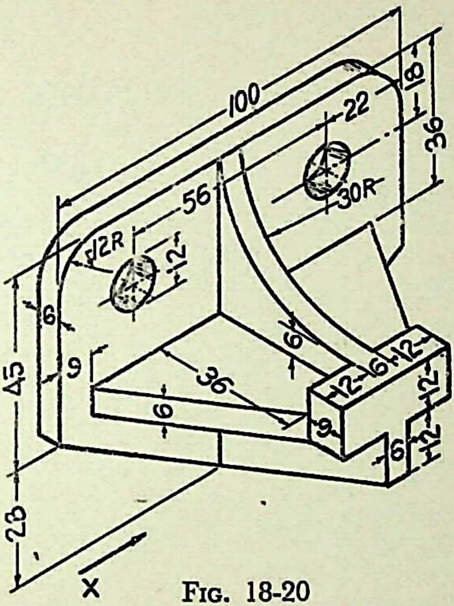


FIG. 18-20

(14) Fig. 18-21: (i) Elevation. (ii) Both end views. (iii) Plan.

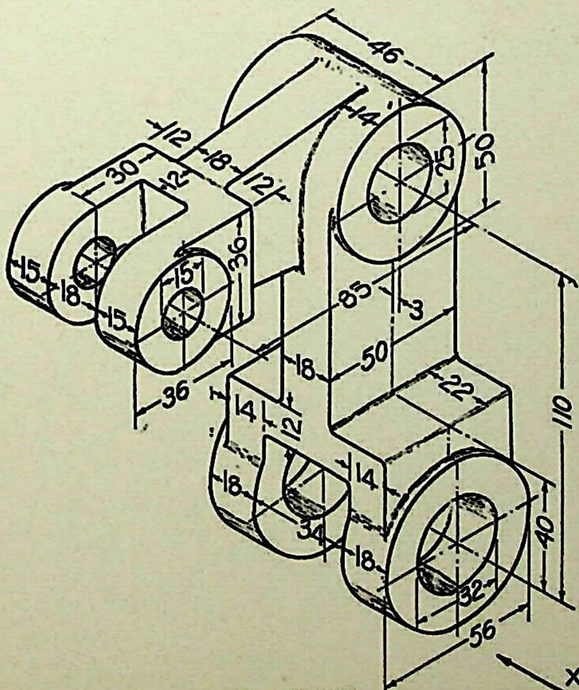


FIG. 18-21

(15) Fig. 18-22: (i) Elevation. (ii) End view. (iii) Plan. Use first-angle projection method.

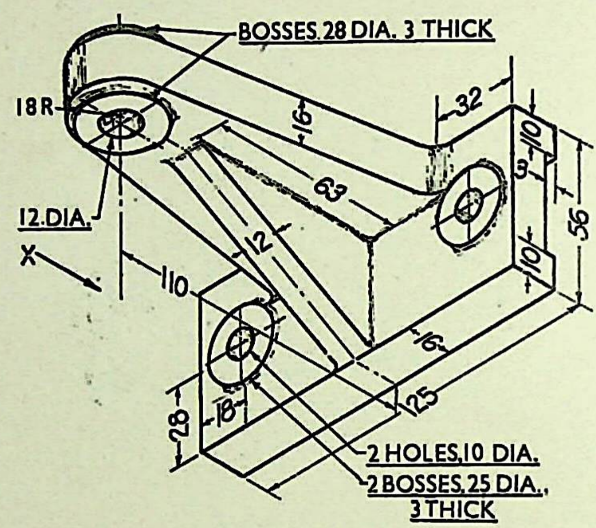


FIG. 18-22

(16) Fig. 18-23: (i) Elevation. (ii) Both end views. (iii) Plan.

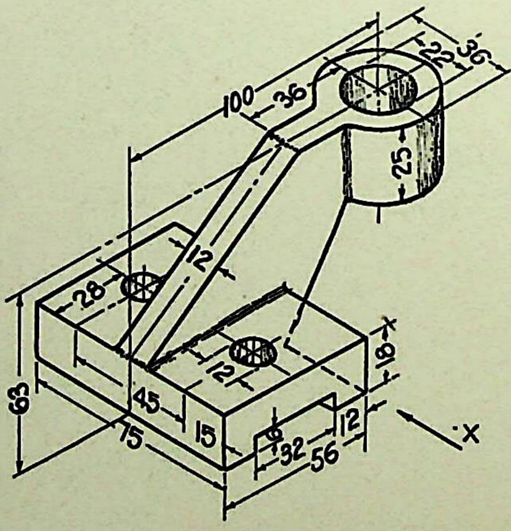


FIG. 18-23

(17) Fig. 18-24: (i) Elevation. (ii) End view from the right. (iii) Plan.

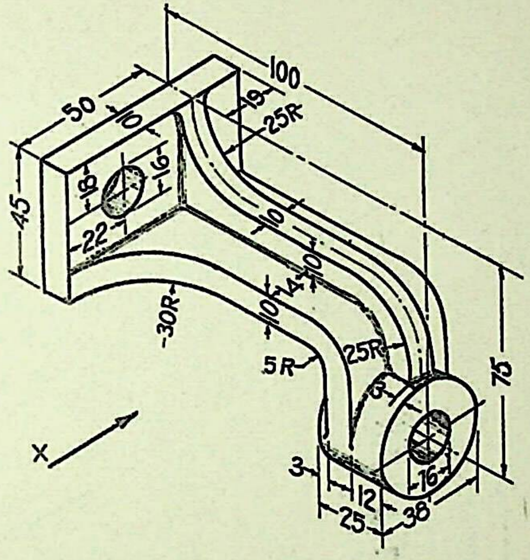


FIG. 18-24

(18) Fig. 18-25: (i) Elevation. (ii) End view from the left. (iii) Plan. Use first-angle projection method.

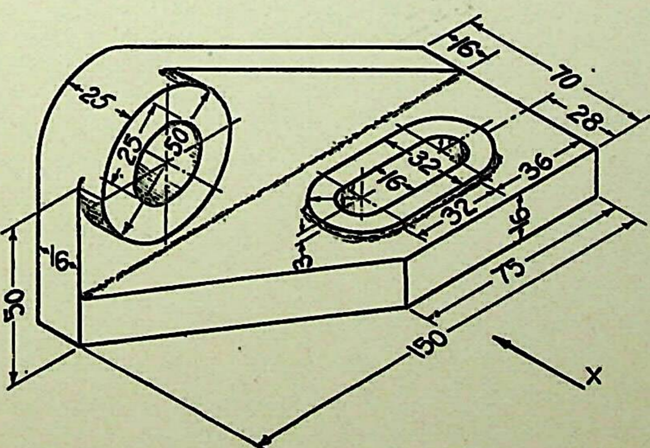


FIG. 18-25

(19) Fig. 18-26: (i) Elevation. (ii) End view from the left. (iii) Plan. Use first-angle projection method.

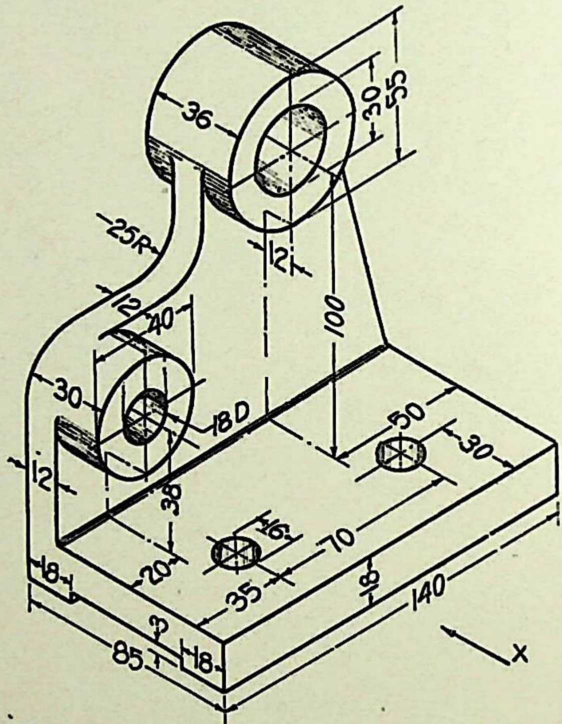


Fig. 18-26

CHAPTER **XIX**

PERSPECTIVE PROJECTION

Perspective projection or Perspective drawing is the representation of an object on a plane surface, called the *picture plane*, as it would appear to the eye, when viewed from a fixed position. It may also be defined as the figure formed on the picture plane when visual rays from the eye to the object cut the picture plane. Perspective is mainly used in Architecture. By means of perspective, the architect is able to show how an object would appear when constructed.

In this chapter, we shall deal with the art of drawing perspectives of simple objects in simple positions.

It is essential to have full knowledge of the principles of orthographic projection before the theory of perspective drawing can be studied.

ORTHOGRAPHIC PROJECTION:

In orthographic projection, the object is assumed to be situated between the two principal planes of projection. The eye or the observer is assumed to be at an infinite distance from the object, and the imaginary rays of sight from the eye to the object are assumed to be parallel to each other and perpendicular to the planes of projection.

Orthographic projection of an object (as described in Chapter XVIII) can be drawn by two methods. In the first-angle projection method, the object is assumed to be situated in the first quadrant, i.e. between the observer and the plane of projection. Hence, the plan of the object is placed vertically below the elevation; while the end view from one side is placed on the other side of the elevation. In the third-angle projection method, the object is assumed to be situated in the third quadrant, i.e. the plane of projection lies between the observer and the object. In this case, the plan lies directly above the elevation, while the end view from a side is placed on the same side of the elevation. Knowledge of the latter method of projection is essential in perspective drawing.

PERSPECTIVE DRAWING:

In perspective projection, the eye is assumed to be situated at a definite position relative to the object. The vertical plane, which (in perspective) is called the picture plane, is usually placed between the object and the eye. Visual rays from the eye to the object converge to a point in the eye and are, therefore, inclined to the picture plane. The rays pierce the picture plane and form an image on it. This image is the perspective of the object. Various elements used in obtaining the perspective view are defined below.

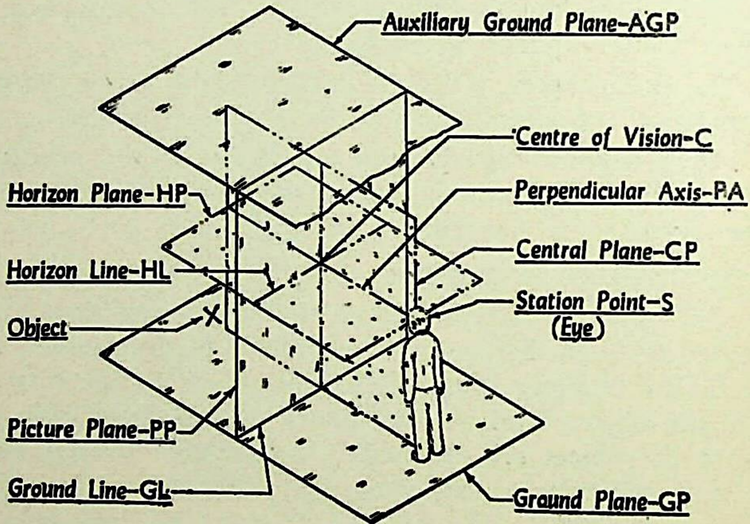


FIG. 19-1

DEFINITIONS OF PERSPECTIVE ELEMENTS (fig. 19-1):

1. **Ground plane** (GP): It is a horizontal plane on which the object is assumed to be situated.

2. **Station point** (S): It is the point where the eye of the observer is located while viewing the object.

3. **Picture plane** (PP): It is a vertical transparent plane located between the station point and the object which is to be viewed. It is the plane on which the perspective is formed. The elevation of perspective elements and of the object (if necessary) is also projected on this plane.

4. **Horizon plane** (HP): This imaginary plane is at the level of the eye, i.e. the station point. It is a horizontal plane, above the ground plane and at right angles to the picture plane.

5. **Auxiliary ground plane** (AGP): It is a horizontal plane placed above the horizon plane. The top view or plan of the object and of the perspective elements is projected on this plane.

6. **Ground line** (GL): The line of intersection of the picture plane with the ground plane is called the ground line.

7. **Horizon line** (HL): It is the line in which the horizon plane intersects the picture plane. It is parallel to the ground line.

8. **Perpendicular axis** (PA): It is the line drawn through the station point, perpendicular to the picture plane. It is, sometimes called the '*Line of sight*' or '*Axis of vision*.'

9. **Centre of vision** (C): The point in which the perpendicular axis pierces the picture plane is called the centre of vision. It lies on the horizon line.

10. **Central plane** (CP): It is an imaginary vertical plane, which passes through the station point and the centre of vision. It contains the perpendicular axis. It is perpendicular to both, the picture plane and the ground plane.

In fig. 19-2, the perspective elements are shown in plan and elevation, drawn according to the third-angle projection method. In plan, the picture plane PP is seen as a horizontal line. The object is above PP, while s the station point is below PP. The line sc is the perpendicular axis and represents the central plane also. The ground plane, the horizon plane and the auxiliary ground plane will be seen as rectangles, but are not shown. In elevation, lines GL (ground line) and HL (horizon line) represent respectively the ground plane and the horizon plane. The station point s' and the centre of vision c' coincide with each other on HL. The central plane CP is seen as a

vertical line through s'. The picture plane will be seen as a rectangle, but is not shown. The perspective view (when drawn) will be seen above or around GL.

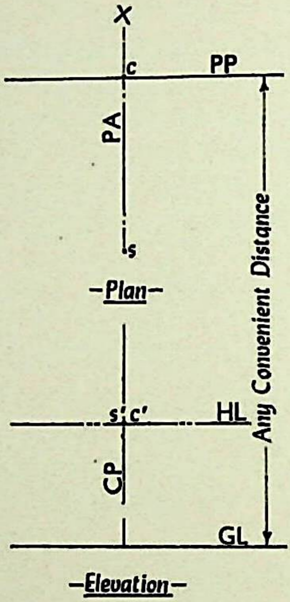


FIG. 19-2

STATION POINT:

The position of the station point is of great importance. Upon its position, the general appearance of the perspective depends. Hence, it should be so located as to view the object in the best manner.

For large objects such as buildings, the station point is usually taken at the eye level of a person of normal height i.e. about 1·5 metres (5 feet). For small objects, the station point should be fixed at such a height as would give a good view of the top as well as side surfaces.

The distance of the station point from the picture plane, when taken equal to about twice the greatest dimension of the object, usually gives good view in the perspective. For objects having heights and widths more or less equal, the location of the station point may be so fixed that the angle between the visual rays from the station point to the outer most boundaries of the object is approximately 30°.

The station point should be so situated in front of the object that the central plane passes through the centre of interest of the object. It may not, necessarily, be placed in front of the exact middle of the object.

PICTURE PLANE:

The position of the picture plane relative to the object, determines the size of the perspective view. The perspective will show the object reduced in size when it is placed behind the picture plane. If the object is moved nearer the picture plane, the size of the perspective will increase. When the picture plane coincides with the object, the perspective of the object will be of its exact size. When the object is placed in front of the picture plane, its perspective, when projected back, will show the object enlarged in size.

In fig. 19-3(*i*), the line *AB* is behind the picture plane. Its perspective *A'B'* is shorter than *AB*. In fig. 19-3(*ii*), *AB* is in the picture plane; its perspective *A'B'* is equal to *AB* and coincides with it. Fig. 19-3(*iii*) shows the line *AB* placed in front of the picture plane; when projected back on the picture plane, its perspective *A'B'* is longer than *AB*.

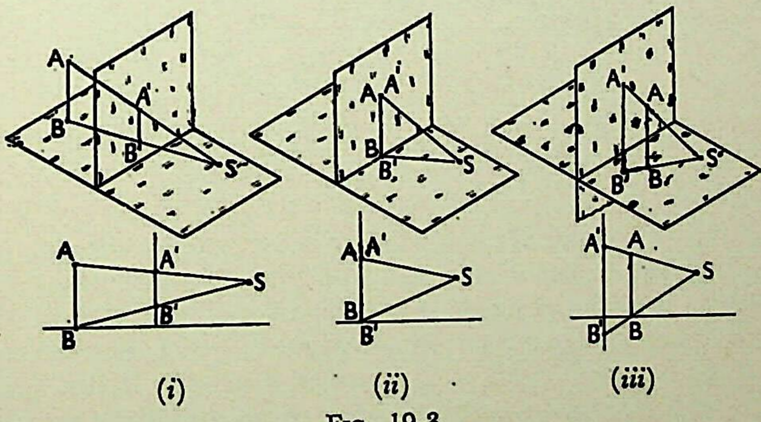


FIG. 19-3

The perspective view of an object may be obtained by either (i) *visual-ray method* or (ii) *vanishing-point method*. In the visual-ray method, points on the perspective are obtained by projecting (i) the plan and (ii) either the elevation

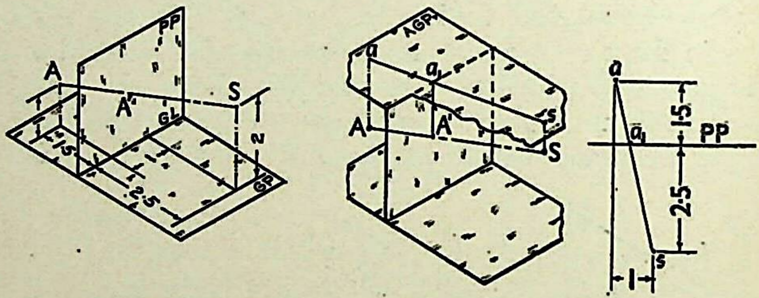
or the end view of the visual rays. Vanishing-point method is comparatively simple. In addition to the plan of the visual rays, use of vanishing points of straight lines is made in this method. An elevation or end view of the object is also required to be drawn, for determining the heights.

VISUAL-RAY METHOD:

This method is explained by means of the following illustrative problems.

Problem 1 (fig. 19-4):

A point A is situated 1·5 cm behind the picture plane and 1 cm above the ground plane. The station point is 2·5 cm in front of the picture plane, 2 cm above the ground plane and lies in a central plane 1 cm to the right of the point. Draw the perspective view of the point A.



(*i*) (*ii*) (*iii*)

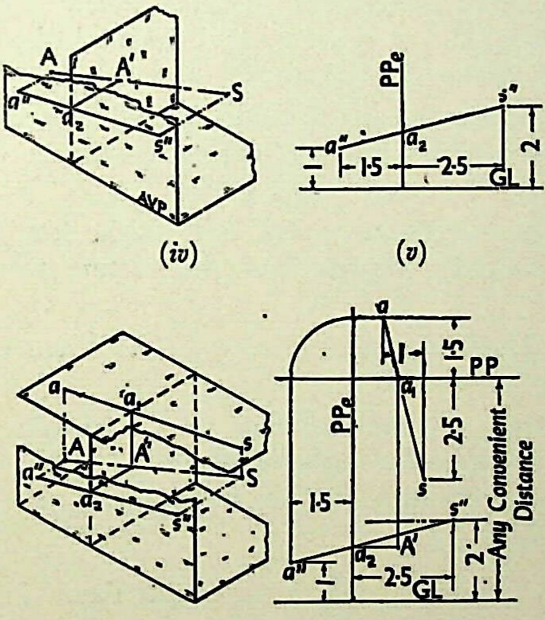
Fig. 19-4

The pictorial view of the ground plane, the picture plane, the given point and the station point in their respective positions is given in fig. 19-4(*i*). The visual ray *AS* from the station point *S* to the point *A* is also shown. It passes through the picture plane. To mark the perspective of *A*, the point *A′* at which *AS* pierces the picture plane should be located.

In fig. 19-4(*ii*), an auxiliary ground plane (AGP) is shown placed above the point *A*, and the visual ray *AS* is shown projected on it. *as* is the plan of *AS* and a_1 is the plan of the point *A′* at which *AS* pierces the picture plane. a_1 shows the position of the point *A′* along the length of the

picture plane. When the auxiliary ground plane, is revolved and brought in the same plane as that of the picture plane, the view will be as shown in fig. 19-4(*iii*).

To obtain the height of A' above the ground plane, an auxiliary vertical plane (AVP) perpendicular to both the picture plane and the ground plane is placed to the left of A and an end view of AS is projected on it [fig. 19-4(*iv*)]. $a''s''$ is the end view of AS and a_2 is the end view of A'. It shows the height of A' above the ground plane. Fig. 19-4(*v*) shows the orthographic view (end view) when AVP is revolved and brought in the same plane as that of the picture plane.



(*iv*) (*v*)

(*vi*) (*vii*)

Fig. 19-4

Fig. 19-4(*vii*) shows the plan [fig. 19-4(*iii*)] and the end view [fig. 19-4(*v*)] combined together. A horizontal line drawn through a_2 and intersecting the vertical line through a_1 gives the point A' which is the perspective view of the point A. It is quite clear from the pictorial view [fig. 19-4(*vi*)] that A' lies in the picture plane on the line AS.

Steps in drawing the perspective view of the point A [fig. 19-4(*vii*)].

(i) Draw a horizontal line PP representing the picture plane in plan.

(ii) Mark a, the plan of A, 1·5 cm above PP.

(iii) Draw a line (representing the central plane) perpendicular to PP and 1 cm to the right of a. On this line, mark s, the plan of the station point, 2·5 cm below PP.

(iv) Draw a line joining a with s and intersecting PP at a point a_1.

(v) At any convenient distance below PP, draw a horizontal line GL. It is the ground line and also represents the ground plane in elevation.

(vi) Draw a line HL parallel to and 2 cm above GL. It is the horizon line and also represents the horizon plane in elevation.

(vii) At any point on GL and to the left of a, draw a vertical line PP_e (representing the picture plane in end view).

(viii) Mark a'', the end view of A, 1 cm above GL and 1·5 cm to the left of PP_e.

(ix) Mark s'', the end view of the station point, on HL and 2·5 cm to the right of PP_e.

(x) Draw a line joining a'' with s'' and intersecting PP_e at a point a_2.

(xi) Through a_1, draw a vertical line. Through a_2, draw a horizontal line intersecting the vertical line at a point A'.

Then A' is the perspective view of the point A.

Alternative method: Instead of the end view of AS, its elevation $a's'$ may be projected on the picture plane (considering it as a vertical plane of projection) as shown in fig. 19-5(*i*). The point A' must lie on this line $a's'$. It can be located by combining the plan and elevation as shown in figs. 19-5(*ii*) and 19-5(*iii*), and as described below.

(i) Draw the line *as* in plan [steps (i) to (iv)].

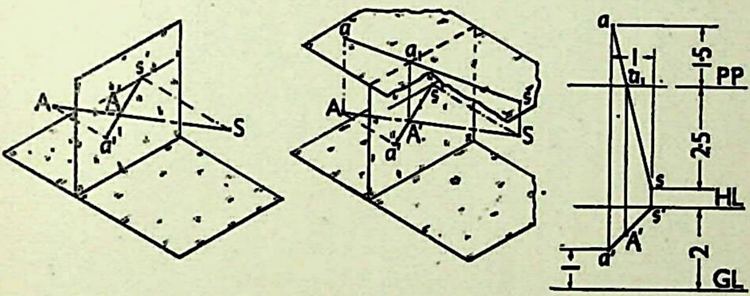
(ii) Draw the ground line GL at any convenient distance below PP and mark *a′*, the elevation of *A*, 1 cm above GL and in projection with *a*.

(iii) Draw the horizon line HL, 2 cm above GL and on it, mark *s′*, the elevation of *S*, in projection with *s*.

(iv) Draw a line joining *a′* with *s′*.

(v) Draw a vertical line through a_1 intersecting *a′s′* at a point *A′*.

Then *A′* is the perspective view of the point *A*.



(*i*) (*ii*) (*iii*)

FIG. 19-5

This method is comparatively simple and is generally adopted. In case of large objects, the perspective view often partly overlaps the elevation. This, sometimes, causes confusion.

Perspective view of a straight line by the visual-ray method is drawn by first marking the perspectives of its ends which are points and then joining them.

Problem 2 (fig. 19-6):

A straight line AB, 4 cm long, is parallel to and 1·5 cm above the ground plane, and inclined at 30° to the picture plane. The end A is 2 cm behind the picture plane. The station point is 4 cm above the ground plane, 5 cm in front of the picture plane and lies in a central plane which passes through the mid-point of AB. Draw its perspective view.

Draw a horizontal line PP. As *AB* is parallel to the ground plane, its plan will show its true length. Therefore, draw a line *ab*, 4 cm long, inclined at 30° to PP and the end *a*, 2 cm above PP.

Draw a vertical line through *m*, the mid-point of *ab* and on it mark *s*, the plan of the station point, 5 cm below PP.

Draw lines joining *s* with *a* and *b*, and intersecting PP at points a_1 and b_1 respectively.

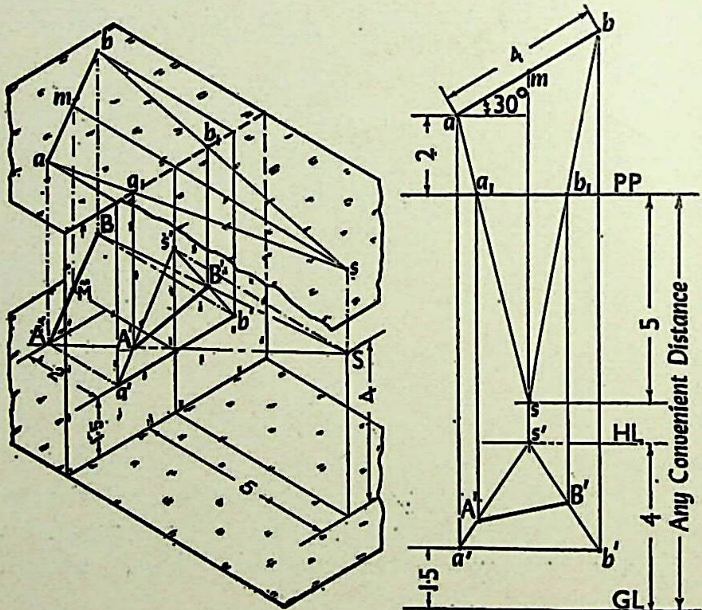


FIG. 19-6

Draw the ground line GL at any convenient distance below PP. Draw the horizon line HL parallel to and 4 cm above GL. Project s', the elevation on HL.

From *ab*, project the elevation $a'b'$, parallel to and 1·5 cm above GL.

Draw lines joining s' with a' and b'.

Through a_1 and b_1, draw verticals to intersect $a's'$ and $b's'$ at points A' and B' respectively.

Join A' with B'. Then $A'B'$ is the required perspective view of AB.

The perspective can also be obtained with the aid of the end view instead of the elevation.

Perspective view of any solid (by visual-ray method) can, similarly, be drawn by first obtaining the perspective of each corner and then joining them in correct sequence, taking care to show the hidden edges by dotted lines.

Problem 3 (figs. 19-7 and 19-8):

A rectangular pyramid, base 3 cm × 2 cm and axis 3·5 cm long, is placed on the ground plane on its base, with the longer edge of the base parallel to and 3 cm behind the picture plane. The central plane is 3 cm to the left of the apex and the station point is 5 cm in front of the picture plane and 2·5 cm above the ground plane. Draw the perspective view of the pyramid.

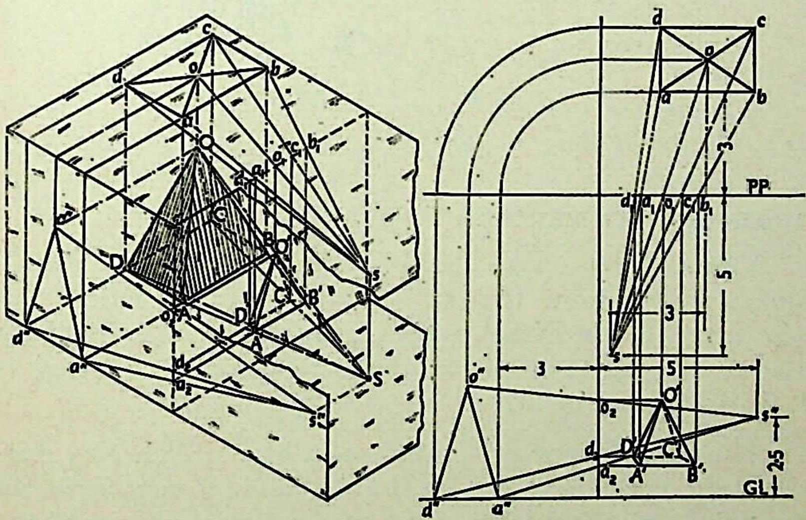


Fig. 19-7

Fig. 19-7 shows the perspective view of the pyramid obtained by means of its plan and end view. The pictorial view shows clearly that points on the perspective lie in the picture plane on respective visual rays.

In fig. 19-8, the perspective view is drawn by means of its plan and elevation. It partly overlaps the elevation.

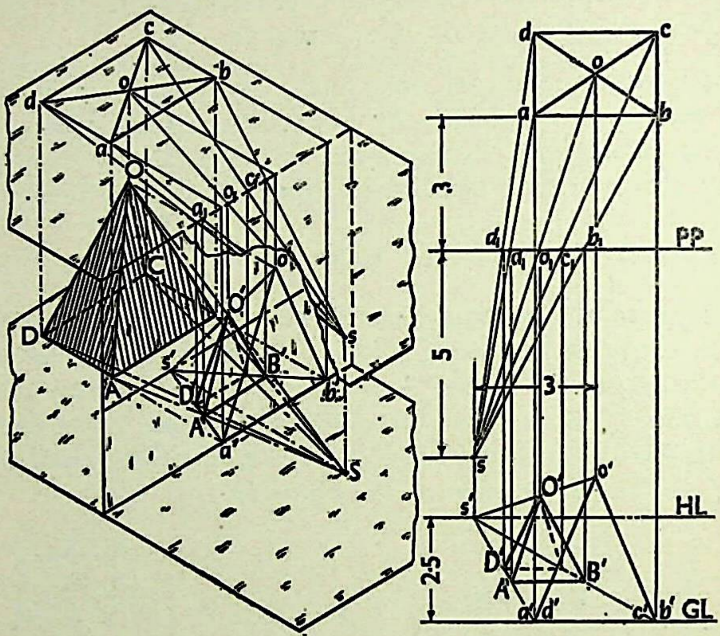


FIG. 19-8

VANISHING-POINT METHOD:

Vanishing points: These are imaginary points infinite distance away from the station point. In practice, the point at which the visual ray from the eye to that infinitely distant vanishing point pierces the picture plane is referred to as the vanishing point.

If we stand between the rails of a long stretch of a railway track, it would appear as if the rails meet very far away at a point just at the level of the eye, i.e. on the horizon line. Even the telegraph and telephone wires running along the track at the sides of the track appear to meet at the same point. This point is a vanishing point.

In fig. 19-9, *ab* is the plan of a line *AB* lying on the ground plane and inclined at angle θ to the picture plane. When viewed from the station point *s*, its intercept on PP is

a_1b_1. If the line is moved along the ground to the right, keeping the same inclination θ with the picture plane, its intercept will go on decreasing. The intercept becomes zero, or the line vanishes in a point at v when ab and the visual ray fall in a straight line. The point v is the plan position of the vanishing point for the horizontal line AB and for all lines parallel to AB, irrespective of their positions. The elevation position V of the vanishing point is obtained by projecting v, vertically on the horizon line.

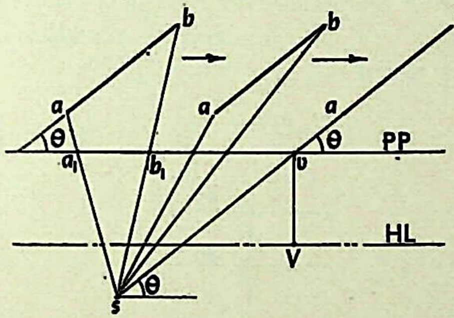


FIG. 19-9

Therefore, the vanishing point for any horizontal line is found by drawing a line parallel to the plan of that line from the plan of the station point. The point at which this line intersects the plan of the picture plane is then projected on the horizon line. This point on the horizon line is the elevation position of the vanishing point.

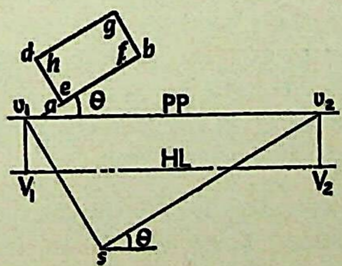


FIG. 19-10

In fig. 19-10, $abcd$ is the plan of a rectangular block placed on the ground plane so that a vertical face is inclined at angle θ to the picture plane. The vanishing points for the

line *ab* and for lines *cd*, *ef* and *gh* (which are parallel to *ab*) is obtained by drawing a line through *s*, parallel to *ab* and intersecting PP at a point v_2. Through v_2, a vertical line is drawn to meet HL at a point V_2. Then V_2 is the elevation position of the vanishing point. In perspective view of the block, edges *AB*, *CD*, *EF* and *GH* will converge to this point V_2. Similarly, V_1 is the vanishing point to which edges *AD*, *BC*, *EH* and *FG* will converge.

Thus, perpsectives of all horizontal lines, if produced, pass through their respective vanishing points on the horizon line. Perspectives of all horizontal parallel lines converge to a vanishing point on the horizon line.

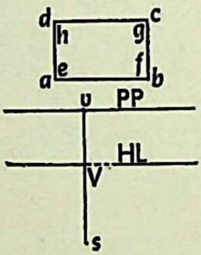


FIG. 19-11

Vanishing point for lines perpendicular to the picture plane is obtained by drawing a line through the plan of the station point, and perpendicular to the picture plane. It lies on the horizon line and coincides with the centre of vision. It is the elevation position of the station point. In fig. 19-11, *V* is the elevation position of the station point and the vanishing point, at which perspectives of lines *AD*, *BC*, *EH* and *FG* will converge. Thus, perspectives of all lines perpendicular to the picture plane converge to the center of vision on the horizon line.

Lines which are parallel to the picture plane will have no vanishing points. They vanish at infinity. Therefore, perspectives of vertical lines are vertical; perspectives of horizontal lines which are parallel to the picture plane, remain horizontal; and perspectives of lines inclined to the ground plane and parallel to the picture plane will be inclined in the same direction (see fig. 19-15).

Problem 4 (fig. 19-12):

A rectangular block, 3 cm × 2 cm × 1·5 cm, is lying on the ground plane on one of its largest faces. A vertical edge is in the picture plane and the longer face containing that edge makes an angle of 30° with the picture plane. The station point is 5 cm in front of the picture plane, 3 cm above the ground plane and lies in a central plane which passes through the centre of the block. Draw the perspective view of the block.

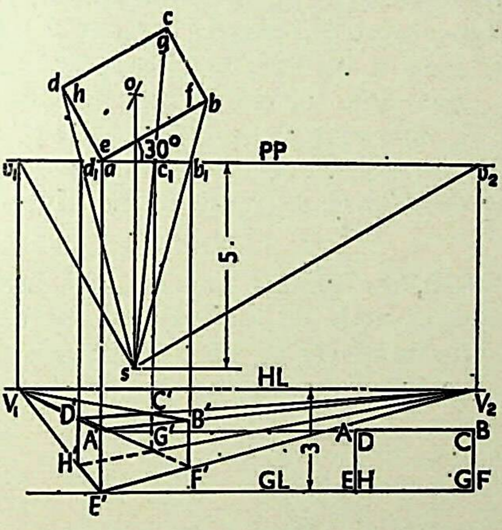


Fig. 19-12

Draw the plan *abcd* with *a* in PP and the longer edge *ab* inclined at 30° to PP. Mark its centre *o*. Mark *s*, the plan of the station point, on a vertical line through *o* and 5 cm below PP. Draw lines joining *s* with corners *a*, *b*, *c* and *d*, and intersecting *PP* at points a_1, b_1 etc.

Draw the ground line GL at any distance below PP and the horizon line HL, 3 cm above GL.

Through *s*, draw lines parallel to *ad* and *ab* cutting PP at points v_1 and v_2 respectively. Project v_1 to V_1 and v_2 to V_2 on HL. V_1 and V_2 are the vanishing points. Perspectives of edges *AD*, *EH*, *BC* and *FG* will converge to V_1 and those of edges *AB*, *CD*, *EF* and *GH* will converge to V_2.

Perspectives of vertical edges *AE*, *BF*, *CG* and *DH* will remain vertical.

As *AE* is in the picture plane, its perspective will be equal to the true length and the end *E* will lie on GL. Therefore, through *a*, draw a vertical line to a point E' on GL and on it, mark A' so that $A'E' = AE$. (This length may be measured directly or may be projected from the elevation, as shown).

Draw lines joining A' and E' with V_1 and V_2. Through b_1, draw a vertical line to intersect $A'V_2$ at B' and $E'V_2$ at F'. Similarly, draw a vertical through d_1 and obtain points D' and H'. Draw lines joining B' and F' with V_1, and D' and H' with V_2, intersecting at points C' and G' respectively. They must lie on the vertical line through c_1. Note that lines meeting at G' are all hidden and therefore, shown dotted.

In fig. 19-13, the ground line GL has been so drawn that HL coincides with PP. Hence, V_1 and V_2 coincide with v_1 and v_2 respectively on PP. The perspective view is obtained in the same manner as described above.

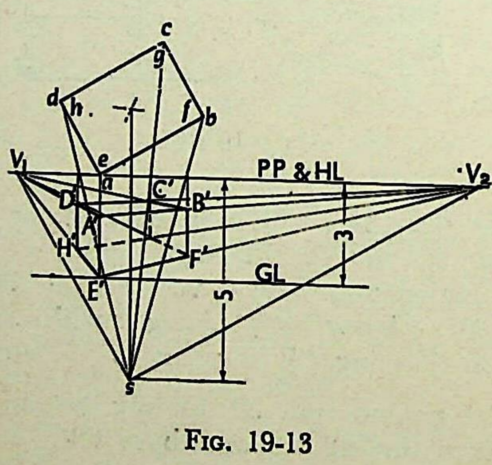


FIG. 19-13

Distance points: Vanishing points for all horizontal lines inclined at 45° to the picture plane are given special name of *distance points* on account of their definite positions. They are equidistant from the centre of vision, the distance of each from the centre of vision being equal to the distance

of the station point from the picture plane. In fig. 18-14, $P_1c' = P_2c' = sc$. Thus, perspectives of all horizontal lines inclined at 45° to the picture plane converge to distance points on the horizon line.

Problem 5 (fig. 19-14):

Draw the perspective view of a cube of 2·5 cm edge, lying on a face on the ground plane, with an edge in the picture plane and all vertical faces equally inclined to the picture plane. The station point is 5 cm in front of the picture plane, 3·5 cm above the ground plane and lies in a central plane which is 1 cm to the left of the centre of the cube.

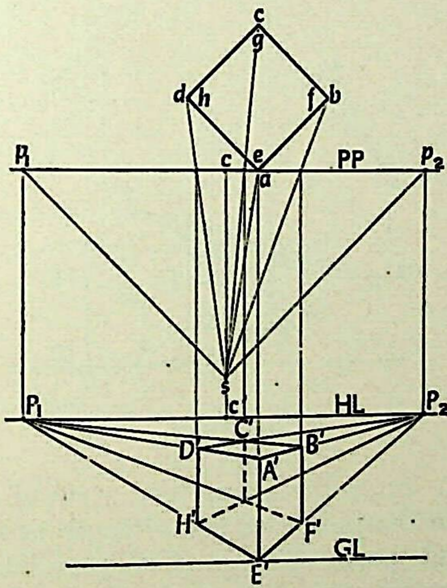


FIG. 19-14

Draw the plan of the cube and of the station point. Draw lines GL and HL. Obtain distance points P_1 and P_2 in the same manner as the vanishing points; or project s to c' on HL and mark points P_1 and P_2 such that $c'P_1 = c'P_2 = sc$. Draw the perspective view as described in the previous problem.

Parallel perspective: When an object has its one or more faces parallel to the picture plane, its perspective is

called parallel perspective. As perspectives of horizontal, vertical or sloping lines parallel to the picture plane remain respectively horizontal, vertical or sloping, faces which are parallel to the picture plane do not get distorted in shape. They change in size only. When a face is in the picture plane, its perspective will be of the true size and shape.

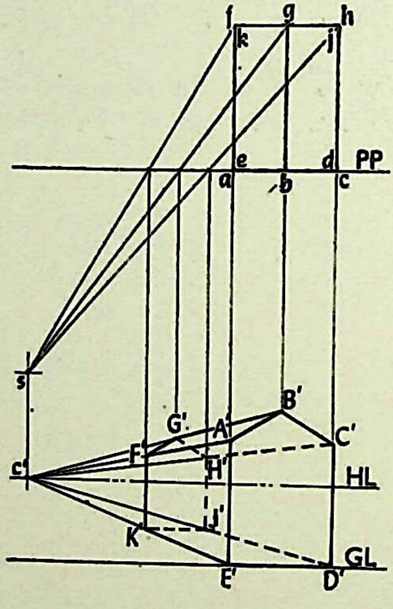


FIG. 19-15

Fig. 19-15 shows the perspective view of a hut having its front face in the picture plane. The front face is seen in its true size and shape, while the back parallel face is of the same shape but reduced in size. As the lines *AF*, *BG*, *CH*, *DJ* and *EK* are perpendicular to the picture plane, their perspectives *A′F′*, *B′G′* etc. converge to the centre of vision *c′* on HL. Note that vertical lines *AE*, *CD* etc. remain vertical in perspective. Similarly, horizontal lines *ED* and *KJ*, and sloping lines *AB*, *BC*, *FG* and *GH* (which are all parallel to the picture plane) remain respectively horizontal and sloping in perspective.

Measuring line or **Line of heights:** We have seen that when a line is in the picture plane, it is seen in its true length

in perspective. When a line is behind the picture plane, it is foreshortened in its perspective view.

In fig. 19-16, *ab* is the plan of a rectangle *ABCD* whose surface is vertical and inclined to the picture plane, the edge *DC* is on the ground plane and the edge *AD* is in the picture plane. In its perspective view *A'B'C'D'*, *A'D'* is equal to the true length *AD*, while *B'C'* is shorter. *A'D'* lies on a vertical line drawn through *a*. The length *B'C'* is derived from *A'D'*.

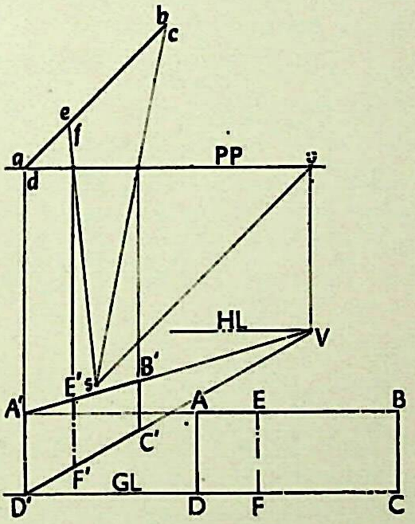


FIG. 19-16

Let us now consider the rectangle *EBCF* within the rectangle *ABCD*. *E'B'C'F'* is its perspective view. *E'F'* is shorter than *EF* or *AD*. Its length has been derived from *A'D'*. Thus, *aD'* is the measuring line or the line of heights for points *E*, *F*, *B* and *C*. It is obtained by producing *eb* to intersect *PP* at the point *a*; through *a*, a perpendicular is dropped to meet *GL* at a point *D'*. In other words, if we imagine that the rectangle *EBCF* is moved along the line *eb* to meet the picture plane, its edge *EF* will strike it at the line *A'D'* showing its true length.

Thus, the measuring line or the line of heights is the trace or the line of intersection with the picture plane, of the

vertical plane containing the point or points whose heights are to be determined. Heights of points lying in different vertical planes can be measured from their respective lines of heights. Heights on this line may be measured directly with a scale or may be projected from the elevation.

Problem 6 (fig. 19-17):

Determine the line of heights for points lying in the line ac which is the plan of a regular hexagon ABCDEF (elevation of which is shown on GL) and then, draw its perspective view from the given position of the station point.

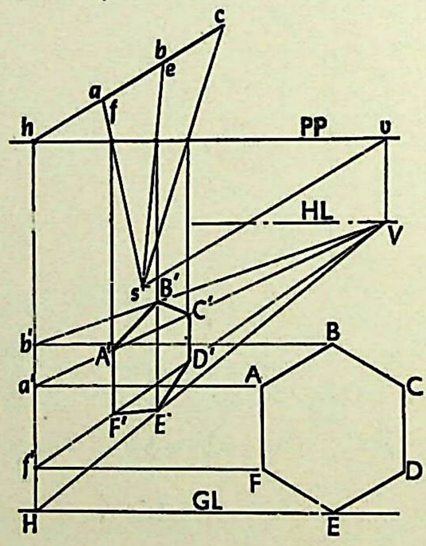


FIG. 19-17

As *ac* is behind the picture plane, all the sides of the hexagon will be foreshortened. To determine the measuring line, produce *ac* to meet *PP* at a point *h*; through *h*, draw a vertical to meet *GL* at a point *H*. Then *hH* is the measuring line for heights of all points in the hexagon.

To draw the perspective view, determine the vanishing point *V*. Project horizontally, points *A*, *B* etc. from the elevation to points *a'*, *b'* etc. on *hH*. Draw lines joining these points with *V*. Draw lines joining *s* with *a*, *b* and *c*. Through the points of intersection of these lines with *PP*,

draw verticals to meet the corresponding lines converging to V, at points $A', B' \ldots F'$. Join these points in correct sequence. The resulting figure is the perspective view of the hexagon.

Perspectives of circles:

A circle will appear as a circle in its perspective view when it is parallel to the picture plane. In other positions its perspective will be an ellipse (except when it lies in the central plane).

To obtain points for the ellipse, the circle is enclosed or 'crated' in a square. The mid-points of the sides of the square are the four points for the ellipse. Points of intersection of the diagonals with the circle are the other four points. Lines are drawn through these points, parallel to the sides of the square. Perspective of the square along with the parallel lines is then drawn, locating the eight points for drawing the ellipse.

Problem 7 (fig. 19-18):

Draw the perspective view of a circle of 5 cm diameter, lying on the ground plane and touching the picture plane. The station point is 8 cm in front of the picture plane and 6 cm above the ground plane. The central plane passes through the centre of the circle.

With *o* as centre, draw the circle touching *PP*. Enclose it in a square *abcd* with *ab* in *PP*. Draw the horizontal and vertical diameters and the diagonals of the square. Mark points 1 to 8 as shown. Through the points on the diagonals, draw lines *ef* and *gh* parallel to *ad*.

Draw lines *GL* and *HL*, and mark *s*, the plan of the station point. Project *s* to c' on *HL*. c' is the centre of vision to which the perspectives of lines perpendicular to *PP* will converge.

Draw the perspective view of the square. Draw the diagonals, intersecting each other at O'. Through O', draw horizontal and vertical lines, intersecting the sides at

points 1′, 2′, 3′ and 4′. Draw perspectives of lines *ef* and *gh*, cutting the diagonals at points 5′, 6′, 7′ and 8′. Draw the ellipse through points 1′, 5′, 2′....8′. The ellipse is the required perspective view of the circle.

It may be noted that the centre of the ellipse does not coincide with *O*′, the centre of the perspective of the square.

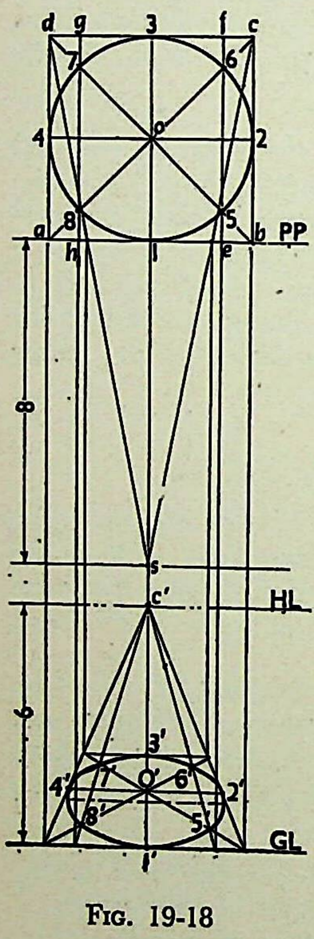


Fig. 19-18

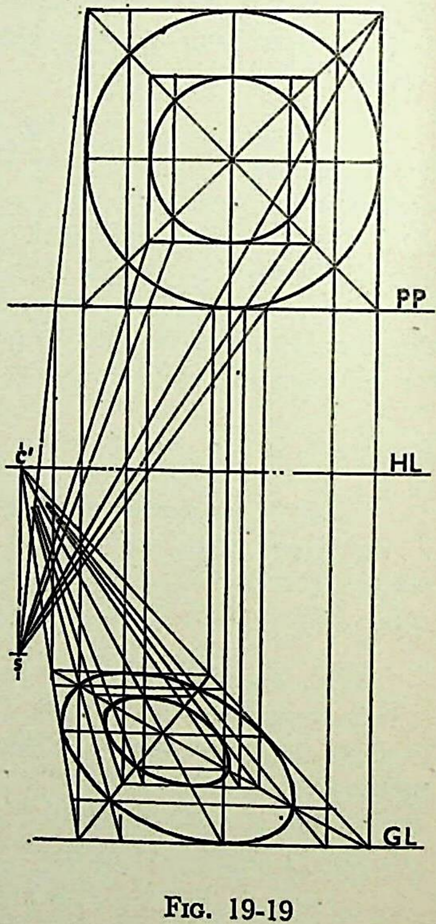


Fig. 19-19

Perspectives of concentric circles are not concentric ellipses as can be seen from fig. 19-19. The circles are on the ground plane. The station point is away to the left of the cnetre of the circles.

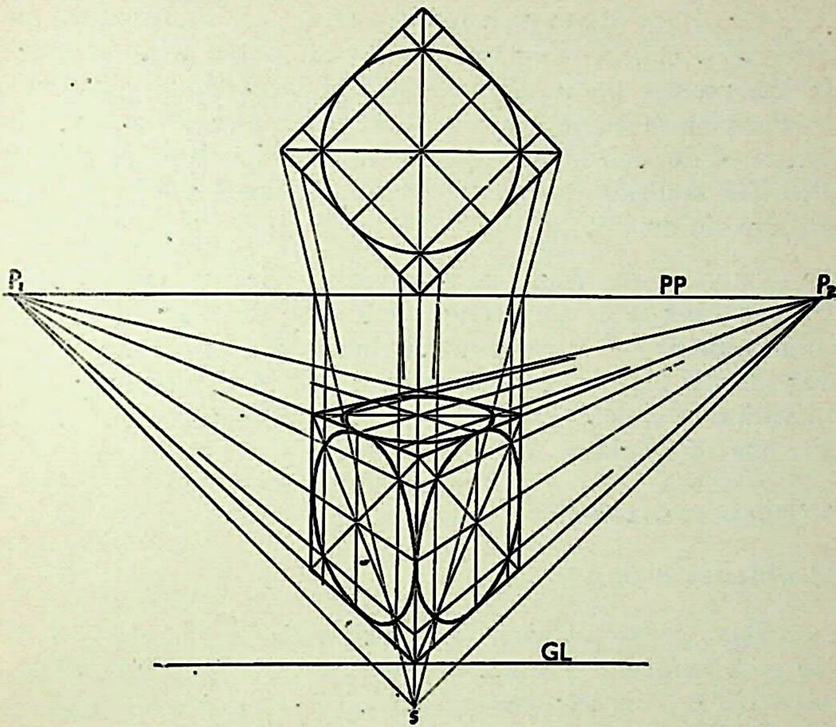


FIG. 19-20

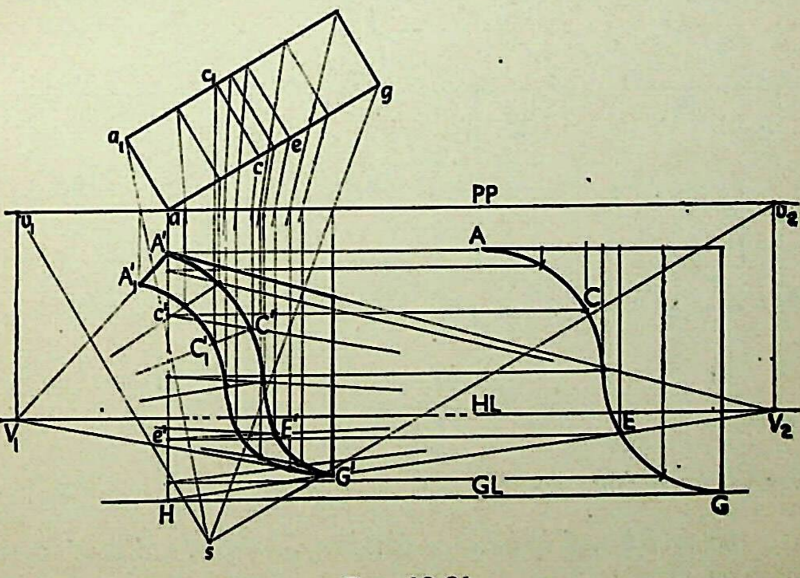


FIG. 19-21

Fig. 19-20 shows perspectives of circles inscribed in the faces of a cube resting on the ground plane with an edge in the picture plane and all vertical faces equally inclined to the picture plane. The central plane passes through the centre of the cube. The ground line *GL* has been so placed that *HL* coincides with *PP*. Hence, distance points P_1 and P_2 also lie on *PP*.

Curve of any shape can, similarly, be drawn in perspective by enclosing it in a rectangle and then drawing horizontal and vertical lines through a number of points on the curve. Fig. 19-21 shows the perspective view of a moulding, the elevation of which is shown on the right-hand side. *aH* is the line of heights.

TYPICAL PROBLEMS:

Problems 8 to 14:

Draw perspective views of a straight line AB, 3 cm long, in the given positions. The station point is 4 cm in front of the picture plane (PP), 3 cm above the ground plane (GP) and lies in a central plane (CP) in the given positions.

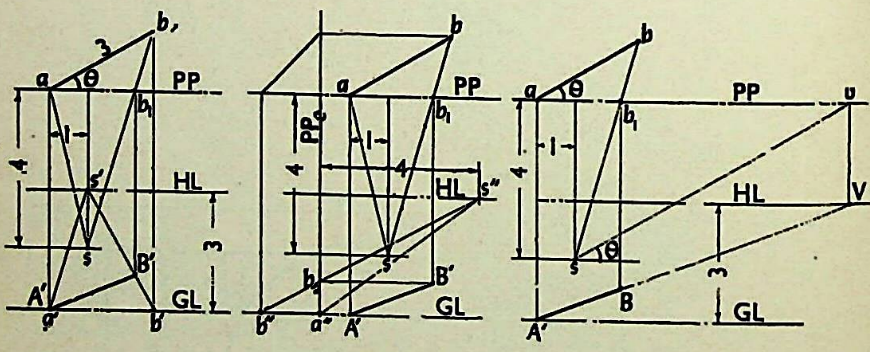


FIG. 19-22

Problem 8 (fig. 19-22):

AB in GP and inclined at θ to PP; A in PP; CP, 1 cm to the right of A.

(i) Visual-ray method—by plan and elevation [fig. 19-22(*i*)]:

Draw the plan *ab* with *a* in *PP*. Draw the elevation *a'b'* in *GL*. Mark *s*, the plan and *s'*, the elevation of the station point. Join *s* with *a* and *b*, and *s'* with *a'* and *b'*. As *A* is in *PP* and on *GP*, its perspective *A'* will coincide with *a'*. *B'* will lie on *s'b'* on a vertical through b_1. Join *A'* with *B'*. Then *A'B'* is the perspective view of *AB*.

(ii) Visual-ray method—by plan and end view [fig. 19-22(*ii*)]:

Draw the plan *ab* and the end view *a"b"*. Mark *s*, the plan and *s"*, the end view of the station point. Join *s* with *a* and *b*, and *s"* with *a"* and *b"*. As *A* is in *PP* and on *GP*, its perspective *A'* will lie on *GL* on the vertical through *a*. *B'* will lie at the point of intersection of the vertical through b_1 and the horizontal through b_2. *A'B'* is the required perspective view.

(iii) Vanishing-point method [fig. 19-22(*iii*)]:

Draw the plan *ab* and mark *s*, the plan of the station point. Draw *sv* parallel to *ab*. Through *v*, draw a vertical and mark *V*, the vanishing point, on *HL*. *A'* will lie on *GL* on a vertical through *a*. Join *A'* with *V*. Draw a vertical through b_1 and obtain *B'* on *A'V*. *A'B'* is the required perspective view.

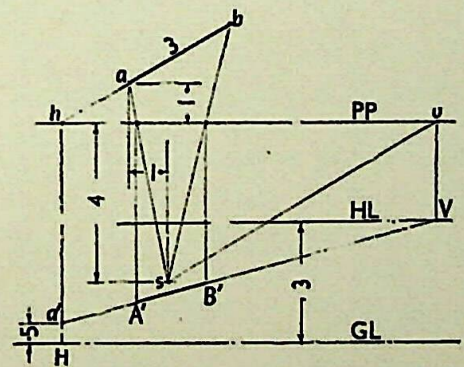


FIG. 19-23

Problem 9 (fig. 19-23):

AB parallel to PP and 0·5 cm above it; inclined at 30° to PP; A, 1 cm behind PP; CP, 1 cm to the right of A.

Draw the plan *ab*, mark *s* and obtain the vanishing point *V*. Join *s* with *a* and *b*. As *A* is behind *PP*, the line of heights will be necessary. Produce *ba* to *h* on *PP*. Draw the line of heights *hH*. Mark a point *a′* on *hH*, 0·5 cm above *GL*. Join *a′* with *V*. Obtain points *A′* and *B′* on the line *a′V*, as shown. *A′B′* is the required perspective view.

Problem 10 (fig. 19-24):

AB, perpendicular to PP; A, 1 cm behind PP and 0·5 cm above GP; CP, 2 cm to the right of AB.

Draw *ab*, the plan and *a′*, the elevation. Mark *s* and *s′*. Join *s* with *a* and *b*, and *s′* with *a′*. Obtain the perspective *A′B′* as shown. This is the visual-ray method.

The same figure can be interpreted as the solution by vanishing-point method. As the line is perpendicular to *PP*, its vanishing point will be at the centre of vision *c′*, i.e. at *s′* on *HL*. The vertical through *a* is the line of heights. *a′*, marked 0·5 cm above *GL*, shows the height of *A* and *B* both. *A′B′*, the perspective, lies on *a′s′*, i.e. on *a′c′*.

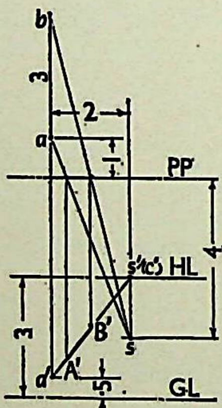


FIG. 19-24

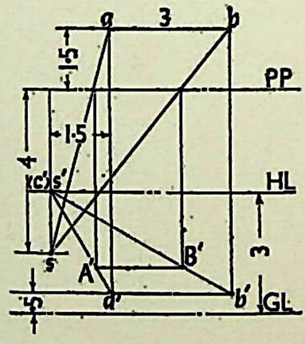


FIG. 19-25

Problem 11 (fig. 19-25):

AB, parallel to both PP and GP; 0·5 cm above GP and 1·5 cm behind PP; CP, 1·5 cm to the left of A.

As the line is parallel to *PP*, it will have no vanishing point. The perspective view *A′B′* is drawn by means of

plan and elevation (visual-ray method). It is parallel to $a'b'$. (See note at the end of problem 13).

Problem 12 (fig. 19-26):

AB, parallel to and 1·5 cm behind PP; inclined at 30° to GP; A, 0·5 cm above GP; CP, 1·5 cm to the left of A.

As AB is parallel to PP, it will have no vanishing point. The perspective $A'B'$ is obtained by means of plan and elevation (visual-ray method). It is parallel to the elevation $a'b'$. (See note at the end of problem 13).

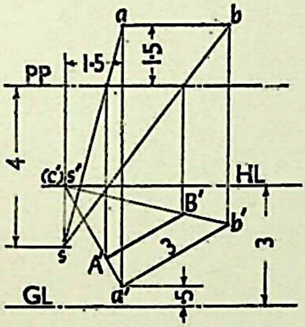


Fig. 19-26

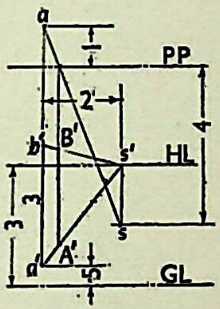


Fig. 19-27

Problem 13 (fig. 19-27):

AB perpendicular to GP and 1 cm behind PP; A, 0·5 cm above GP; CP, 2 cm to the right of AB.

As AB is parallel to PP, it will have no vanishing point. a is the plan and $a'b'$ is the elevation. The perspective $A'B'$ is obtained by visual-ray method. It is perpendicular to GP.

Note: Problems 11, 12 and 13 may also be interpreted as solved by vanishing-point method. The ends A and B of the line may each be assumed to be the ends of two separate lines perpendicular to the PP and which in perspective view would vanish at the centre of vision c′, i.e. at s′.

These problems may also be termed as cases of parallel perspective. Horizontal, inclined and vertical lines are each parallel to the PP. In their perspective views they remain respectively horizontal, inclined and vertical.

Problem 14 (fig. 19-28):

A rectangle ABCD, 3 cm × 2 cm, has its surface parallel to and 1 cm above the ground plane. Its shorter edge AD is inclined at 60° to the picture plane which passes through the rectangle so that the corner A is 1 cm in front of it. The station point is 6 cm in front of the picture plane, 4 cm above the ground plane and lies in a central plane which passes through A. Draw the perspective view of the rectangle.

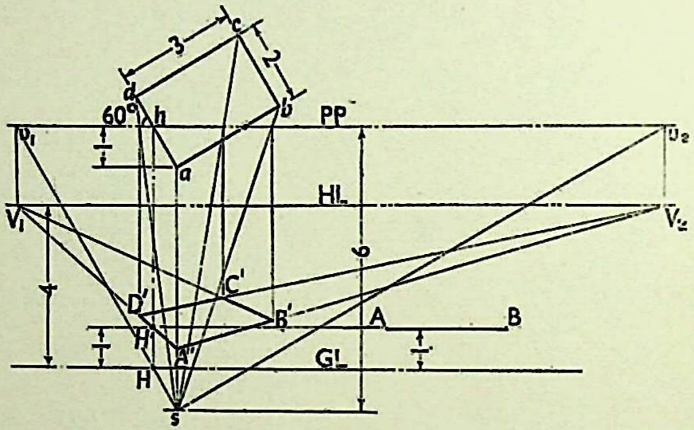


Fig. 19-28

Draw the line PP and the plan *abcd* in the given positions. Mark *s* and join it with the corners *a*, *b*, *c* and *d*. Obtain the vanishing points V_1 and V_2. Through *h* (the point at which PP cuts *ad*), draw the line of heights *hH*. Draw the elevation of the rectangle. It is the line *AB*, parallel to and 1 cm above GL. Project the line to a point *H'* on *hH*. Draw a line through *H'* and V_1 and on it, obtain points *A'* and *D'* as shown. Draw lines joining V_2 with *A'* and *D'*, and on them, obtain points *B'* and *C'*. Join *B'* with *C'*. Then *A'B'C'D'* is the perspective view of the rectangle. Note that *B'C'*, if produced, will pass through V_1.

It may also be noted that *H'* can be marked directly 1 cm above *H*. Hence, it is not absolutely essential to draw the elevation in this particular case. (A line of heights may also be drawn through the point of intersection of *ab* with

PP. In that case, H' will be joined with V_2 and the point B' obtained on $H'V_2$.)

Problem 15 (fig. 19-29):

Draw the perspective view of a circle of 4 cm diameter, having its surface vertical and inclined at 45° to the picture plane. The centre of the circle is 2·5 cm above the ground plane. The positions of the station point and the horizon level are as shown in the figure.

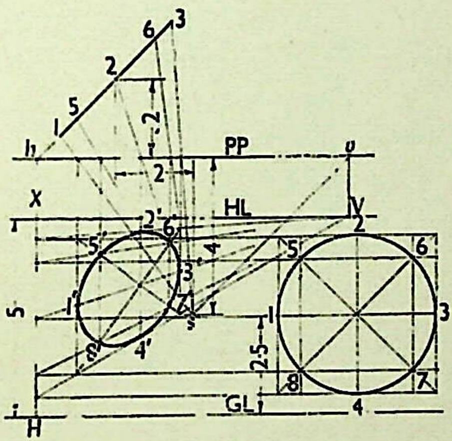


Fig. 19-29

Draw the plan of the circle. Obtain the vanishing point V. Draw the elevation of the circle (on the right-hand side) with its centre 2·5 cm above GL and mark eight points on it. Mark these points on the plan also. Draw the line of heights hH and on it, project horizontally, the eight points from the elevation. Join s with the points in the plan, and join V with the points on hH. Draw verticals through the points of intersection of $s1$, $s5$ etc. with PP, and obtain points 1′, 5′ etc. Draw the ellipse through these points. It is the perspective view of the circle. Note that points 5′, 6′, 7′ and 8′ lie on the diagonals of the perspective view of the square in which the circle is enclosed.

Problem 16 (fig. 19-30):

Draw the perspective view of a pentagonal prism, lying on the ground plane on one of its rectangular faces, the axis being inclined at 30° to the picture plane, and a corner of the base touching the picture plane. The station point is 6·5 cm in front of the picture plane, and lies in a central plane which bisects the axis. The horizon is at the level of the top edge of the prism.

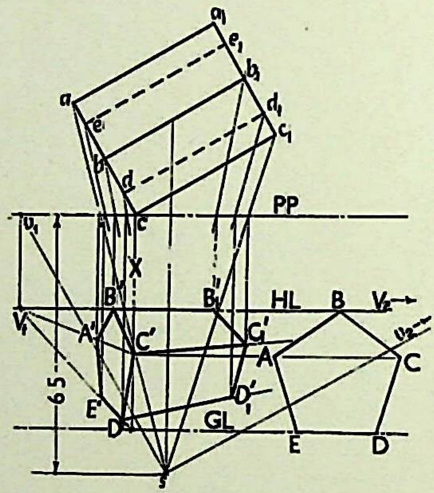


Fig. 19-30

Draw the plan and end elevation of the prism. Mark *s* and obtain the two vanishing points V_1 and V_2. The line *X*, through *c*, is the line of heights for points *A*, *B*....*E*. Join *s* with all the points in the plan.

Project horizontally, all the corners in the elevation to points on the line *X* and join these points with V_1. Obtain points A', B'....E' on these lines. Draw lines joining B', C' and D' with V_2, and obtain points B_1', C_1' and D_1' on these lines. Complete the perspective view as shown The vanishing point V_2 is not shown in the figure. Hidden edges of the prism are also not shown in the perspective view.

Problem 17 (fig. 19-31):

Draw the perspective view of the frustum of a hexagonal pyramid (top 1·5 cm side, base 2·5 cm side and axis 4 cm long), the plan of which is given in the figure, along with the positions of the station point and the horizon.

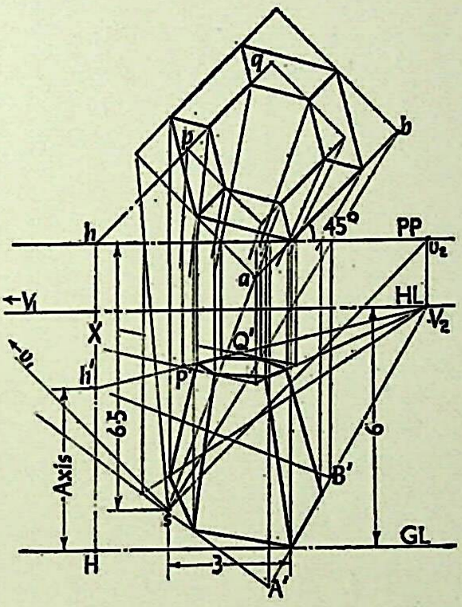


FIG. 19-31

Enclose the two hexagons in two rectangles as shown in the figure. (They can be enclosed in different ways also.) Determine the vanishing points V_1 and V_2. Draw the perspective of the larger rectangle. Note that the corner *a* is projected back on *PP*. Draw *hH* the line of heights for points *p* and *q*. On this line, mark the length of the axis viz. *h'H*. Join *h'* with V_2 and obtain the perspective of the line *pq* and the whole (small) rectangle. Mark the corners of top of the frustum and complete the perspective view as shown. Hidden edges have not been shown.

The data in the problem on perspective view is generally given in the form of a figure showing the plan and elevation of the object together with the position of the station point or the observer.

In the following more advanced problems, the data of each problem along with the solution is given in the same figure. All construction lines are shown to make the solutions self-explanatory. Hints are given only where deemed necessary.

Students are advised to copy only the data of each object, draw its perspective view and then compare with the given solution.

Lines of heights are marked with letter X. All dimensions, unless otherwise stated, are given in centimetres.

Problem 18 (fig. 19-32): *Letter P.*

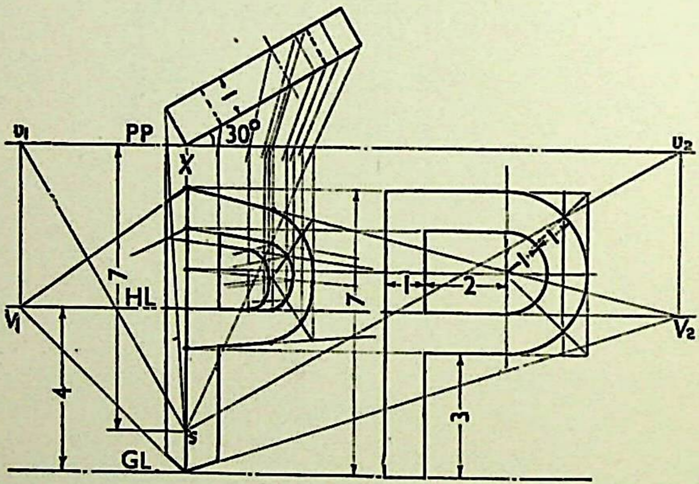


FIG. 19-32

Perspectives of semi-circles are drawn by enclosing the outer semi-circle in a rectangle and then marking points at which the diagonals cut the semi-circles.

Problem 19 (fig. 19-33): *Guide-block.*

This is a problem on parallel perspective. The front face is in *PP* and hence, it is seen in its true shape and size. It is above the ground plane and, therefore, above *GL*.

Circles appear as circles. Lines perpendicular to *PP* converge to the centre of vision *c'*.

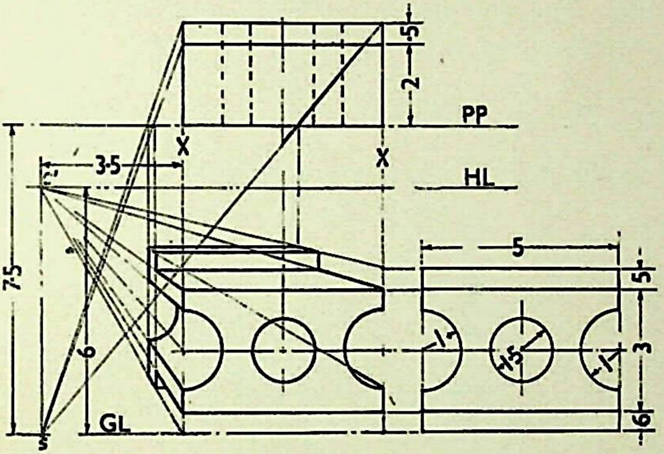


FIG. 19-33

Problem 20 (fig. 19-34): *Mile-stone.*

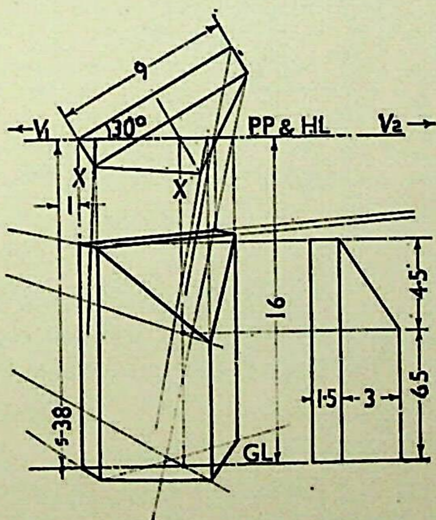


FIG. 19-34

PP passes through the stone; hence, the perspective of the front part will appear bigger. A separate line of heights is needed for the front vertical edge. The station point and vanishing points are not shown in the figure. *HL* is shown coinciding with *PP*.

Problem 21 (fig. 19-35):

A square prism with a square pyramid placed centrally on its top.

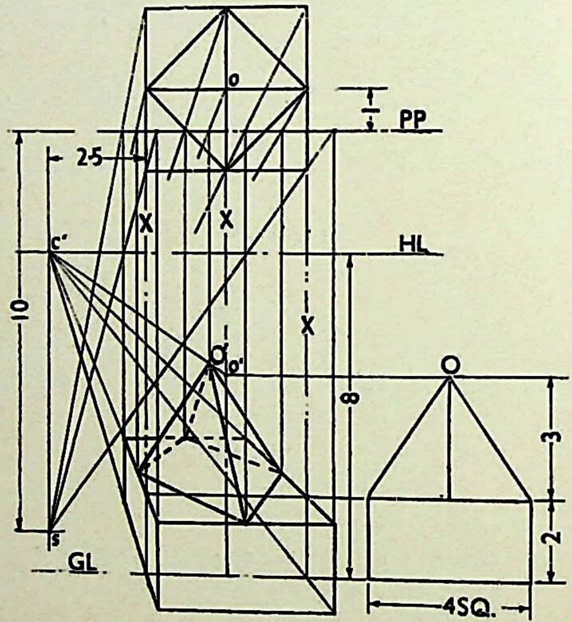


FIG. 19-35

Again a case of parallel perspective, but some portions of the solids are in front of *PP*. The front corners are projected back on *PP*; hence, these portions appear bigger in size.

Two lines of heights have been used for the prism (though even one is sufficient). Corners of the base of the pyramid are at the mid-points of the sides of top of the prism. A separate line of heights is needed for the apex.

Problem 22 (fig. 19-36): *Window-frame placed in a wall.*

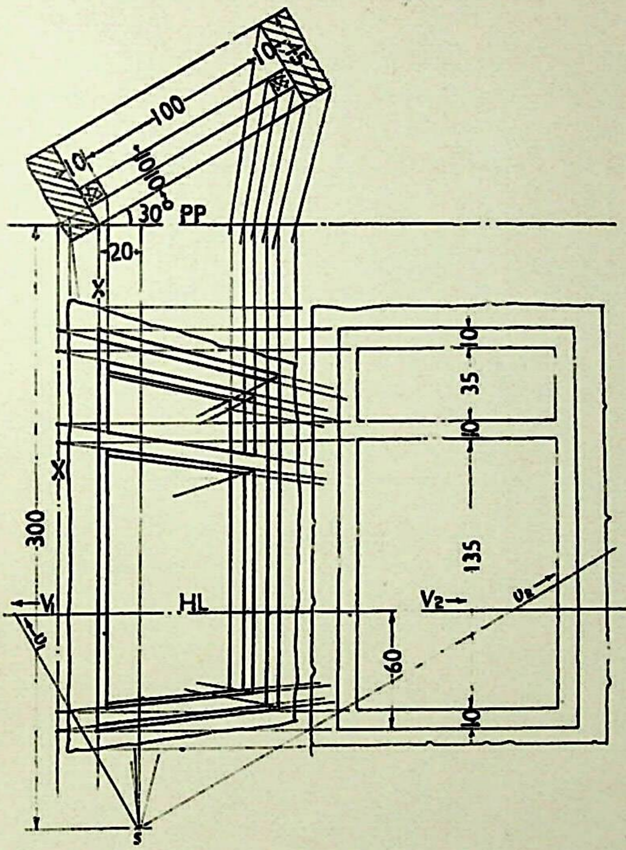


FIG. 19-36

Two lines of heights — one for the frame and the other for the wall — have been used.

Problem 23 (fig. 19-37): *Casting.*

GL has been so placed that *HL* coincides with *PP*. It may be taken lower. A separate line of heights is required for the central rib.

Problem 24 (fig. 19-38):

Hut with a door and a side window. All dimensions are in metres. Take scale 2 cm = 1 m.

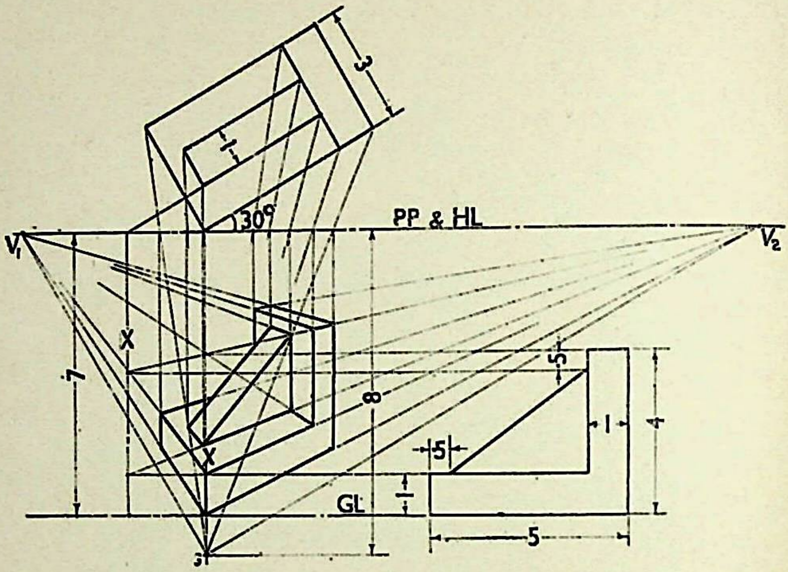


FIG. 19-37

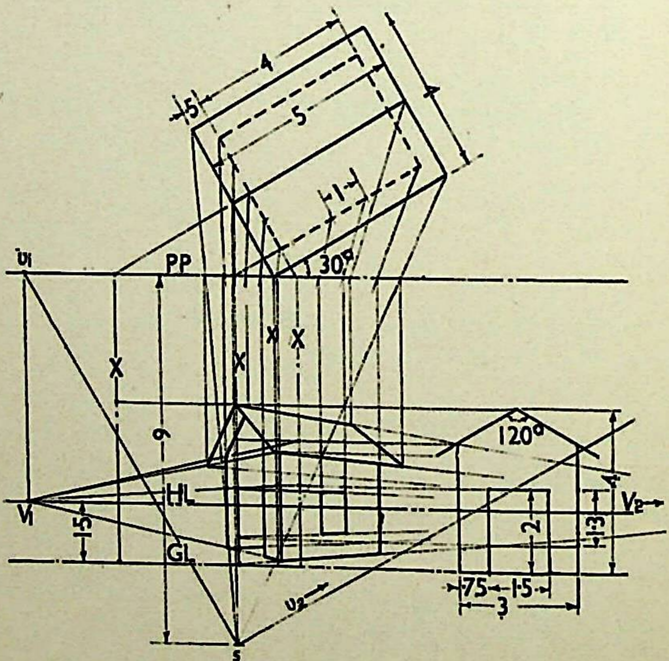


FIG. 19-38

Note that a corner of the roof is in *PP*. Four lines of heights are needed.

Problem 25 (fig. 19-39): *Ogee arch in a wall.*

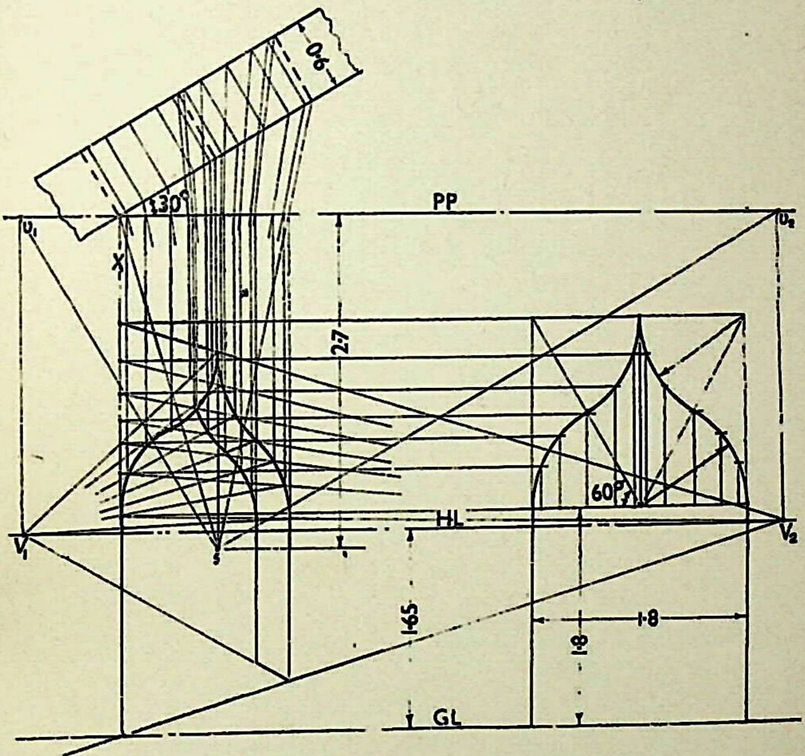


FIG. 19-39

A number of points are marked on the arch and the curves are drawn through the perspectives of these points.

Problem 26 (fig. 19-40): *Model of steps.*

This is also a problem on parallel perspective, but the front face is behind *PP*; hence, that face will be smaller in size.

Problem 27 (fig. 19-41): *Sign-post.*

(*All dimensions are in decimetres.*)

Four lines of heights have been used. Vanishing points are not shown in the figure.

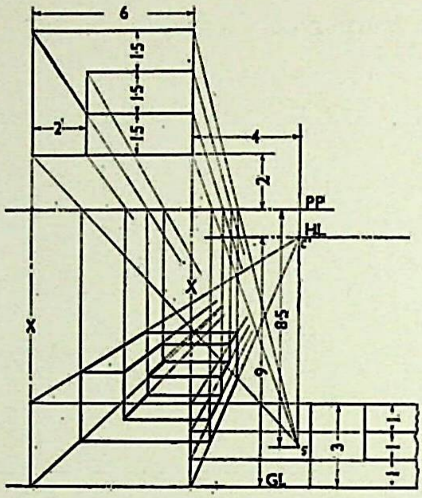


FIG. 19-40

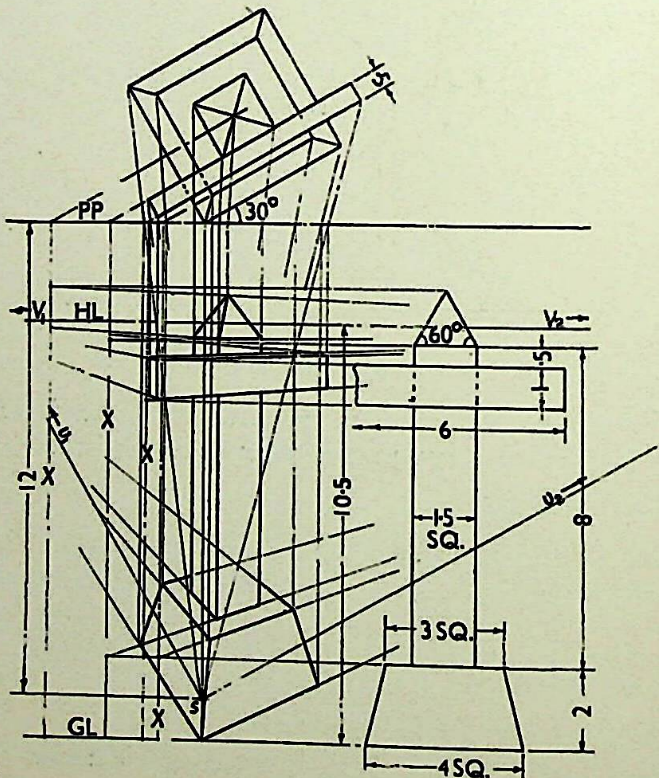


FIG. 19-41

Problem 28 (fig. 19-42): *Lamp-post.*

All faces of the lamp casing are of glass (except the bottom). The base is cylindrical.

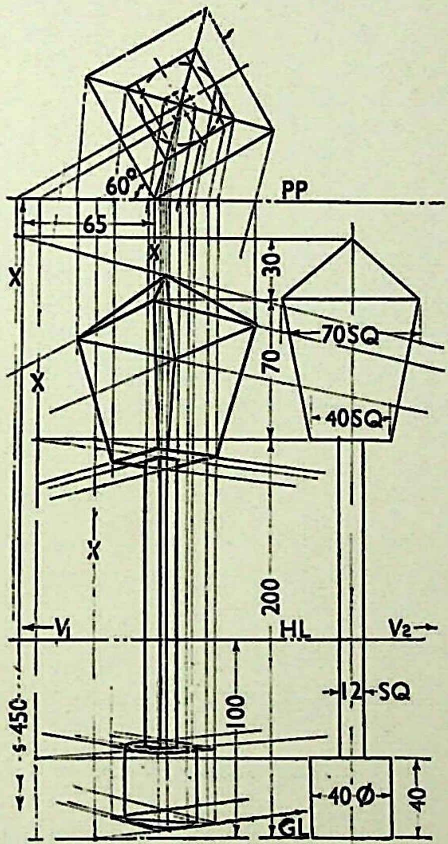


FIG. 19-42

The back edges (of the lamp casing) which are visible through the glass are shown thinner.

Problem 29 (fig. 19-43): *Steps with guards at sides.*

Two lines of heights are used for convenience.

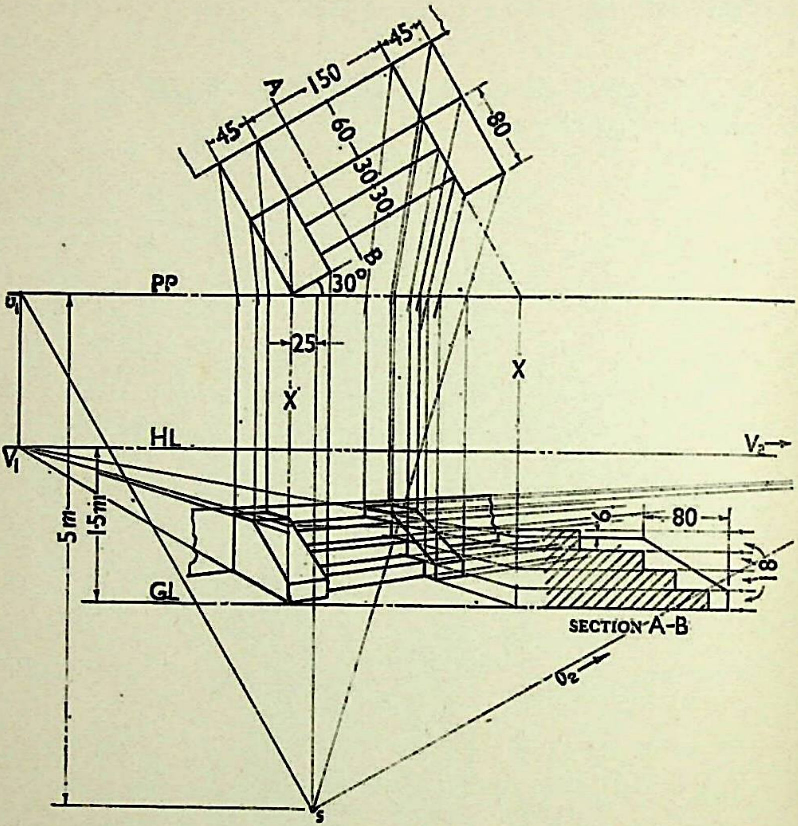


FIG. 19-43

EXERCISES XIX

(1) A man stands at a distance of 5 m from a flight of four stone steps, having a width of 2 m, tread 0·3 m and rise 0·2 m. The flight makes an angle of 45° with the picture plane and touches the same at a distance of 2 m to the right of the centre of vision. Draw the perspective view of the flight.

(2) Draw the perspective view of a square pyramid of base 10 cm side and height of the apex 12 cm. The nearest edge of the base is parallel to and 3 cm behind the picture plane. The station point is situated at a distance of 30 cm from the picture plane, 6 cm above the ground plane and 20 cm to the right of the apex.

(3) Draw the perspective view of the model of steps shown in fig. 15-19. The position of the steps relative to the picture plane is shown in fig. 19-44. The station point is 200 mm from the picture plane. Take horizon level to be 100 mm above the ground level.

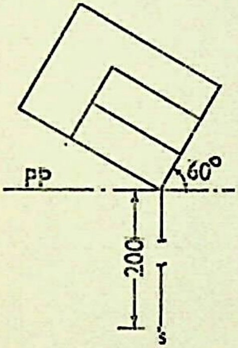


FIG. 19-44

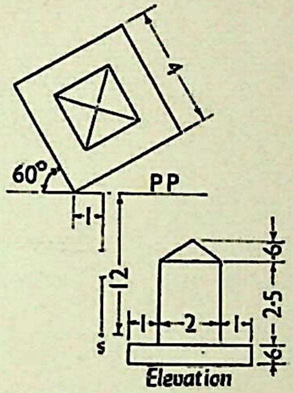


FIG. 19-45

(4) Draw the perspective view of the memorial shown in fig. 19-45. The horizon level is 8 m above the ground plane, while the observer is stationed at a distance of 12 m from the picture plane. (All dimensions are in metres.)

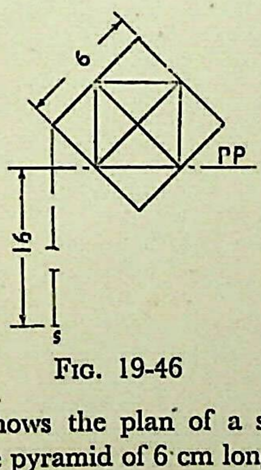


FIG. 19-46

(5) Fig. 19-46 shows the plan of a square prism of 4 cm thickness with a square pyramid of 6 cm long axis, placed centrally on it. The station point is 16 cm from the picture plane and 10 cm above the ground plane. Draw the perspective view of the solids.

(6) Draw the perspective view of a semi-circular arched opening in a wall having the length of 4 m, thickness 0·5 m and height 3·5 m. The opening is 1·5 m wide and the springing points are at a height of 2 m. The wall makes an angle of 45° with the picture plane. Select a suitable position of the spectator.

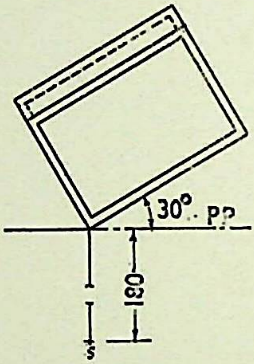


FIG. 19-47

(7) Draw the perspective view of the box shown in fig. 15-23. The plan of the box with the lid 90° open, along with the picture plane and the station point is shown in fig. 19-47. Assume horizon level to be 200 mm above the ground level.

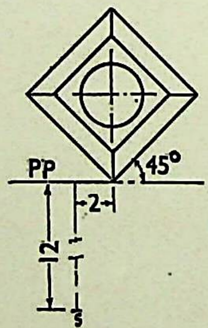


FIG. 19-48

(8) Draw the perspective view of the pedestal for a statue (neglecting the sphere) shown in fig. 15-50(4). The edges of the base, as shown in fig. 19-48, are equally inclined to the picture plane. Assume the horizon plane to be 15 cm above the ground plane.

(9) Draw the perspective view of the wooden cabinet shown in fig. 19-49. The thickness of wood is 2 cm. The cabinet is placed with one edge touching the picture plane and 100 cm to the left of the centre of vision. The station point is 250 cm away from the picture plane and 150 cm above the ground plane.

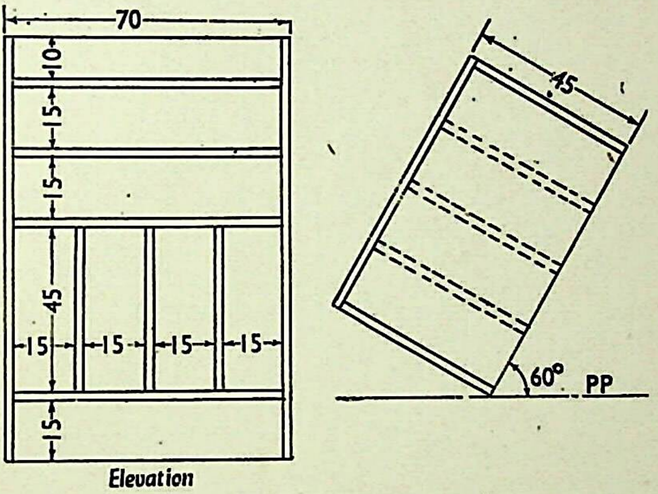


Fig. 19-49

(10) The casting shown in fig. 15-50(2) is placed behind the picture plane as shown in fig. 19-50. Draw its perspective view, assuming the horizon plane to be 8 cm above the ground plane.

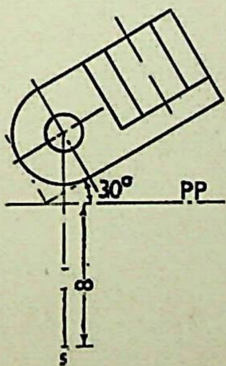


Fig. 19-50

(11) Draw the perspective view of a steel cupboard 1 m × 2 m × 0·75 m deep, having four shelves. One shutter is open. The front of the cup-board makes an angle of 30° with the picture plane, with a vertical edge touching it. Assume suitable position of the spectator at a height of 4 m above the ground.

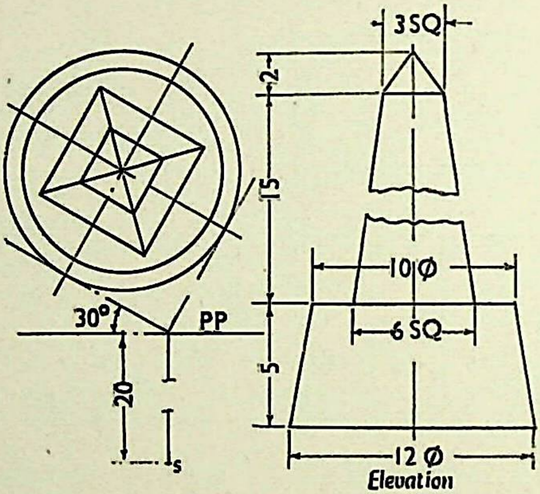


FIG. 19-51

(12) Draw the perspective view of the model of a memorial shown in fig. 19-51. The station point is 20 cm in front of the picture plane and 16 cm above the ground plane. (All dimensions are in centimetres.)

MISCELLANEOUS EXERCISES

Note: *For conversion of inches into centimetres refer to the table given on page 508.*

(1) A bolt consists of a conical part 2·5″ long, 1″ diameter at the end and increasing to 1·5″ at the head. The head is a square block 2·5″ × 2·5″ × 1″ thick. Draw an isometric projection when the axis of the bolt is vertical, the head being at the bottom. (G. U.)

(2) A person on the top of a tower 80′ high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects H and K on the plane to be 15° and 25°; the direction of H and K from the tower being due north and due west respectively. Draw a plan to a scale of 1″ ≐ 100′, showing the relative positions of the person and the two objects. Measure and state in feet the distance between H and K. (G. U.)

(3) The frustum of a cone appears in elevation as a trapezium of 2″ and 3″ parallel sides, $3\frac{1}{2}$″ apart. A cylindrical pipe joins the frustum and in the end view, the circle representing the pipe is found to touch the 2″ side and the sides of the frustum. Draw the three views of the frustum and portions of the pipe joined as stated. (D.T.E.)

(4) A 60° set-square of 10″ longest side is so kept that the longest side is on the H.P. making an angle of 30° with xy, and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square. (P. U.)

(5) A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 6″ and 4″ respectively and its length is 6″. It is situated in such a way that its axis is parallel to the H.P. and lies in a plane inclined at 60° to the V.P. Draw plan and elevation of the duct, assuming the thickness of the duct-sheet to be negligible. (D.T.E.)

(6) PQ is a diameter of a circle and is 3″ long. A piece of string is tied tightly round the circumference of the semi-circle starting at P and finishing at Q. The end Q is then untied and the string, always kept taut, is gradually unwound from the circle, until it lies along the tangent at P. Draw the curve traced by the moving extremity of the string. (G. U.)

(7) Two pegs *A* and *B* are fixed in each of two adjacent side walls of a rectangular room which meet in a corner. Peg *A* is $4\frac{1}{2}'$ above the floor, $3\frac{1}{2}'$ from the side wall and is protruding $1'$ from the wall. Peg *B* is $6'$ above the floor, $3'$ from the other side wall and is also protruding $1'$ from the wall. Find the distance between the ends of the pegs. (P. U.)

(8) A pipe $1\frac{1}{2}''$ diameter and $4\frac{3}{4}''$ long (along the axis) is welded to the vertical side of a tank. Show the development of the pipe, if it makes an angle of 60° with the side to which it is welded, the other end of the pipe making an angle of 30° with its own axis. Neglect thickness of the pipe. (P. U.)

(9) A pentagonal pyramid base $1\frac{1}{4}''$ edge, axis $3''$ long, stands upon a circular block, $3''$ diameter and $1''$ thick, so that their axes are in a straight line. Draw plan and elevation of the solids when the base of the block is inclined at 30° to the ground, an edge of the base of the pyramid being parallel to the V. P. (G. U.)

(10) A vertical straight line *AB* is at a distance of $3\frac{1}{2}''$ from the centre of a circle of $3''$ diameter. A straight line *PQ* passes through the centre of the circle and makes an angle of 60° with the vertical. Draw circles having their centres on *PQ* and to touch the straight line *AB* and the circle. Measure the radius of each circle. (D. T. E.)

(11) Two objects *A* and *B*, $30'$ above and $20'$ below the ground level respectively, are observed from the top of a tower $100'$ high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between *A* and *B* is $60'$. Draw to scale $1'' = 20'$, the projections of the objects and the tower and find (a) the true distance between *A* and *B*, and (b) the angle of depression of another object *C* situated on the ground midway between *A* and *B*. (P. U.)

(12) Draw a semi-circle of 12·5 cm (5″) diameter and inscribe in it the largest equilateral triangle having a corner at the centre. The semi-circle is the development of a cone, and the triangle that of a line on its surface. Draw the plan and elevation of the cone resting on its base in the H.P., showing the line in both views.

(13) A room measures $25'$ long, $15'$ wide and $12'$ high. An electric point hangs in the centre of the ceiling and $3'$ below it.

A thin straight wire connects the point to a switch kept in one of the corners of the room and 6′ above the floor. Draw the elevation and plan of the wire, and find the length of the wire and its slope-angle with the floor. (G. U.)

(14) An ordinary (single-start) square-threaded screw has an outside diameter of 4″ and an inside (core) diameter of 3″. Draw the correct shape of the screw in plan and elevation for a length of at least 2″ of the screw. (P. U.)

(15) A cone of $2\frac{1}{4}''$ diameter and 3″ height is resting on the H.P. on one of its generators in such a way that, the generator is parallel to the V.P. It is cut by a plane parallel to the V.P. and inclined at 90° to the H.P. and passing through a point $\frac{9}{16}''$ in front of its axis. Draw the sectional elevation and plan of the cone. (B. U.)

(16) A bucket, 30 cm (12″) diameter at the top and 22·5 cm (9″) diameter at the bottom has a circular ring 22·5 cm (9″) diameter and 5 cm (2″) wide attached at the bottom. The total height of the bucket is 30 cm (12″). Draw plan and elevation of the bucket when its axis is inclined at 60° to the H.P. and as a vertical plane makes an angle of 45° with the V.P. Assume the thickness of the plate of the bucket to be equal to that of your line.

(17) The vertex-angle of the cone just touching the edges of a vertical hexagonal pyramid 5″ in height is 45°. Draw the projections of the pyramid on a 45° inclined plane when the former is truncated by a plane making 45° with the axis and bisecting the axis. (D.T.E.)

(18) The true section of a vertical square prism cut by an inclined plane is a rectangle of $3'' \times 1\frac{1}{2}''$. The plane cuts one of the side faces at a height of $1\frac{1}{2}''$ from the base. Draw three views of the cut prism when it rests on the cut face in the H.P. with its axis remaining parallel to the V.P. (P. U.)

(19) A rectangular tank 10′ high is strengthened by four stay rods one at each corner, connecting the top corner to a point in the bottom 2′ and $3\frac{1}{2}'$ from the sides of the tank. Find graphically the length of the rod required and the angle it makes with the surface of the tank. (D.T.E.)

(20) A knob of a machine handle consists of $\frac{1}{2}''$ diameter $\times$ 6" long cylindrical portion and $1\frac{1}{2}''$ diameter spherical portion. The centre of the sphere lies on the axis of the cylindrical portion. Draw a plan and elevation if its axis is inclined at 45° to the horizonal plane. (D.T.E.)

(21) Fig. M-1 gives the elevation of a cylinder having a slot. Draw the given view and add an end elevation looking from the left. Unit = 2·5 cm (1").

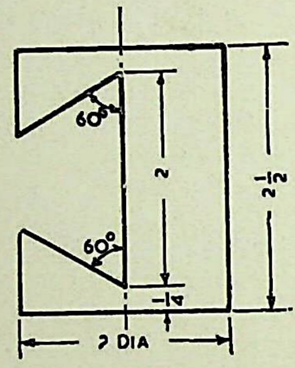


FIG. M-1

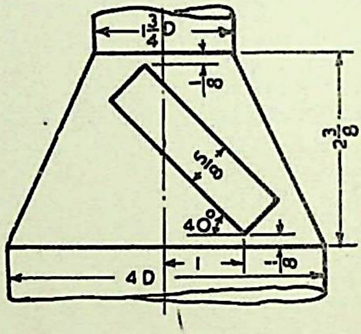


FIG. M-2

(22) A rectangular slot for a cotter is cut in a tapering portion of a piston rod as shown in fig. M-2. Draw three views of the rod, showing the curves of penetration in each view. (P. U.)

(23) The body diagonal of a cube is 3 inches long. The cube has a central 1" square hole. The faces of the hole make 45° with the side faces of the cube. Draw the projections of the cube when a body diagonal is perpendicular to the H.P. (P. U.)

(24) The inside of a hopper of a floor mill is to be lined with tin sheet. The top and bottom of the hopper are regular pentagons with each side equal to $1\frac{1}{2}'$ and 1' respectively (internally). The height of the hopper is $1\frac{1}{2}'$. Draw the shape to which the tin sheet is to be cut so as to fit in the hopper. (G. U.)

(25) An equilateral triangular prism, base 5 cm (2") side and height 10 cm (4") is standing in the H.P. on its triangular face with one of the sides of that face inclined at 90° to the V.P. It is cut by an inclined plane in such a way that the true shape of the section is a trapezium of 5 cm (2") and 1·2 cm ($\frac{1}{2}''$) para-

llel sides. Draw elevation, plan and true shape of the section and find the angle which the cutting plane makes with the H.P.

(26) A 2″ cylindrical pipe branches off at 90° from a 3″ cylindrical main pipe as shown in fig. M-3. Draw the developments of both the pipes at the joint. Assume suitable lengths for the main pipe as well as for the branch pipe. (P. U.)

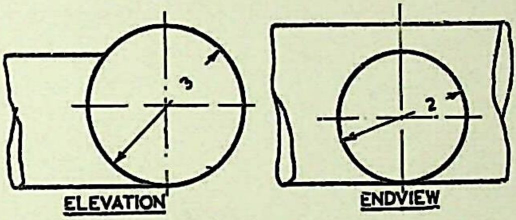


Fig. M-3

(27) A cylinder, 10 cm (4″) diameter and 15 cm (6″) long, has a rectangular slot 5 cm (2″) × 3 cm ($1\frac{3}{16}$″) cut through it. The axis of the slot bisects the axis of the cylinder at right angles and the 5 cm (2″) side of the slot makes an angle of 60° with the base of the cylinder. Draw three views of the cylinder.

(28) A horizontal cylinder, $1\frac{1}{4}$″ diameter and length $2\frac{1}{2}$″, is placed centrally on the top of a truncated cone, diameter of the base $1\frac{3}{4}$″, diameter of the top 1″ and height $1\frac{3}{4}$″. Draw a sectional elevation of the two solids on a vertical plane, distant $\frac{1}{2}$″ from the axis of the cone and making 50° with the axis of the cylinder. (G.U.)

(29) A cone, base 7·5 cm (3″) diameter, axis 10 cm (4″) long, has its base in the H.P. A section plane, parallel to one of the end generators and perpendicular to the V.P., cuts the cone intersecting the axis at a point 7·5 cm (3″) from the base. Draw the sectional plan and project another plan on a plane parallel to the section plane, showing the shape of the section clearly.

(30) A very thin glass shade for a table lamp is the portion of a sphere 5″ diameter included between two parallel planes at $\frac{1}{2}$″ and $2\frac{1}{4}$″ from the centre, making the height $2\frac{3}{4}$″. If the axis of the shade is inclined at 30° to the vertical, obtain the projections of the shade. (P. U.)

(31) Six equal spheres rest on the H.P. in contact with each other and also with the slanting faces of a regular upright

hexagonal pyramid, 1″ edge of base and 5″ height of axis. Draw the projections and find the diameter of the sphere. (D.T.E.)

(32) A cylinder, 2″ diameter and 4″ long, and a square pyramid, base 2″ side and axis 3″ long, leaning mid-way between the ends of the cylinder are shown in elevation in fig. M-4. Draw the plan of the solids and project a second elevation on x_1y_1.

(G. U.)

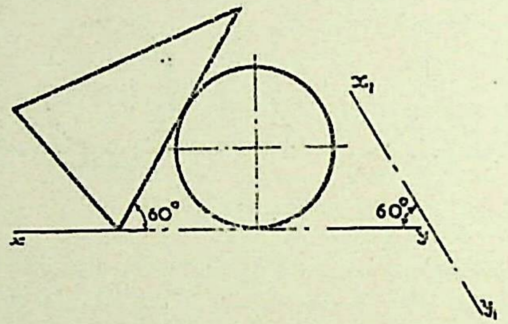


FIG. M-4

(33) A solid is made up of a cylinder, 3 cm ($1\frac{3}{16}$″) diameter and 7·5 cm (3″) long, which joins another cylinder of 7·5 cm (3″) diameter and 2·5 cm (1″) long, by a fillet of 2·2 cm ($\frac{7}{8}$″) radius, the axes of the two cylinders being in a straight line. Draw the plan of a horizontal section of the solid made by a plane parallel to and 1·6 cm ($\frac{5}{8}$″) above the axis.

(34) A circular block, 7·5 cm (3″) diameter and 2·5 cm (1″) thick, is pierced centrally through its flat faces by a square prism, base 3·5 cm ($1\frac{3}{8}$″) side and 12·5 cm (5″) long, which comes out equally on both sides of the block. Draw plan and elevation of the solids when the combined axis is parallel to the H.P. and inclined at 30° to the V.P., a face of the prism making an angle of 30° with the H.P.

(35) A cube of 7·5 cm (3″) edge is pierced centrally through all its faces by 5 cm (2″) square holes, the sides of which are parallel to the edges of the cube. It is resting in the H.P. on one of its faces with a vertical face inclined at 30° to the V.P. A section plane parallel to the V.P. cuts the cube at a distance of 0·6 cm ($\frac{1}{4}$″) from the axis. Draw the sectional elevation and plan of the cube.

(36) Draw the isometric projection of a 3" hexagonal nut and washer (without chamfer of the nut). The washer is to be of $6\frac{1}{2}$" diameter and $\frac{1}{4}$" thickness. (G. U.)

(37) In a circle of 5 cm (2") radius, draw a square of 3 cm ($1\frac{3}{16}$") side symmetrically around its centre. The figure is the plan of a sphere completely penetrated by a 7·5 cm (3") long square prism. From this plan, project an elevation on a ground line *xy* parallel to a diagonal of the square. From this elevation, project another plan on a ground line making 45° angle with the axis of the prism.

(38) A cone frustum, base 7·5 cm (3") diameter, top 3·5 cm ($1\frac{3}{8}$") diameter and height 6·5 cm ($2\frac{1}{2}$") has a hole of 3 cm ($1\frac{3}{16}$") diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base in the H.P. and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional plan and sectional end view of the frustum.

METRIC CONVERSION TABLE

INCHES WITH FRACTIONS OF AN INCH TO CENTIMETRES

BASIS: 1 INCH = 2·54 CENTIMETRES

INCHES	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$
0	0·00	0·16	0·32	0·48	0·63	0·79	0·95	1·11
1	2·54	2·70	2·86	3·02	3·17	3·33	3·49	3·65
2	5·08	5·24	5·40	5·56	5·71	5·87	6·03	6·19
3	7·62	7·78	7·94	8·10	8·25	8·41	8·57	8·73
4	10·16	10·32	10·48	10·64	10·79	10·95	11·11	11·27
5	12·70	12·86	13·02	13·18	13·33	13·49	13·65	13·81
6	15·24	15·40	15·56	15·72	15·87	16·03	16·19	16·35
7	17·78	17·94	18·10	18·26	18·41	18·57	18·73	18·89
8	20·32	20·48	20·64	20·80	20·95	21·11	21·27	21·43
9	22·86	23·02	23·18	23·34	23·49	23·65	23·81	23·97
10	25·40	25·56	25·72	25·88	26·03	26·19	26·35	26·51

INCHES	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$
0	1·27	1·43	1·59	1·75	1·90	2·06	2·22	2·38
1	3·81	3·97	4·13	4·29	4·44	4·60	4·76	4·92
2	6·35	6·51	6·67	6·83	6·98	7·14	7·30	7·46
3	8·89	9·05	9·21	9·37	9·52	9·68	9·84	10·00
4	11·43	11·59	11·75	11·91	12·06	12·22	12·38	12·54
5	13·97	14·13	14·29	14·45	14·60	14·76	14·92	15·08
6	16·51	16·67	16·83	16·99	17·14	17·30	17·46	17·62
7	19·05	19·21	19·37	19·53	19·68	19·84	20·00	20·16
8	21·59	21·75	21·91	22·07	22·22	22·38	22·54	22·70
9	24·13	22·49	24·45	24·61	24·76	24·92	25·08	25·24
10	26·67	26·83	26·99	27·15	27·30	27·46	27·62	27·78

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$\frac{560}{\text{number of sides}} =$